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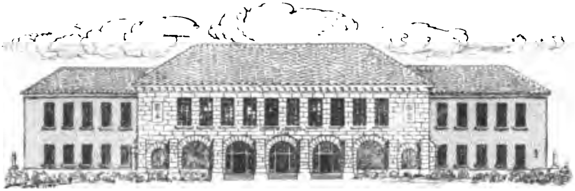

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WENTWORTH-SMITH MATHEMATICAL SERIES

ESSENTIALS OF ARITHMETIC

INTERMEDIATE BOOK

BY
GEORGE WENTWORTH
AND
DAVID EUGENE SMITH

DEPARTMENT OF
EDUCATION
SEP 1916
LELAND STANFORD
JUNIOR UNIVERSITY

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PREFACE

This book covers the work in arithmetic that is usually assigned to the fifth and sixth school years. In Grade V there is usually a review of the fundamental operations with integers and United States money, a thorough study of the fractions of ordinary business, and a study of the denominate numbers used in practical life. The central feature of the year's work, in substantially all schools, is common fractions.

In Grade VI the central feature is usually decimal fractions, the other topics being practical measurements, a study of the best ways of solving problems, and an introduction to percentage. These are the topics discussed in the second half of this book, thus adapting it to the work of the sixth school year in all parts of the country.

Having always advocated the elimination of obsolete matter from arithmetic, as well as from other branches of mathematics, and having repeatedly urged the reduction of the number of separate topics and the avoidance of subjects in which the child has no immediate or remote interest, the authors take pleasure in presenting a series of books containing only those essentials of arithmetic which are needed in common life. More complete courses are offered in other books, but the teacher who sympathizes with the modern trend in education will find in this series all of arithmetic that the pupil must thoroughly know.

The arrangement of the book is topical, so that a pupil stays long enough with a subject at one time to acquire that feeling of mastery which is his right and privilege. Along with this sequence of topics, however, are two new features which are noteworthy: One is the Little Examinations, a brief series of tests covering each chapter in turn; the other is the Review and Drill section, also placed near the end of each chapter, and furnishing a cumulative review of all preceding work. By the aid of these features a teacher may be assured that the pupil is kept refreshed upon those essentials of computation without which he cannot hope to succeed.

In the theory of the work the authors have no more sympathy with the idea that a pupil should be told to do a thing in a certain way, with no knowledge of why this way is the right one, than they have with the notion that he must explain every operation with all the care that a textbook writer would show. They believe that every process should be learned as a reasonable one, with an appeal to the pupil's understanding, and that thereafter it should become entirely mechanical; and in this way each operation has been presented in this book.

As to the applications, the effort has been to select the problems with reference to the life and interests of our people to-day. Groups of related problems are frequently given, and the isolated problems are, in general, typical of those which the pupil will need in the ordinary walks of life. The applications are more numerous than is usually the case in textbooks, but these applications are not permitted to exclude the abstract drill work without which no pupil has ever become a good computer. To balance adequately the abstract and the concrete, the drill work and the applied problem, the review work and the new material, has been one of the earnest endeavors of the authors in the preparation of this series.

When a pupil reaches this stage, certain features needed in the primary grades become unnecessary. He is now quite as interested in the process itself as he would be if a fictitious concrete dress were given to it, and he is quite prepared to abandon the use of objects, except in such cases as the introduction to percentage and the theory of fractions.

The authors hope that their effort to prepare a perfectly usable textbook for Grades V and VI, free from all eccentricities, will prove to be helpful to teachers and to pupils alike.

GEORGE WENTWORTH
DAVID EUGENE SMITH

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BRIEF TITLES OF RELATED PROBLEMS

In Grades I and II no problem should be given which involves more than a single step. In Grade III, only an occasional problem involving two steps, and this of the simplest kind, should be attempted. In Grade IV the problem involving two steps is gradually introduced. In general, in the primary grades, groups of related problems should be of the simplest character, and the relation should not be such as to require the pupil to hold many details in mind. This was the policy followed in the primary book of this series.

In Grades V and VI, for which this book is intended, more attention may properly be given to narrative problems of various kinds — those which are grouped about some incident or interest. These should be introduced occasionally, but the chief work of these years must necessarily be computation. In Grades VII and VIII related problems, those in which the work of one or two pages centers about some special interest, become more prominent, and this feature is seen in the advanced book of this series.

The following are the brief titles of many of the related problems to be found in this book:

Blood tells in the dairy	45
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ESSENTIALS OF ARITHMETIC

INTERMEDIATE BOOK

CHAPTER I

FUNDAMENTAL OPERATIONS

1. What you have Learned of Arithmetic. You have already learned much about arithmetic. You have learned to count and to write numbers, at least to a million; how to add, subtract, multiply, and divide whole numbers, or integers; how to solve many problems; how to keep simple accounts; and how to do some easy work with fractions.

You have also learned how to write United States money, how to measure things, and how to use the ordinary tables needed in making these measurements. Besides all this, you have seen that you have need for arithmetic at the store, in your games, in reading, and in many other ways.

2. What there is still to Learn. There is, however, much more to learn about the uses of arithmetic, and more to learn about how to work with numbers. Every one of us is interested in what the world is doing and in our country and its products. We all like to know about our farms, our mines, our forests, our great industries, the accounts that we must sometimes keep, and the customs of business life, and we find much of interest about these matters in arithmetic. We shall now learn how to work with larger numbers, and how to solve problems about our individual interests and about the great industries of the country.

3. Numerals. Symbols used to represent numbers are called *numerals.* The common numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are called *Arabic*, or *Hindu-Arabic, numerals.* The symbol 0 is called *zero*, *naught*, or *cipher*, or by the name of the letter *o*.

Little attention is now given to the distinction between *notation* (the writing of numbers) and *numeration* (the reading of numbers) or to the memorizing of such definitions as those given on this page.

Schools should recognize the modern reading of numbers in telephoning, as "three–nine–six–o" for 3960, and in rapidly comparing entries in bookkeeping, as "nineteen seventy-five point fifty" for $1975.50.

4. Period. A group of figures into which a number is separated for ease in reading is called a *period.*

We separate into periods by commas or, in print, by spaces, thus: 14,375,286, 14 375 286, $756,283.75, and $756 283.75.

In separating into periods, begin at the right unless there is a decimal point, in which case begin at the decimal point. The left-hand period may have one, two, or three figures. We do not separate into periods the part at the right of the decimal point. Periods are not commonly used in written numbers of four figures, in dates, or in telephone and car numbers. In this book we shall use them only with numbers of five figures or more.

5. Place Value. The value of a period or figure depends upon the place it occupies. Therefore each has a *place value.*

For example, in the number 5427, the 7 is in *units' place* or *units' order*, and means 7 ones; the 2 is in *tens' place* or *tens' order* and means 2 tens; and so on.

6. Reading Numbers. In the number 62,124,706,871 we have four periods. In each period there are three orders: units, tens, and hundreds. We read this number "sixty-two billion, one hundred twenty-four million, seven hundred six thousand, eight hundred seventy-one."

$$\overbrace{62}^{\text{Billions}},\overbrace{124}^{\text{Millions}},\overbrace{706}^{\text{Thousands}},\overbrace{871}^{\text{Units}}$$

ORAL EXERCISE

1. What name do you give to 3 tens? to 10 tens?

2. What name do you give to 1000 thousands?

3. Looking at your fingers, do you see why people came to count by tens? Why was it?

4. Read these numbers:

296
1296
5280
9999
10,000
21,236
34,007
90,009
354,200
400,576
148,392
900,009
4,246,384
5,027,050
15,125,000
75,248,309

5. What advantage has our method of writing nine (9) over the method the Romans used, as shown on a clock face (IX)?

6. In the number 234, what place is occupied by the 4? by the 3? by the 2?

7. In the number 123,456,789, what name is given to the period 789? to the period 456? to the period 123?

WRITTEN EXERCISE

Write the following in words:

1. 15,756.
2. 24,908.
3. 137,005.
4. 230,230.
5. 500,005.
6. 999,999.
7. 2,342,756.
8. 31,427,642.
9. 141,141,141.

Write the following in figures:

10. Four million, twenty-five thousand, five.
11. Seventy-seven thousand, five hundred eighty-six.
12. Ninety-eight thousand, seven hundred fifty-four.
13. One hundred six thousand, seven hundred twenty.
14. Two hundred million, fourteen thousand, sixty-eight.

7. Roman Numerals. Roman numerals are used chiefly for numbering chapters, for dates, and on clocks. They consist of seven capital letters, as follows:

Letters,	I	V	X	L	C	D	M
Values,	1	5	10	50	100	500	1000

The first nine numbers are written thus:

I, II, III, IV or IIII, V, VI, VII, VIII, IX.

The tens are written thus:

X, XX, XXX, XL, L, LX, LXX, LXXX, XC.

The hundreds are written thus:

C, CC, CCC, CD, D, DC, DCC, DCCC, DCCCC or CM.

The numbers from eleven to nineteen are written thus:

XI, XII, XIII, XIV, XV, XVI, XVII, XVIII, XIX.

The following are examples of other numbers:

XXIII = 23
XXXVI = 36
XLVIII = 48
XCVIII = 98
LXXVI = 76
LXXXII = 82
CLXVI = 166
CCCLIX = 359
MDCCXCV = 1795
MDCCCCXXII or MCMXXII = 1922

WRITTEN EXERCISE

Write in common numerals:

1. XI.
2. LXIV.
3. LXXX.
4. XXVI.
5. LXVI.
6. DCCX.
7. LXXII.
8. XCVII.
9. MDCX.
10. LXXV.
11. XXXIV.
12. MDCCCCXX.

Write in Roman numerals:

13. 31.
14. 175.
15. 42.
16. 1916.
17. 125.
18. 1919.
19. 150.
20. 1928.

WRITTEN EXERCISE

1. Write in Arabic numerals: MDCLXIX, MCCCXLIV.

2. Write in Roman numerals: 49, 79, 94, 96, 99, 146.

3. Write in Roman numerals the number of the present year; of last year; of fifty years ago.

4. Write in common figures the number twenty-one million, one thousand, four hundred seventy-five.

5. Write in common figures the number one billion, one hundred seven million, one thousand, one.

6. Write in common figures the number of Ex. 5 decreased by 42,675; by 12,245; by 295,001.

Write in words:

7. 127,007.
8. 236,036.
9. 417,000.
10. 2,176,144.
11. 3,981,426.
12. 5,237,826.
13. 12,123,123.
14. 42,307,400.
15. 100,000,000.

Write in common numerals:

16. Six million, six; seven million, twelve.
17. Seventy-seven million, seventy-seven.
18. Five hundred fifty-five thousand, five.
19. Seven hundred seventeen thousand, one hundred one.
20. Six hundred sixty-six thousand, eighteen.
21. One hundred million, five hundred seventy-five.

Write in Roman numerals:

22. 240.
23. 375.
24. 126.
25. 1930.

Write in common numerals:

26. MC.
27. MDC.
28. MDI.
29. MCII.

8. Addition. The process of finding the number which is equal to two or more numbers taken together is called *addition.* The numbers added are called *addends* and the result is called the *sum.*

9. Unit. A standard by which we measure or count is called a *unit.*

Thus, in speaking of $4 the unit is $1; in speaking of 5 eggs the unit is 1 egg, but in speaking of 10 dozen eggs the unit is 1 dozen eggs; in speaking of 40 thousand bricks the unit is a thousand bricks; in counting 1, 2, 3, 4, and so on, the unit is 1; in counting by thousands the unit is 1000; and in measuring long distances the unit may be 1 mile.

10. Like Numbers. Numbers applied to the same unit are called *like numbers.*

Thus $5 and $17 are like numbers, and 27 and 36 are also like numbers.

Only like numbers can be added, and the sum is like the addends.

That is, we cannot add $2 and 3 yd. or 7 lb. and 4 books. If we add $12 and $13, the sum must be like the addends; that is, it must be $25, not 25 ft. It is true that we can put 4 books with 7 crayons, but we have to think of them only as *things,* and say that we have 11 things.

Teachers may, if they wish, be guided by the note on abstract and concrete numbers given on page 18, where the subject first assumes some importance.

11. How to Add. In adding a column of figures, try to read a column like a word. Try especially to see the 10's, the 5's, or the 15's.

In this column, for example, you should see the two 10's and the 15, and write the answer, 35. However, it is not wise to skip numbers in order to group by 10's, because you may forget the numbers that you have skipped and thus make a serious error.

$$\begin{array}{r}
\left.\begin{array}{r}4\\2\\9\end{array}\right\}\\
\left.\begin{array}{r}8\\2\end{array}\right\}\\
\left.\begin{array}{r}7\\3\end{array}\right\}\\
\hline
35
\end{array}$$

12. Check. As a *check* or *proof* of the work, add the column from the top downwards, if you first added it upwards.

WRITTEN SPEED TEST

Copy the numbers in lines 1 to 8 inclusive, and add; then add the columns numbered 9 to 16 inclusive:

	9.	**10.**	**11.**	**12.**	**13.**	**14.**	**15.**	**16.**
1.	3	4	6	4	3	3	9	2
2.	4	4	1	9	8	1	4	3
3.	2	9	5	3	4	8	9	5
4.	4	7	4	9	4	8	2	8
5.	8	3	3	4	1	3	7	6
6.	8	2	2	1	9	7	3	6
7.	9	5	4	2	8	1	6	3
8.	6	4	8	4	3	9	9	2

Proceed in the same way for lines numbered 17 to 24 inclusive, and for columns numbered 25 to 32 inclusive:

	25.	**26.**	**27.**	**28.**	**29.**	**30.**	**31.**	**32.**
17.	5	9	8	7	3	2	6	8
18.	2	5	9	2	8	5	7	6
19.	9	7	6	5	4	6	9	7
20.	2	7	4	1	7	6	4	9
21.	7	7	5	8	8	3	9	4
22.	1	3	6	6	2	8	4	7
23.	2	4	6	3	9	5	8	7
24.	4	2	3	8	5	7	9	6

Such tests are of great value, especially if the pupils keep records of their speed and accuracy and review the exercise from time to time to see how the records improve.

13. Addition of Larger Numbers. You have already learned, in primary arithmetic, how to add numbers of two or more figures, so that we need consider only one simple example. To add 67, 86, and 34, we write them with units under units, and so on.

$$\begin{array}{rl} 67 & \\ 86 & \\ \underline{34} & \\ 187 & \text{sum} \end{array}$$

Then 4 + 6 + 7 = 17, or 1 ten + 7 units. We therefore write the 7 in the units' column and add the 1 to the tens.

Then 1 ten + 3 tens + 8 tens + 6 tens = 18 tens, or 1 hundred + 8 tens. We therefore write the 8 in the tens' column and the 1 in the hundreds' column.

The sum is 187.

Arrange units of the same order in the same column.

Add each column by itself, beginning at the right, and write the sum, if it is less than 10, below the column added.

If the sum of any column is 10 or more, write the unit figure below the column added, and add the tens with the next column.

WRITTEN SPEED TEST

Copy the numbers in lines 1 to 6 inclusive, and add; then add the columns numbered 7 to 14 inclusive, and time yourself:

	7.	**8.**	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
1.	12	20	23	14	86	77	80	78
2.	6	10	15	17	83	85	90	94
3.	7	9	12	23	87	88	91	93
4.	13	15	14	36	42	23	31	46
5.	21	31	21	41	28	17	16	20
6.	42	24	33	23	14	23	41	26

ORAL EXERCISE

Look at each example and state only the sum:

1.	2.	3.	4.	5.	6.	7.
17	27	37	67	97	29	42
2	2	22	72	32	10	30

8.	9.	10.	11.	12.	13.	14.
15	21	32	10	21	11	12
13	12	14	6	10	11	10

15.	16.	17.	18.	19.	20.	21.
20	33	42	81	16	15	17
25	12	63	40	10	10	20

22.	23.	24.	25.	26.	27.	28.
21	32	41	10	21	31	40
35	70	88	73	20	31	50

29.	30.	31.	32.	33.	34.	35.
75	31	41	21	32	50	51
32	92	21	36	40	60	62

36.	37.	38.	39.	40.	41.	42.
41	23	20	21	20	30	32
62	40	42	42	96	40	43
10	10	20	20	10	50	54

This is a type of the rapid oral work that should be given daily in arithmetic. From three to five minutes of such rapid work in each recitation will do much to remove the charge that pupils are usually slow and inaccurate. The most important part of arithmetic is found in the four fundamental operations. Particular attention should be given to oral drill in addition by endings. The pupil should think of 15 + 13 as 15 + 10 + 3, or 25 + 3, and of 25 + 17 as 25 + 10 + 7, or 35 + 7.

WRITTEN EXERCISE

See how long it takes to copy and add each of these columns, and to check each result. Write the time with the results.

1.	2.	3.	4.
$147.50	$748.06	$375.42	$298.04
35.70	126.53	104.41	127.63
61.80	39.40	630.67	701.96
130.50	86.51	49.60	872.37

5.	6.	7.	8.
$359.68	$496.48	$396.04	$982.69
298.76	127.92	72.63	604.05
400.92	600.00	129.87	81.97
640.32	503.52	871.64	631.83
701.24	872.08	280.93	400.00

9.	10.	11.	12.
$1482.63	$4761.42	$3147.29	$4268.75
689.74	3917.63	2873.41	3942.35
2908.37	5238.58	4192.64	2065.08
8517.37	6082.37	128.46	5095.02
9310.26	4098.07	3029.00	2711.40
7091.63	5901.93	7081.00	1029.60

13.	14.	15.	16.
$3246.48	$2763.48	$2983.96	$1430.00
4139.72	298.43	4198.73	280.00
863.41	4200.75	2299.80	7557.75
96.37	8342.66	4763.81	263.48
2800.42	528.93	989.95	2986.75
437.00	2743.66	2263.48	3945.50

14. Subtraction. The operation of taking one number from another is called *subtraction.*

The number thus obtained is called the *difference* between the two given numbers; that is, the difference between 563 and 127 is 436. The difference is sometimes called the *remainder.*

563 minuend
127 subtrahend
436 difference

15. Subtrahend. The number which is subtracted is called the *subtrahend.*

16. Minuend. The number from which we subtract is called the *minuend.* The minuend is the sum of the difference and the subtrahend.

Subtraction may also be defined as the operation of finding the number which when added to the smaller of two numbers makes the larger. Since a great deal of subtraction is "making change," where the work is based on addition, the latter definition is often preferred.

The teacher is advised to spend but little time in asking for such definitions as those given above, since the terms are rarely used in business. The business man says, "*Deduct* $10 and pay me the *balance,*" or "*Take out* $10 and pay me the *rest,*" instead of "*Subtract* $10 and pay me the *difference.*"

The teacher should lead the pupil to see that 100 − 37 is the sum of 90 − 30 and 10 − 7, or 63.

ORAL EXERCISE

Subtract the following numbers:

1.	17	27	16	26	13	43	15	75
	3	3	2	2	1	1	2	2
2.	27	27	36	36	43	43	85	85
	10	13	20	22	30	31	40	42
3.	67	48	96	78	57	39	75	87
	21	23	35	46	27	18	66	78

17. How to Subtract. If one village has 1632 inhabitants and another has 756, and we wish to find how many more persons live in the first village than in the second, we need to subtract 756 from 1632. We cannot subtract 6 from 2, but we can subtract 6 from 12, for 12 – 6 = 6. We have now used 10 of the 30, leaving 20. We cannot subtract 50 from 20, but 120 – 50 = 70. We have now used 100 of the 1600, and 1500 – 700 = 800. Therefore the result is 876.

$$\boxed{\begin{array}{r}1632\\ \underline{756}\\ 876\end{array}}$$

Check. Since 876 + 756 = 1632, the work is evidently correct.

WRITTEN EXERCISE

Subtract the following numbers:

1.	**2.**	**3.**	**4.**	**5.**
1796	1796	1796	1746	1746
384	394	398	399	587
6.	**7.**	**8.**	**9.**	**10.**
3234	3234	3234	3234	3234
872	815	845	1856	2978
11.	**12.**	**13.**	**14.**	**15.**
4368	5721	2933	4203	4307
299	4186	1768	2192	2358
16.	**17.**	**18.**	**19.**	**20.**
3628	4023	5127	5962	6014
969	3648	3675	3183	4235
21.	**22.**	**23.**	**24.**	**25.**
6271	8123	8207	8190	9235
4632	4637	5620	6391	7687

18. Making Change. If you owe 70¢ to a merchant and give him $1, he says, "70 and 5 are 75, and 25 more makes a dollar," at the same time laying down 5¢ and 25¢. He thus sees that $1 - $0.70 = $0.30.

Teachers may use real or toy money in teaching this subject, but pupils are now old enough to imagine the coins, as they will do in actual business methods.

ORAL EXERCISE

Make change for $1, when you owe:

1. 25¢ 50¢ 96¢ 85¢ 63¢ 39¢
2. 35¢ 40¢ 94¢ 87¢ 37¢ 29¢

Make change for 50¢, when you owe:

3. 25¢ 22¢ 36¢ 17¢ 27¢ 43¢
4. 35¢ 32¢ 38¢ 16¢ 19¢ 24¢

Make change for 25¢, when you owe:

5. 10¢ 8¢ 7¢ 6¢ 21¢ 13¢
6. 12¢ 18¢ 17¢ 16¢ 19¢ 15¢

Make change for $2, when you owe:

7. $1 $1.25 $1.40 $1.45 $1.57
8. $1.50 $1.75 $1.60 $1.85 $1.78

Imagine yourself buying goods at a store and handing the clerk the first amount given, the goods costing the second amount. State the amount of change due you in each case:

9. $1; 72¢.
10. $2; $1.37.
11. $2; $1.67.
12. $3; $2.17.
13. $4; $3.41.
14. $5; $1.27.
15. $5; $2.40.
16. $10; $6.55.
17. $5; $3.05.
18. $5; $1.70.
19. $10; $7.05.
20. $10; $6.05.

WRITTEN EXERCISE

See how long it takes to copy these numbers and to subtract and check. Write the time with the last result:

1.	**2.**	**3.**	**4.**
42,350	14,762	36,431	50,000
21,175	9,987	21,596	29,872
5.	**6.**	**7.**	**8.**
63,527	27,998	60,000	70,900
41,962	19,999	48,762	56,907
9.	**10.**	**11.**	**12.**
$175.62	$129.87	$208.09	$400.12
81.98	67.98	98.78	69.84
13.	**14.**	**15.**	**16.**
$170.08	$370.42	$209.18	$302.78
69.99	91.64	87.69	62.95
17.	**18.**	**19.**	**20.**
$802.65	$200.00	$608.75	$672.93
69.81	93.74	35.82	98.94
21.	**22.**	**23.**	**24.**
$409.98	$682.70	$200.00	$325.34
23.49	93.81	187.60	128.92
25.	**26.**	**27.**	**28.**
$429.81	$308.72	$298.05	$408.00
149.92	129.85	109.72	129.60

Teachers will do well to return to these efficiency tests occasionally, comparing the records of the pupils with their earlier records.

19. Cash Accounts. In keeping a cash account it is customary to write the receipts on the left side (called the *debit* side), and the expenditures on the right side (called the *credit* side), thus:

	19—					*19—*					
a	*May*	*4*	*Cash on hand*	*$1*	*50*	*May*	*5*	*Paid for ball*		*20*	*g*
b		*5*	*From father*		*20*		*6*	*Ball game*		*25*	*h*
c		*6*	*From mother*		*10*		*6*	*Marbles*		*15*	*i*
d		*7*	*Earned, errands*		*25*		*8*	*Balance*	*1*	*45*	*j*
e				*2*	*05*				*$2*	*05*	*k*
f	*May*	*8*	*Balance*	*$1*	*45*						

Here *a* shows cash on hand when this page of the account is begun; *b*, *c*, and *d* are receipts from father and mother, and for errands; *e* is the sum of these items on the day that the account is added to see how it stands. On the right side *g*, *h*, and *i* are bills paid. To find how much is now on hand we subtract the sum of 20¢, 25¢, and 15¢, or 60¢, from $2.05, and we find that the remainder (in accounts called the *balance*) is $1.45, which is written on the right, at *j*, and on the left, at *f*. To check the work, *g*, *h*, *i*, and *j* are added, and the sums, at *e* and *k*, must agree.

WRITTEN EXERCISE

Make out cash accounts and balance them as above:

1. Receipts: April 3, cash on hand, $2.10; April 4, caring for furnace, $0.75; April 7, selling papers, $0.15. Expenses: April 4, picture show, $0.10; April 6, book, $0.45. Balanced on April 8.

2. Receipts: Feb. 6, cash on hand, $135.60; Feb. 6, R. S. Roberts, $75.50; Feb. 7, M. L. Richards, $87.90; Feb. 8, R. P. Jasper, $42.25. Expenses: Feb. 6, rent, $75; Feb. 7, salaries, $60; Feb. 8, coal, $15.30; Feb. 8, account book, $2.25; Feb. 8, telephone rent, $6.50; Feb. 8, gas bill, $3.75. Balanced on Feb. 9.

Make out cash accounts and balance them:

3. Receipts: May 1, cash on hand, $1.25; May 3, earned by errands, 35¢. Expenses: May 3, pencil, 5¢. Balanced May 4.

4. Receipts: Sept. 15, cash on hand, 35¢; Sept. 16, earned by delivering papers, 16¢; Sept. 17, earned by cleaning automobile, 25¢; Sept. 18, earned by errands, 15¢. Expenses: Sept. 16, ball, 18¢; Sept. 18, dog collar, 15¢. Balanced Sept. 19.

5. Receipts: Oct. 7, cash on hand, $2.10; Oct. 7, earned by caring for horse, 30¢; Oct. 8, from Uncle John, $1; Oct. 9, earned by errands, 20¢. Expenses: Oct. 8, football pants, $1.50; Oct. 9, share in football, 50¢; Oct. 10, football shoulder pads, 30¢. Balanced Oct. 11.

6. Receipts: Dec. 4, cash on hand, 62¢; Dec. 4, earned by caring for furnace, 15¢; Dec. 5, earned by cleaning walks, 20¢; Dec. 6, earned by caring for furnace, 15¢. Expenses: Dec. 6, pair of skates, 94¢. Balanced Dec. 7.

7. Receipts: May 1, cash on hand, $36.50; May 2, R. S. Jacobs, $20; May 4, M. R. Randall, $32.60. Expenses: May 3, groceries, $7.86; meat, $3.50. Balanced May 5.

8. Receipts: April 3, cash on hand, $25.30; April 4, J. B. Fuller, $30; April 7, R. H. Bates, $4.30. Expenses: April 4, groceries, $9.32; April 6, meat, $7.28. Balanced April 8.

9. Receipts: Feb. 8, cash on hand, $154.25; Feb. 9, John Smith, $62.75; Feb. 11, M. L. Tilton, $85.44; Feb. 12, J. R. Haskins, $51.10. Expenses: Feb. 9, rent, $75; Feb. 12, clerks, $72; Feb. 15, coal, $15.30; Feb. 17, account book, $1.75; Feb. 22, telephone, $4.50; Feb. 23, gas, $4.25. Balanced Feb. 23.

Problems of this kind should be made by the pupils. The children should also be urged to keep their personal accounts, although these should not, as a rule, be submitted to general class inspection.

10. This boy is comparing his father's cash account with his own. He finds that he received $1.00 from his father on Oct. 15, 20¢ for work on Oct. 16, 35¢ for picking apples on Oct. 17, 15¢ for work on Oct. 18, 25¢ for work on Oct. 19, and 50¢ for some chestnuts that he gathered on Oct. 20. He paid 35¢ for a book on Oct. 16, 5¢ for some marbles on Oct. 17, 20¢ for paper and ink on Oct. 18, 40¢ for a knife on Oct. 20, and on Oct. 21 he put the balance that he had on hand in the bank, where he already had $17.50. Make out the cash account and find how much he then had in the bank.

11. For his father he made out a statement of the farm property, which showed the following items: value of the land and buildings, $7250; standing wood and timber, $1600; wagons and machinery, $750; cows and horses, $1975; oats, corn, and wheat on hand, $490; hay and straw on hand, $230; poultry, $35; sewing machine and household furniture, $325; due from creamery for milk furnished, $72.75; due from Mr. Young for hay, $28.50. The father still owed $2250 on the farm, and there was a grocery bill of $12.80 due. Make a statement like a cash account, showing the value of the property after deducting what was owed on the farm and at the grocery.

20. Multiplication. The process of taking one number as many times as there are units in another is called *multiplication.*

This means multiplication by a whole number. Thus, to multiply $3 by 4 means that $3 is taken 4 times.

The number multiplied is called the *multiplicand.*

The number by which the multiplicand is multiplied is called the *multiplier.*

The result of multiplying is called the *product.*

$325	multiplicand
3	multiplier
$975	product

21. Abstract and Concrete Numbers. It is common to speak of a number that does not refer to any particular kind of object or measure as an *abstract number.* It is also customary to speak of a number that refers to some particular kind of object or measure as a *concrete number.*

Thus 4 is commonly called an abstract number, and 4 feet a concrete number.

This distinction is not observed so much as it formerly was, for all *number* is essentially abstract. We may specify the units (feet, yards, apples, etc.), but the *number* is still abstract. The custom of specifying the units in solving a problem is also tending to disappear, the pupil keeping the units in mind just as the business man does when he solves his problems.

For example, consider the following cases:

If one book costs 20¢ and another book costs 30¢, both books cost *as many cents* as 20 + 30, or 50¢.

If one book costs 20¢ and another book costs 30¢, the second book costs *as many cents* more than the first as 30 — 20, or 10¢.

If 1 orange costs 5¢, 4 oranges will cost *as many cents* as 4 × 5, or 20¢.

If 1 orange costs 5¢, one can buy *as many oranges* for 20¢ as 20 ÷ 5, or 4 oranges.

If 4 oranges cost 20¢, each orange costs *as many cents* as 20 ÷ 4, or 5¢.

If desired, the above method may be used throughout the entire book.

In school, however, it adds to the clearness of the solution if the more important numbers are labeled with their proper units, and therefore this is done in the solutions in this book. For example, in 3 × $325 = $975, the multiplicand and product are properly given the same label.

22. Nature of the Numbers. It is therefore seen that

The multiplier must be thought of as abstract, and the product is like the multiplicand.

That is, if we have 3 times 17 ft., the 3 is abstract, and the product is feet, like the multiplicand.

23. Multiple. The product of two abstract integers is called a *multiple* of either. For example, 35 is a multiple of 7 and of 5.

24. Factors. The numbers which multiplied together make another number are called its *factors*. For example, the factors of 30 are 2, 3, and 5.

25. Power. The result of taking a number any number of times as a factor is called a *power* of the number.

For example, $2 \times 2 = 4$, and 4 is called the *second* power, or the *square*, of 2, and the product 2×2 is written 2^2. So $2 \times 2 \times 2$, or 2^3, is the *third* power, or the *cube*, of 2.

ORAL EXERCISE

1. State four multiples of 7; of 3; of 9; of 11; of 15.

2. State the factors of 35; of 77; of 21; of 49; of 121.

3. State the squares of 7, 6, 9, 8, 10, 11, 20, 100, 1000.

4. What is the square of 5? the cube of 3? the fourth power of 2? the cube of 5? the cube of 2? the cube of 10?

5. What is the square of 12? the cube of 4? the fourth power of 1? the fifth power of 2?

6. Multiply, in turn, by 2, 3, 4, and so on to 9:

7 4 6 3 8 5 9 2
11 20 21 30 50 70 40 60

26. How to Multiply. You have already learned how to multiply one number by another. For example, to multiply $635.52 by 215, 3247 by 1305, and 3247 by 1350 we have:

$$\begin{array}{r} \$635.52 \\ 215 \\ \hline 3177\ 60 \\ 6355\ 2 \\ 127104 \\ \hline \$136636.80 \end{array} \qquad\qquad \begin{array}{r} 3247 \\ 1305 \\ \hline 16235 \\ 9741 \\ 3247 \\ \hline 4237335 \end{array} \qquad\qquad \begin{array}{r} 3247 \\ 1350 \\ \hline 162350 \\ 9741 \\ 3247 \\ \hline 4383450 \end{array}$$

In multiplying $635.52 by 215 we multiply as if the problem were 215 × 63,552¢. Then 5 × 63,552 = 317,760. We next have to multiply 63,552 by 1 ten, and the result is 63,552 tens, which we write so that the 2 is in tens' place. We next have to multiply 63,552 by 2 hundreds, and the result is 127,104 hundreds, which we write so that the right-hand figure is in hundreds' place. Adding, we have 13,663,680 cents, or $136,636.80.

WRITTEN EXERCISE

Multiply each number by each of the numbers below it in the column; thus, 376 × 243, 405 × 243, and so on:

I	II	III	IV	V	VI
243	307	442	581	226	822
376	476	208	622	642	344
405	921	179	389	379	296
221	330	643	427	496	473
392	407	827	462	577	892
470	920	906	330	640	487
689	821	432	448	807	745
573	139	507	229	488	307

This will give 168 examples, from which the teacher should select only as many as necessary. Upon this whole question of efficiency tests it would be well at this time to read the suggestions on page 41.

WRITTEN EXERCISE

Multiply the following:

1. 24 × 325.
2. 32 × 526.
3. 41 × 578.
4. 53 × 482.
5. 67 × 573.
6. 78 × 409.
7. 89 × 277.
8. 96 × 483.
9. 523 × 832.
10. 624 × 421.
11. 532 × 409.
12. 677 × 505.
13. 752 × 327.
14. 581 × 826.
15. 823 × 983.
16. 435 × 481.
17. 632 × 408.
18. 563 × 481.
19. 408 × 662.
20. 507 × 836.
21. 802 × 906.
22. 532 × 848.
23. 783 × 927.
24. 860 × 342.
25. 83 × 1765.
26. 59 × 2763.
27. 23 × 4126.
28. 58 × 3279.
29. 63 × 4081.
30. 49 × 7756.
31. 82 × 3793.
32. 41 × 8872.
33. 64 × 8709.
34. 33 × 4981.
35. 25 × 4444.
36. 62 × 3436.
37. 83 × 7070.
38. 49 × 6236.
39. 83 × 8192.
40. 54 × 7762.
41. 36 × 8087.
42. 53 × 8129.
43. 66 × 6666.
44. 91 × 3473.
45. 17 × 4646.
46. 33 × 8308.
47. 46 × 12,481.
48. 34 × 36,408.
49. 43 × 21,761.
50. 56 × 29,376.
51. 58 × 42,807.
52. 64 × 80,009.
53. 123 × 1407.
54. 236 × 3143.
55. 823 × 4062.
56. 347 × 8096.
57. 823 × 7743.
58. 629 × 3648.
59. 426 × 3140.
60. 408 × 3776.
61. 567 × 9873.
62. 228 × 4362.
63. 503 × 8742.
64. 621 × 8263.
65. 987 × 7890.
66. 602 × 3348.
67. 672 × 4815.
68. 360 × 4572.
69. 666 × 7883.
70. 565 × 8432.
71. 821 × $248.75.
72. 633 × $336.90.

ORAL EXERCISE

1. Multiply by 10: 10¢, 50¢, $1.20, $3.50.

2. Multiply by 100: 3, 42, 565, $1.50, $21.75.

3. State a short way of multiplying a whole number (integer) by 10; by 100; by 1000; by 10,000.

4. State a short way of multiplying a number representing United States money by 10; by 1000.

27. Multiplying by Powers of 10. *To multiply an integer by 10, annex a zero; to multiply by 100, annex two zeros; to multiply by 1000, annex three zeros.*

To multiply a number representing United States money by 10, move the decimal point one place to the right; to multiply by 100, move the decimal point two places to the right; to multiply by 1000, move the decimal point three places to the right.

Hence to multiply 27 by 40, multiply by 4 and annex 0.

To multiply $25.16 by 300, multiply by 3 and move the decimal point two places to the right.

WRITTEN EXERCISE

Multiply the following:

1. 210 × $346.
2. 800 × 932 ft.
3. 9600 × $4300.
4. 7000 × $15.42.
5. 6520 × $12.50.
6. 320 × $4215.
7. 700 × 864 rd.
8. 4300 × $8250.
9. 9000 × $21.75.
10. 2750 × $31.02.

11. If a railway train goes 48 mi. an hour, how far will it go in 20 hr., at the same rate?

12. If it costs $14.75 a year to educate you, how much does it cost to educate 300 pupils, at the same rate?

ORAL EXERCISE

1. At $4 each, how much will 20 desks cost?
2. At $9 each, how much will 30 tables cost?
3. At $12 each, how much will 20 tables cost?

Multiply the following:

4. 60 × 8 70 × 9 40 × 11 50 × 7
5. 20 × 13 30 × 12 80 × 10 90 × 10

WRITTEN EXERCISE

Multiply, in turn, by 2, 20, and 200:

1. 143 286 409 527 683
2. $1.43 $2.86 $40.90 $52.70 $6.83

Multiply, in turn, by 3, 30, and 300:

3. 482 525 633 840 900
4. $4.82 $52.50 $63.30 $8.40 $9.00

Multiply, in turn, by 40, 400, 50, and 500:

5. 736 821 963 742 893
6. $5.35 $8.93 $7.34 $6.81 $9.75
7. Multiply $234.65 by 9; by 90; by 900; by 982.
8. Multiply $275.40 by 8; by 80; by 800; by 874.
9. Multiply $324.45 by 7; by 35; by 305.
10. Multiply $846.60 by 6; by 60; by 3; by 300.
11. At $85 each, how much will 60 mules cost?
12. At $55 each, how much will 70 cows cost?
13. At $135 each, how much will 80 horses cost?
14. At $575 each, how much will 90 automobiles cost?

ORAL EXERCISE

1. On the blackboard, multiply 68 by 5. Divide 680 by 2.

2. On the blackboard, multiply $12.25 by 5. Also divide $122.50 by 2. How do the results compare? Tell a short way of multiplying by 5.

3. Because $5 = 10 \div 2$, to multiply an integer by 5 is the same as to annex 0 and divide by what number?

4. On the blackboard, multiply $42.74 by 50. Also divide $4274 by 2. Compare the results.

5. Because $50 = 100 \div 2$, to multiply by 50 we may first multiply by what number, and then divide by what other number?

6. On the blackboard, multiply $32.56 by 25. Also divide $3256 by 4. Compare the results.

Multiply the following by 5, 50, and 25, in turn:

7. 4 12 16 28 40 80 120

8. 8 24 32 48 88 96 160

28. Aliquot Part. A number that will exactly divide another is called an *aliquot part* of that number.

For example, 2 and 5 are aliquot parts of 10, and 2, 5, 10, 20, 25, and 50 are aliquot parts of 100.

Therefore the aliquot parts of $1 are 2¢, 5¢, 10¢, 20¢, 25¢, and 50¢. To these may be added $33\frac{1}{3}$¢, $16\frac{2}{3}$¢, and certain other aliquot parts which will be considered later.

29. Multiplying by Aliquot Parts. From the above exercise we have the following:

To multiply by 5, multiply by 10 and divide by 2.
To multiply by 50, multiply by 100 and divide by 2.
To multiply by 25, multiply by 100 and divide by 4.

ORAL EXERCISE

1. Multiply 20 by 5; divide 200 by 2.
2. Multiply 40 by 5; divide 400 by 2.
3. Multiply 4 by 25; divide 400 by 4.
4. At 24¢ each, what will 5 pineapples cost?
5. At 28¢ a yard, what will 5 yd. of ribbon cost?
6. At $80 each, what will 50 ponies cost?
7. State a short way of multiplying by 5; by 50; by 500.
8. How much is 25 × 4? 25 × 8?
9. State a short way of multiplying by 25.

WRITTEN EXERCISE

1. Multiply by 50: 237, 425 ft., $68.50, $987.40.
2. Multiply by 25: 134, 802 ft., $49.60, $384.40.
3. Multiply by 500: 16, 64, 726, $42.80, $124.40.

Multiply the following by 5:

4. 48 74 88 82 448 1264 3642
5. 64 38 78 96 826 2488 8177

Multiply the following by 50:

6. 66 38 96 76 224 2172 4242
7. 88 46 98 42 338 3108 6543

8. Multiply the following by 25:

32 36 64 56 224 4844 4036

9. Multiply, in turn, 50¢, 25¢, 20¢, and 10¢ by:

40 200 160 240 360 480 2800

WRITTEN EXERCISE

1. At $72 an acre, how much must a man pay for 25 acres of land? for 250 acres?

2. At $55 each, how much must a farmer pay for 25 cows? for 50 cows? for 5 cows?

3. At $1425 each, how much will a manufacturer receive for 25 automobiles? for 50 automobiles?

4. If 2 M (2 thousand) feet of pine cost $96.50, how much will 8 M cost?

Pupils should appreciate the importance of recognizing the *relations of numbers*. In this example they should see that 8 M is 4 × 2 M, and should simply multiply $96.50 by 4.

5. If 6 thousand square feet of building paper cost $13.68, how much will 18 thousand square feet cost?

6. If 5 lb. of butter cost $1.50, how much will 15 lb. cost? How much will 25 lb. cost? How much will 50 lb. cost?

7. If a man grinds 35 bu. of corn, each bushel weighing 56 lb., how many pounds does he grind?

8. A farmer adds to his property a field of 65 acres, paying $75 an acre for it. How much does the field cost?

9. A dealer sells 15 automobiles for $13,875. How much does he receive for 45 automobiles at this rate?

10. If 38 cows average 408 lb. of milk apiece a month, how many pounds of milk do the cows produce in a month?

11. If a dealer sells 8 building lots for $13,875, how much does he receive for 24 lots at this rate?

Teachers should accustom the pupils always to estimate an answer in advance and to write it down. Then if they find a great difference between their result and their estimate they have either shown poor judgment or made an error in their work. Many absurd answers can be avoided in this way.

30. Nature of Division. In multiplication two factors are given to find the product. In division the product and one of two factors are given to find the other factor.

That is, in the case of $2 \times \$5 = \$10$ we have given the factors 2 and $5, from which to find the product $10.

In division we have given the product $10 and the factor 2, to find the other factor $5; or the factor $5, to find the other factor 2.

Upon the question of abstract and concrete numbers the teacher should consult § 21.

31. Division. If the product and one of two factors are given, the process of finding the other factor is called *division.*

There are two common symbols used in division. Besides the symbol ÷ we use the fraction form. Thus, we may write $15 \div 5 = 3$, or $\frac{15}{5} = 3$. That is, regarded as operations to be performed, $15 \div 5$ and $\frac{15}{5}$ have the same meaning.

32. Dividend. The given product, that is, the number to be divided, is called the *dividend.*

33. Divisor. The given factor, that is, the number by which we divide, is called the *divisor.*

34. Quotient. The required factor, that is, the number found by division, is called the *quotient.*

35. Remainder. Often a divisor does not exactly divide the dividend. Then that part of the dividend which is left is called the *remainder.*

Thus $14 \div 3 = 4$ with remainder 2. The quotient is usually written $4\frac{2}{3}$, meaning 4 with 2 still to be divided into 3 equal parts.

Such terms have long been familiar to you, and they are given at this time only for the purpose of review.

36. Two Kinds of Division. Since 2 × \$4 = \$8, we see that \$8 ÷ \$4 = 2 and \$8 ÷ 2 = \$4. That is,

If the dividend and the divisor are concrete, they must be like numbers, and the quotient is abstract.

If the dividend is concrete and the divisor abstract, the quotient is concrete and like the dividend.

Concerning this distinction, however, the teacher should consult § 21.

37. Check. To check the work in division, we multiply the quotient by the divisor; this product added to the remainder, if any, should equal the dividend.

For example, $45 \div 5 = 9$; then $5 \times 9 = 45$. Again, $48 \div 5 = 9$ with remainder 3; then $5 \times 9 = 45$, and $45 + 3 = 48$.

38. Short Division. When the divisor is so small that the work may be done mentally, the process is called ***short division.***

1. Required to divide 342 by 3.

If 342 is separated as shown, we can divide each part by 3. This is what we really do mentally. We write the numbers as shown; then 3 is contained in 3 once (that is, 100 times in 300); 3 is contained in 4 once with remainder 1 (that is, 10 times in 40 with remainder 10); this 1 is 10 units, and with the 2 it makes 12. Then 3 is contained in 12 four times. Therefore the quotient is 114. We think, "3 in 3, 1; in 4, 1; in 12, 4."

$$\begin{array}{rl}3) & \underline{300+30+12} \\ & 100+10+4=114\end{array}$$

$$\begin{array}{rr}3) & \underline{342} \\ & 114\end{array}$$

2. Required to divide 42,364 by 7.

Here 7 is not contained in 4, but 7 is contained in 42 six times. Then 7 is not contained in 3, so we write a 0 below the 3. Then 7 is contained in 36 five times with remainder 1, and in 14 twice. Therefore the quotient is 6052.

$$\begin{array}{rr}7) & \underline{42364} \\ & 6052\end{array}$$

3. Required to divide 35,282 by 9.

$$\begin{array}{r} 9\,\overline{)\,35282} \\ 3920\frac{2}{9} \end{array}$$

Here 9 is contained in 35 three times with remainder 8; in 82 nine times with remainder 1; in 18 twice; in 2 no times with remainder 2. Therefore the quotient is 3920 with remainder 2, and this is written 3920$\frac{2}{9}$.

4. Required to divide \$27.53 by 4.

$$\begin{array}{r} 4\,\overline{)\,\$27.53} \\ \$6.88\frac{1}{4} \end{array}$$

Dividing as before, we have \$27 ÷ 4 = \$6 with \$3 remaining; \$3.50 ÷ 4 = \$0.80 with \$0.30 remaining; \$0.33 ÷ 4 = \$0.08 with \$0.01 remaining. Therefore the quotient is \$6.88$\frac{1}{4}$.

ORAL EXERCISE

Divide the following:

1. 48 ÷ 2.
2. 39 ÷ 3.
3. 84 ÷ 4.
4. 65 ÷ 5.
5. 72 ÷ 6.
6. 84 ÷ 7.
7. 96 ÷ 8.
8. 54 ÷ 9.
9. 38 ÷ 2.
10. 54 ÷ 3.
11. 52 ÷ 4.
12. 75 ÷ 5.
13. 78 ÷ 6.
14. 91 ÷ 7.
15. 96 ÷ 6.
16. 33 ÷ 2.
17. 41 ÷ 3.
18. 53 ÷ 4.
19. 63 ÷ 5.
20. 75 ÷ 6.

WRITTEN EXERCISE

Divide the following:

1. 8242 ÷ 2.
2. 4122 ÷ 3.
3. 7004 ÷ 4.
4. 4015 ÷ 5.
5. 4032 ÷ 6.
6. 3073 ÷ 7.
7. 8368 ÷ 8.
8. 4019 ÷ 9.
9. 7555 ÷ 2.
10. 9015 ÷ 3.
11. \$38.72 ÷ 4.
12. \$92.75 ÷ 5.
13. \$41.30 ÷ 6.
14. \$99.47 ÷ 7.
15. \$65.68 ÷ 8.
16. \$83.61 ÷ 9.
17. 81,004 ÷ 4.
18. 39,008 ÷ 5.
19. 41,269 ÷ 6.
20. 80,047 ÷ 7.
21. 42,345 ÷ 8.
22. 76,209 ÷ 9.
23. \$35.21 ÷ 2.
24. \$72.43 ÷ 3.
25. \$61.23 ÷ 4.
26. \$72.36 ÷ 5.
27. \$82.02 ÷ 6.
28. \$42.07 ÷ 7.
29. \$75.20 ÷ 8.
30. \$73.18 ÷ 9.

39. Dividing by Powers of 10. To divide by 10 is merely to find how many tens there are in the number.

Thus in 1230 there are 123 tens and no units. In 2373 there are 237 tens and 3 units remainder; that is, $2373 \div 10 = 237\frac{3}{10}$.

To divide an integer by 10, cut off the right-hand figure of the dividend. The result is the quotient, and the right-hand figure expresses the remainder.

To divide an integer by any power of 10, cut off as many figures from the right of the dividend as there are zeros at the right of the divisor. The result is the quotient, and the right-hand figures express the remainder.

Thus $12{,}400 \div 100 = 124$; $736{,}117 \div 1000 = 736\frac{117}{1000}$.

40. Dividing by Multiples of Powers of 10. In this case we divide by the power of 10, as in § 39, and then by the number indicating the multiple.

For example, to divide 42,800 by 200, we cut off two zeros from the divisor and the dividend and then divide by 2.

So, to divide 27,407 by 300, we cut off two zeros from the divisor and two figures from the right of the dividend. We then divide by 3, and the quotient is 91 with remainder 1 in the hundreds' place, and to this we join the 07 cut off, obtaining the remainder 107.

$$\boxed{\begin{array}{l} 2\cancel{0}\cancel{0}\overline{)\,428\cancel{0}\cancel{0}} \\ \quad\;\; 214 \\ 3\cancel{0}\cancel{0}\overline{)\,274\cancel{0}\cancel{7}} \\ \qquad\; 91\frac{107}{300} \end{array}}$$

WRITTEN EXERCISE

Find the quotient, and the remainder if any:

1. $860 \div 10$.

2. $940 \div 20$.

3. $630 \div 30$.

4. $840 \div 40$.

5. $950 \div 50$.

6. $720 \div 60$.

7. $840 \div 70$.

8. $960 \div 80$.

9. $450 \div 90$.

10. $810 \div 90$.

11. $375 \div 10$.

12. $625 \div 20$.

Find the quotient, and the remainder if any:

13. 625 ÷ 30.
14. 475 ÷ 40.
15. 336 ÷ 50.
16. 527 ÷ 70.
17. 642 ÷ 70.
18. 495 ÷ 80.
19. 873 ÷ 90.
20. 888 ÷ 90.
21. 41,300 ÷ 100.
22. 37,800 ÷ 200.
23. 54,300 ÷ 300.
24. 92,400 ÷ 400.
25. 72,501 ÷ 500.
26. 67,803 ÷ 600.
27. 49,521 ÷ 700.
28. 32,449 ÷ 800.
29. 378,367 ÷ 900.
30. 438,000 ÷ 3000.
31. 228,000 ÷ 4000.
32. 745,000 ÷ 5000.
33. 663,600 ÷ 6000.
34. 569,800 ÷ 7000.
35. 848,623 ÷ 8000.
36. 729,542 ÷ 9000.

37. There being 60 min. in an hour, how many hours are there in 1080 min.?

38. There being 60 sec. in a minute, how many minutes are there in 1680 sec.?

39. At $40 a set, how many sets of harness must a manufacturer sell to receive $1240?

40. A farmer pays $6650 for some land at $70 an acre. How many acres does he buy?

41. A dealer sells some city lots for $44,100 at $700 a lot. How many lots does he sell?

42. A dealer has $2750 with which to buy some village lots at $500 each. What is the greatest number he can buy, and how much money will he have left?

43. An automobile dealer has $112,000 to invest in automobiles at $1200 each. What is the greatest number he can buy for this sum, and how much money will he have left?

44. A ton is 2000 pounds. If a coal pocket contains 597,350 pounds of coal, how many tons does it contain, and how many pounds are left over?

41. Long Division. When the divisor has two or more figures, except in such cases as are given in §§ 39 and 40, it is customary to use a process known as *long division*.

We shall first consider the simplest case, that in which the units' figure of the divisor is 1.

1. Required to divide 7182 by 21.

If all the work is written, it would appear as in the first of these forms, but for practical purposes the second is used.

(1)

$$\begin{array}{r l} 342 & \\ 21\overline{)7182} & \\ \underline{6300} & = 300 \text{ times the divisor} \\ 882 & \\ \underline{840} & = 40 \text{ times the divisor} \\ 42 & \\ \underline{42} & = 2 \text{ times the divisor} \end{array}$$

(2)

$$\begin{array}{r} 342 \\ 21\overline{)7182} \\ \underline{63} \\ 88 \\ \underline{84} \\ 42 \\ \underline{42} \end{array}$$

First we will consider (1), the complete form. We see that 21 evidently is not contained in 7, so there are no thousands in the quotient. The divisor is contained three times in 71, and since the 71 is hundreds, the quotient figure 3 is written over the hundreds' place. Then 300 × 21 = 6300, and there remains 882 still to be divided; 21 is contained in 88 four times, and since the 88 is tens, the 4 of the quotient is written over the tens' place. Then 40 × 21 = 840, and there remains 42 to be divided; 21 is contained in 42 twice, and 2 is written over the units' place. The quotient is therefore 342. It will be seen that 6300, 840, and 42 make up 342 × 21, as should be the case.

Consider now the second form, which is always used in practice. Since at any time in this example we need only two figures of the dividend or two figures of such a partial dividend as 882, we may omit the zeros in 6300 and 840, and use only the 88 in the remainder 882, thus making the work shorter.

We may check our result by multiplying 342 by 21. Then we have 21 × 342 = 7182, the dividend.

2. Required to divide 8547 by 21.

$$\begin{array}{r} 407\\ 21\overline{)8547}\\ 84\\ \hline 147\\ 147\\ \hline \end{array}$$

Here we have 85 ÷ 21 = 4, and since the 84 is hundreds, the 4 is written over the hundreds' place. Then 4 × 21 = 84, and when 84 is subtracted, there is a remainder 1. Bringing down one figure as before, we have 14 tens to be divided by 21. Since 21 is not contained in 14, we write 0 in the tens' place to show that there are no tens, and bring down the next figure, 7. Then 147 ÷ 21 = 7, and 7 × 21 = 147, exactly, so there is no remainder.

WRITTEN EXERCISE

Divide the following:

1. 462 ÷ 11.
2. 561 ÷ 11.
3. 693 ÷ 11.
4. 979 ÷ 11.
5. 869 ÷ 11.
6. 968 ÷ 11.
7. 748 ÷ 11.
8. 924 ÷ 11.
9. 273 ÷ 21.
10. 294 ÷ 21.
11. 462 ÷ 21.
12. 693 ÷ 21.
13. 798 ÷ 21.
14. 819 ÷ 21.
15. 987 ÷ 21.
16. 899 ÷ 31.
17. 779 ÷ 41.
18. 816 ÷ 51.
19. 1098 ÷ 61.
20. 1353 ÷ 41.
21. 1633 ÷ 71.
22. 2673 ÷ 81.
23. 3367 ÷ 91.
24. 2331 ÷ 21.
25. 2583 ÷ 21.
26. 3472 ÷ 31.
27. 2121 ÷ 21.
28. 6789 ÷ 31.
29. 8568 ÷ 21.
30. 3317 ÷ 31.
31. 10,731 ÷ 21.
32. 14,322 ÷ 31.
33. 16,461 ÷ 31.
34. 25,297 ÷ 41.
35. 18,879 ÷ 31.
36. 28,741 ÷ 41.
37. 41,004 ÷ 51.
38. 49,410 ÷ 61.
39. 59,696 ÷ 41.
40. 52,582 ÷ 61.
41. 62,267 ÷ 71.
42. 63,048 ÷ 71.
43. 71,280 ÷ 81.
44. 90,090 ÷ 91.
45. 90,909 ÷ 91.
46. 50,337 ÷ 51.
47. 50,898 ÷ 51.
48. 62,937 ÷ 81.
49. 71,928 ÷ 81.
50. 60,939 ÷ 61.
51. 34,587 ÷ 61.
52. 73,629 ÷ 81.
53. 65,448 ÷ 81.
54. 73,619 ÷ 91.

42. Long Division Continued. We shall now consider the more general case of a two-figure divisor.

Required to divide 19,224 by 54.

The divisor is not contained in 1 or in 19, so there are no ten-thousands and no thousands in the quotient. But since 3 × 54 = 162, the divisor is contained three times in 192 with remainder 30. Since this is hundreds, the quotient figure 3 is written over the hundreds' place. Bringing down the next figure of the dividend, 2, we have 302 tens to be divided. Since 5 × 54 = 270, the divisor is contained five times in 302 with remainder 32. Since this is tens, the quotient figure 5 is written over the tens' place. Bringing down the next figure of the dividend, 4, we have 324 still to be divided. Since 324 is exactly 6 × 54, the units' figure is 6 and there is no remainder. Hence the quotient is 356.

$$\begin{array}{r} 356 \\ 54\overline{)19224} \\ 162 \\ \hline 302 \\ 270 \\ \hline 324 \\ 324 \\ \hline \end{array}$$

43. Product Tables. It is sometimes helpful for beginners to prepare a table of products of the divisor, as follows:

2 × 54 = 108
3 × 54 = 162
4 × 54 = 216
5 × 54 = 270
6 × 54 = 324
7 × 54 = 378
8 × 54 = 432
9 × 54 = 486

With such a table made out in advance, we can tell at once, without making several trials, what the next figure of the quotient will be.

WRITTEN EXERCISE

Divide the following:

1. 3848 ÷ 52.
2. 8556 ÷ 92.
3. 6572 ÷ 62.
4. 4599 ÷ 73.
5. 8815 ÷ 43.
6. 2275 ÷ 35.
7. 2793 ÷ 57.
8. 6006 ÷ 77.
9. 2470 ÷ 19.
10. 8580 ÷ 66.
11. 22,311 ÷ 67.
12. 31,950 ÷ 75.
13. 63,204 ÷ 69.
14. 74,907 ÷ 87.
15. 89,001 ÷ 99.
16. 41,890 ÷ 59.
17. 301,184 ÷ 64.
18. 891,891 ÷ 99.

WRITTEN EXERCISE

1. If $1548 is divided equally among a dozen persons, how much is the share of each?

2. If $2193.75 is divided equally among 15 persons, how much is the share of each?

3. If a piece of land containing 289 acres is cut into 17 equal lots, what is the area of each?

4. If the dividend is 420, the quotient 32, and the remainder 4, what is the divisor?

5. If the quotient is 24, the divisor 12, and the remainder 4, what is the dividend?

6. At 260 mi. a day, how long would it take a steamer to go from San Francisco to Guam, 5200 mi.?

7. At a certain temperature sound travels 1132 ft. a second. How long will it take sound to travel 11,320 ft.?

8. The product of two numbers is 98,441, and one of them is 49. What is the other number?

9. A man paid $20,280 for a farm, the price being $65 an acre. How many acres did he buy?

10. A canning factory ships 30,000 cans of fruit in boxes containing 4 dozen cans each. How many boxes does it require?

11. There being 27 cu. ft. in a cubic yard, how many loads (cubic yards) of earth must be removed to excavate 108,675 cu. ft.?

12. One of the routes around the world is 18,424 mi. long. If a man makes the trip in 56 days, what is the average distance traveled per day?

13. If the circumference of a wagon wheel is 15 ft., and there are 5280 ft. in a mile, how many times will the wheel turn in going 9 mi.? how many times in going 25 mi.?

44. Long Division Continued. We shall now consider a case like that of 341,597 divided by 248.

The divisor is not contained in 3 or in 34, so there are no hundred-thousands or ten-thousands in the quotient. But it is contained in 341 once, with remainder 93. Since this is thousands, the quotient figure, 1, is written over the thousands' place. Bringing down the next figure of the dividend, 5, we have 935 hundreds to be divided. It might seem as if the next figure of the quotient would be 4, and here a product table (§ 43) would be helpful; but 4 × 248 = 992, and this is larger than 935. Trying 3 for the next figure, we have 3 × 248 = 744 with 191 remaining. Since we have divided hundreds by 248, the 3 is hundreds and is written over the hundreds' place. Bringing down the next figure, we find that 1919 ÷ 248 = 7 with 183 remaining. Since the 7 is tens, we write it over the tens' place. Then, bringing down the last figure, we have 1837 ÷ 248 = 7 with 101 remaining. Therefore the quotient is $1377\frac{101}{248}$.

$$\begin{array}{r@{}c@{}c@{}c@{}c@{}c@{}c}
 & & & 1 & 3 & 7 & 7 \\
\cline{2-7}
248\big) & 3 & 4 & 1 & 5 & 9 & 7 \\
 & 2 & 4 & 8 & & & \\
\cline{2-4}
 & & 9 & 3 & 5 & & \\
 & & 7 & 4 & 4 & & \\
\cline{3-5}
 & & 1 & 9 & 1 & 9 & \\
 & & 1 & 7 & 3 & 6 & \\
\cline{3-6}
 & & & 1 & 8 & 3 & 7 \\
 & & & 1 & 7 & 3 & 6 \\
\cline{4-7}
 & & & & 1 & 0 & 1
\end{array}$$

We may check the work by adding 101 to the product of 248 × 1377, the result being 341,597, the dividend.

45. General Directions. Therefore, in long division

Write the divisor to the left of the dividend.

Take for the first partial dividend the least number of left-hand figures that will contain the divisor, and write the quotient over the right-hand figure of this partial dividend.

Multiply the divisor by this quotient, and place the product under the partial dividend used.

Subtract this product, and to the remainder annex the next figure of the dividend for a new partial dividend.

Divide as before, and continue this process until all the figures of the dividend have been used.

WRITTEN EXERCISE

Divide the following:

1. 25,585 ÷ 301.
2. 52,373 ÷ 631.
3. 47,971 ÷ 623.
4. 153,397 ÷ 211.
5. 976,950 ÷ 501.
6. 972,471 ÷ 111.
7. 161,784 ÷ 321.
8. 324,360 ÷ 306.
9. 140,850 ÷ 225.
10. 183,750 ÷ 175.
11. 148,555 ÷ 407.
12. 234,901 ÷ 503.
13. 208,575 ÷ 309.
14. 870,672 ÷ 204.
15. 367,848 ÷ 786.
16. 102,465 ÷ 495.
17. 472,932 ÷ 604.
18. 349,206 ÷ 407.
19. 117,440 ÷ 320.
20. 419,640 ÷ 538.
21. 287,712 ÷ 216.
22. 615,000 ÷ 328.
23. 685,542 ÷ 286.
24. 997,108 ÷ 478.
25. 965,496 ÷ 852.
26. 7753 ÷ 69.
27. 8598 ÷ 76.
28. 8585 ÷ 23.
29. 76,452 ÷ 56.
30. 18,753 ÷ 64.
31. 71,797 ÷ 57.
32. 19,140 ÷ 46.
33. 26,523 ÷ 47.
34. 27,559 ÷ 59.
35. 47,078 ÷ 69.
36. 50,602 ÷ 82.
37. 101,160 ÷ 232.
38. 110,370 ÷ 124.
39. 252,150 ÷ 412.
40. 120,830 ÷ 653.
41. 260,380 ÷ 422.
42. 192,892 ÷ 232.
43. 211,428 ÷ 5873.
44. 287,712 ÷ 5328.
45. 313,205 ÷ 8465.
46. 411,075 ÷ 5481.
47. 616,693 ÷ 8009.
48. 965,496 ÷ 9852.
49. 1,599,560 ÷ 2836.
50. 2,402,728 ÷ 6257.

WRITTEN EXERCISE

1. If 11 acres of land are worth $1485, what is the average price per acre? How much are 4 acres worth?

2. If a dealer pays $336 a dozen for suits of ready-made clothes, how much does he pay per suit? for 7 suits?

3. A man pays $1276 for the rent of a house in the city for 11 mo. At this rate, how much does he pay for 5 mo.?

4. A wagon dealer pays $1215 for 9 wagons. At this rate, how much do 4 wagons cost? How much do 7 wagons cost?

5. At the rate of $784 for 8 acres of land, how much will a man have to pay for 3 acres? for 5 acres? for 100 acres?

6. If a stock dealer buys some cattle for $72,012, paying $51 a head, how many head of cattle does he buy?

7. If a jobber sells during the year $82,411 worth of shoes at $61 a case, how many cases does he sell?

8. If a dealer sells $164,598 worth of linseed oil during a year at $42 a barrel, how many barrels does he sell?

9. If a factory sells 3684 ready-made suits of a certain grade for $92,100, what is the price per suit?

10. If a state pays $2,325,798 for constructing 471 mi. of model road, what is the average cost per mile?

11. If it costs $5,236,602 to construct 287 mi. of railroad, what is the average cost per mile?

12. If a company employs 8009 men on the same schedule of wages, and the pay roll for these men is $6,247,020 a year, what is the annual wages of each?

13. If a company employs 250 men on the same schedule of wages, and the pay roll for these men is $270,000 a year, what is the monthly wages of each?

46. Long Division of United States Money. Division of United States money does not involve any new principle. We divide as with other numbers, placing the decimal point in the quotient above that in the dividend.

Required to divide $21,375.12 by 372.

Since $2, $21, and $213 cannot be divided by 372 with an integer for a quotient, we divide $2137 by 372, and the quotient is $5 with remainder $277. Since the $2137 is in the tens' place of dollars, we write the $5 over the tens' place of the dividend.

The rest of the division is as in cases already studied. The decimal point in the quotient is placed over that in the dividend, and the quotient is $57.46.

$$\begin{array}{r|r} & \$57.46 \\ \hline 372 & \$21375.12 \\ & 1860 \\ \hline & 2775 \\ & 2604 \\ \hline & 1711 \\ & 1488 \\ \hline & 2232 \\ & 2232 \\ \hline \end{array}$$

WRITTEN EXERCISE

Divide the following:

1. $973.47 ÷ 111.
2. $573.12 ÷ 199.
3. $1967.50 ÷ 50.
4. $6882.92 ÷ 11.
5. $12,018.30 ÷ 21.
6. $96,899.23 ÷ 23.
7. $36,147.06 ÷ 53.
8. $33,328.80 ÷ 54.
9. $33,747.20 ÷ 64.
10. $39,913.56 ÷ 81.
11. $27,891.25 ÷ 125.
12. $14,225.40 ÷ 420.
13. $734,986.12 ÷ 836.
14. $501,738.80 ÷ 538.
15. $102,465.00 ÷ 2070.
16. $692,356.00 ÷ 8764.
17. $616,693.00 ÷ 8009.
18. $388,591.00 ÷ 9037.

19. A shop has 664 men on its pay roll, each receiving the same wages. For 6 days this pay roll amounts to $9960. How much does each man receive a day?

LITTLE EXAMINATIONS

I. 1. MDXC = (?).
2. 987,654 + 456,789.
3. 900,123 − 456,789.
4. 7096 × 98,307.
5. 25 × 427,386.
6. $1780.02 ÷ 99.
7. $42,346.20 ÷ 7.
8. 52,720 ÷ 500.

II. 1. MCCXCIV = (?).
2. 427,625 + 49,836.
3. 302,123 − 56,877.
4. 4009 × 92,638.
5. 75 × 326,004.
6. 500 × 24,876.
7. $180.18 ÷ 91.
8. 46,500 ÷ 250.

III. 1. CLXXXIX = (?).
2. 697,439 + 89,876.
3. 711,234 − 68,098.
4. 386 × $562.75.
5. 50 × 486,368.
6. 75 × 544,664.
7. $446.22 ÷ 67.
8. 52,320 ÷ 125.

IV. 1. LXXXVIII = (?).
2. 587,629 + 89,987.
3. 834,213 − 96,789.
4. 7237 × 41,826.
5. 936 × $700.99.
6. 25 × 336,404.
7. 500 × 27,460.
8. $176.30 ÷ 43.
9. 56,140 ÷ 70.
10. 57,235 ÷ 130.

V. 1. MCCCIV = (?).
2. 829,968 + 73,875.
3. 501,230 − 66,463.
4. 5009 × 80,008.
5. 672 × $899.50.
6. 75 × 42,764.
7. 250 × 6744.
8. $55.86 ÷ 57.
9. 12,012 ÷ 77.
10. 57,620 ÷ 400.

REVIEW AND DRILL

47. Nature and Purpose of Review and Drill. At the end of each chapter in this book there occurs a section like this one. The work is called Review and Drill, and it is optional with the teacher whether or not it shall be used and how much of it shall be taken. None of these sections is merely a review of the chapter immediately preceding it, for review work is already provided in the chapter itself; on the contrary, the review is general and covers all the important processes in arithmetic that have been studied. This renders possible a continual return to fundamental and first principles, a constant brushing up in the essentials of arithmetic. This constant brushing up, drilling, and reviewing of the essentials is necessary for the attainment of good results. It is easily made interesting; it contributes to efficiency in computation; and it will, if carried out in the right spirit, help to remove the charge that most pupils are not quick and accurate in their calculations.

When the length of the school year permits, the study of these Review and Drill sections is strongly recommended; but the work being supplementary and optional, it doubtless will often be desirable for the teachers to select from these exercises only what is specially adapted to their immediate needs, omitting the rest. A certain flexibility is thus offered that will make the textbook more adaptable to varying local requirements than has heretofore been the case.

Each section consists of three parts: first, a drill in operations, emphasizing the *mechanical* side of arithmetic; second, an exercise in problems without numbers, emphasizing the *analytic* side; and finally, live industrial problems that photograph real situations, emphasizing the *practical* side.

It will at once occur to every teacher that in these Review and Drill sections, with their threefold, inclusive emphasis, there is ready at hand a series of *efficiency tests* which will make it easy at frequent intervals to check up the work of the school as well as the work of the individual pupils.

I. DRILL SECTION

ORAL EXERCISE

1. State the sum and also the difference of:

98	73	66	80	88	35	45
11	21	22	30	30	28	36

2. Multiply, in turn, by 2, 3, 4, and so on to 9:

20 30 40 70 60 21 31

3. Divide, in turn, by 2, 3, 4, and so on to 9:

10 20 50 70 30 25 48

WRITTEN EXERCISE

Add the following:

1. 273 + 421 + 68 + 9026 + 34,379 + 75 + 286.

2. 3972 + 8298 + 4000 + 17,625 + 31 + 9 + 407.

Subtract the following:

3. 876,481 − 298,327.

4. 173,209 − 106,492.

5. 298,008 − 43,661.

6. 381,200 − 179,699.

Multiply the following:

7. 628 × 325.

8. 872 × 896.

9. 1268 × 3472.

10. 2081 × 6235.

11. 469 × 832.

12. 4287 × 3009.

Divide the following:

13. 65,664 ÷ 456.

14. 221,336 ÷ 102.

15. 7,605,920 ÷ 112.

16. 77,112 ÷ 189.

17. 328,320 ÷ 360.

18. 7,913,576 ÷ 209.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. How do you add a number of two figures to another number of two figures when the sum of the units is more than ten? Give your reasons.

2. How do you subtract a number of two figures from a greater number when the units' figure to be subtracted is greater in value than the units' figure of the minuend?

3. How do you multiply by a number of three figures, the middle figure being zero?

4. How do you check your work in addition? How do you check your work in subtraction?

5. How do you check your work in division? How do you check your work in multiplication?

6. If you know the cost of a certain number of acres of land, how do you find the average cost per acre?

7. If you know the weight of each member of your class, how do you find the average weight?

8. If you have the product of two numbers and one of the two numbers given, how do you find the other number?

9. If you have the dividend, remainder, and quotient given, how do you find the divisor?

10. If you have the multiplicand and product given, how do you find the multiplier?

11. How do you multiply by a power of ten? How do you multiply by a multiple of a power of ten?

12. How do you divide by a power of ten? How do you divide by a multiple of a power of ten?

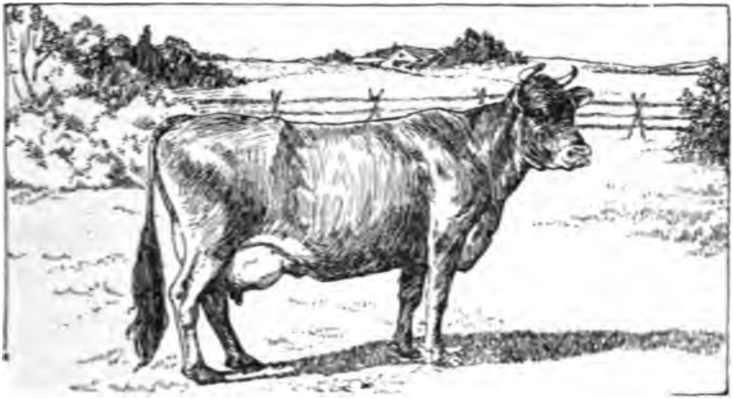
III. INDUSTRIAL PROBLEMS

THE COST OF WASTEFULNESS

1. If a farm wagon that cost $60 is left out in the yard instead of being kept in the shed, it will last about 6 yr., but if kept under cover, it will last about twice as long. How much does a farmer pay for his carelessness per year if he leaves the wagon out?

2. Each of two neighbors bought for his farm work a two-horse cultivator, and each paid $25. One of them took good care of his cultivator, and at the end of 15 yr. it was still giving good service. The other left his out during all kinds of weather and had to buy a new one at the end of every 5 yr. How much did the careless one lose in 15 yr. by his neglect?

3. A farmer after threshing his wheat had 16 tons of straw left. A ton of this straw contains 12 lb. of nitrogen, worth 20¢ a pound, and 10 lb. of potash and 2 lb. of phosphoric acid, each worth 5¢ a pound. If the farmer wastes the straw instead of using it on the soil as fertilizer, how much money does he waste?



DOES BLOOD TELL IN THE DAIRY?

1. Ralph's father explained to him what was meant by grading the cows, keeping their records for milk, and disposing of those with poor records. He showed him the records of twelve farms, as follows:

With Systematic Grading			*Without Systematic Grading*		
Herd	Annual cost of food per cow	Annual profit per cow	Herd	Annual cost of food per cow	Annual profit per cow
a	$34.28	$34.02	*g*	$31.65	$13.34
b	33.37	36.19	*h*	40.58	8.13
c	47.11	25.83	*i*	38.30	22.22
d	36.72	38.27	*j*	37.40	22.33
e	31.19	48.86	*k*	33.78	38.93
f	31.57	42.81	*l*	32.88	14.91

Ralph and his father then figured out the average annual cost of food and profit per cow, in each class of farms. They did this by dividing each of the four totals by six. What were the results?

2. They then found the differences between the two cost averages and the two profit averages. What are these two differences?

RICHARD'S VEGETABLE GARDEN

1. Richard's father has rented him a vegetable garden. Richard is to have the profits for a camping trip. On April 16 he plants some early potatoes. He counts on these potatoes being ready to dig in 79 da. When must he be home to begin to dig them?

2. If Richard allows 18 da. after the date found in Ex. 1 in which to let all the potatoes mature, and to sell them, what is the earliest day on which he can go camping?

3. On April 24 Richard sets out some early cauliflower, and he has to allow 72 da. for this to be ready to sell. When must he be on hand to sell the cauliflower?

4. Richard raised $4\frac{1}{2}$ bu. of early potatoes and sold them at $1 a bushel. How much did he receive for the potatoes?

5. Richard sold 25 heads of early cauliflower at 10¢ a head. How much did he receive for the cauliflower?

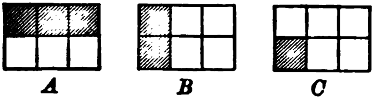
6. Richard pays his father 75¢ rent for his garden and for his materials. Make out a statement showing Richard's income and expenses, and the amount that he has left for his camping trip.

CHAPTER II

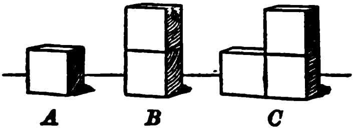
COMMON FRACTIONS

ORAL EXERCISE

1. When anything is divided into 3 equal parts what is each part called? What is each part called when the object is divided into 5 equal parts? into 12 equal parts?

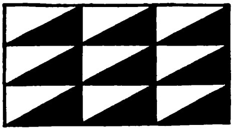


2. In the above figure, what part of *A* is shaded? What part of *B* is shaded? What part of *C* is shaded? From *A* we see that $\frac{1}{2}$ equals how many sixths? From *B* we see that $\frac{1}{3}$ equals how many sixths?



3. In this figure of the cubes, *A* is what part as large as *B*? as *C*? *B* is what part as large as *C*?

4. In the above figure of the cubes, if *C* is called 1, what are *A* and *B* called? If *B* is called 1, what are *A* and *C* called? If *A* is called 1, what are *B* and *C* called?



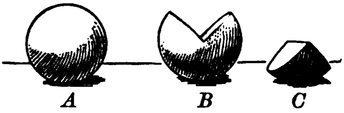
5. Each of these small rectangles, or oblongs, is what part of the whole rectangle? Each shaded triangle is what part of the whole rectangle? What does this tell you about $\frac{1}{2}$ of $\frac{1}{9}$? What does it tell you about $\frac{1}{9}$ of $\frac{1}{2}$?

48. Fraction. A number which shows what number of equal parts of a unit is taken is called a *fraction.*

Thus, $\frac{1}{2}$ shows that one of the two equal parts of a unit is taken; $\frac{3}{4}$ shows that three of the four equal parts of a unit are taken.

49. Denominator. The number which shows into how many equal parts a unit has been divided to make a fraction is called the *denominator* of the fraction.

50. Numerator. The number which shows how many equal parts of a unit have been taken to make a fraction is called the *numerator* of the fraction.



For example, in this figure, B contains three of the four equal parts of a sphere; that is, B is $\frac{3}{4}$ of a sphere, 4 is the denominator of the fraction $\frac{3}{4}$, and 3 is the numerator.

51. Terms. The numerator and the denominator together are called the *terms* of the fraction.

52. Common Fraction. A fraction which has both terms expressed is called a *common fraction.*

53. Proper Fraction. A fraction whose numerator is less than the denominator is called a *proper fraction.*

54. Unit Fraction. A fraction whose numerator is 1 is called a *unit fraction.*

55. Improper Fraction. A fraction whose numerator equals or exceeds the denominator is called an *improper fraction.*

In the above figure, A is $\frac{4}{4}$ of a sphere, and $A + C$ is $\frac{5}{4}$ of a sphere.

56. Integer. A whole number, or a number expressed without fractions, is called an *integer.*

57. Mixed Number. A number composed of an integer and a fraction taken together is called a *mixed number.*

For example, if the above sphere A is 1, $A + B$ is $1\frac{3}{4}$ and $A + C$ is $1\frac{1}{4}$.

ORAL EXERCISE

Read aloud the following:

1. $\frac{3}{8}$ $\frac{7}{8}$ $\frac{1}{16}$ $\frac{3}{5}$ $\frac{7}{16}$ $\frac{9}{16}$
2. $\frac{5}{8}$ $\frac{2}{3}$ $\frac{3}{16}$ $\frac{15}{16}$ $\frac{11}{16}$ $\frac{7}{32}$
3. $2\frac{4}{5}$ $3\frac{7}{8}$ $9\frac{2}{3}$ $8\frac{3}{16}$ $2\frac{5}{32}$ $25\frac{2}{3}$
4. $12\frac{1}{2}$ $33\frac{1}{3}$ $15\frac{4}{25}$ $9\frac{3}{32}$ $6\frac{7}{64}$ $157\frac{7}{8}$
5. An inch is $\frac{1}{12}$ ft., $\frac{1}{36}$ yd., or $\frac{1}{198}$ rd.

Fractions having denominators larger than 8 are rarely used in computation, but in measuring we use fractions with larger denominators, usually not exceeding 64.

WRITTEN EXERCISE

Write the following fractions:

1. Seven sixteenths.
2. Nine sixteenths.
3. Nineteen thirty-seconds.
4. Eight ninths.
5. Seven thirty-seconds.
6. Seventeen eighteenths.
7. Nine sixty-fourths.
8. Five thirty-seconds.

Write in words the following:

9. $\frac{1}{2}$ $\frac{1}{8}$ $\frac{5}{8}$ $\frac{7}{8}$ $\frac{5}{16}$ $\frac{5}{24}$ $\frac{15}{16}$ $\frac{17}{32}$
10. $\frac{1}{5}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{3}{8}$ $\frac{7}{16}$ $\frac{23}{24}$ $\frac{19}{32}$ $\frac{21}{64}$
11. $\frac{1}{3}$ $\frac{2}{3}$ $1\frac{2}{3}$ $5\frac{4}{5}$ $62\frac{7}{8}$ $27\frac{1}{2}$ $6\frac{2}{3}$ $15\frac{5}{8}$

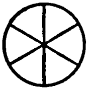
In practical business $\frac{1}{2}$¢ or over, in the final result, is called 1¢; less than $\frac{1}{2}$¢ is neglected in many cases, although in making out bills any fraction of a cent usually counts as a cent.

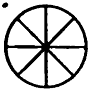
Write the following in words, and also write the number of dollars and cents that would be taken in making out bills:

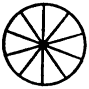
12. $\$125.12\frac{1}{2}$ $\$6.66\frac{2}{3}$ $\$333.33\frac{1}{3}$ $\$275.16\frac{2}{3}$
13. $\$135.87\frac{1}{2}$ $\$14.37\frac{1}{2}$ $\$112.16\frac{7}{8}$ $\$235.45\frac{1}{3}$

ORAL EXERCISE

1. Look at figure *A* and state how many sixths you see in one half.







2. Look at figure *B* and state how many eighths you see in one half; in three quarters.

3. Look at figure *C* and state how many tenths you see in one half; in one fifth; in four fifths.

4. How many fifths do you see in two tenths? in four tenths? in six tenths? in eight tenths?

58. Reducing Fractions. We see that $\frac{2}{4}$ may be obtained from $\frac{1}{2}$ by multiplying both terms of the fraction by 2, and that $\frac{1}{2}$ may be obtained from $\frac{5}{10}$ by dividing both terms by 5. That is,

Both terms of a fraction may be multiplied by the same number without changing the value of the fraction.

Both terms of a fraction may be divided by the same number without changing the value of the fraction.

When we change the terms without changing the value of a fraction we *reduce* the fraction.

When both terms cannot be divided by the same number, the fraction is said to be *in lowest terms.*

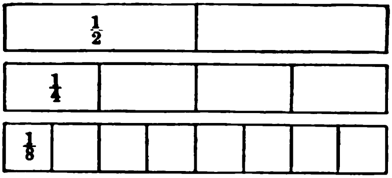
To reduce a fraction to lowest terms, divide by the largest number that will divide both terms without a remainder.

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}, \text{ lowest terms.}$$

$$\boxed{\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}} = \frac{2}{3}}$$

In this example the common factor 4 is said to be *canceled* from both terms when the work is written as here shown.

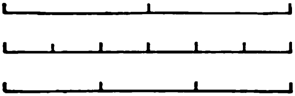
ORAL EXERCISE



1. Calling the large rectangle *one*, how many halves do you see in 1?

2. How many fourths do you see in $\frac{1}{2}$? in 1?

3. How many eighths do you see in $\frac{1}{4}$? in $\frac{1}{2}$? in 1?



4. Calling the line *one*, how many sixths do you see in $\frac{1}{2}$? in 1?

5. $1\frac{1}{2}$ = how many sixths?

6. How many sixths do you see in $\frac{1}{3}$? in $\frac{2}{3}$? in $\frac{3}{3}$? in 1?

WRITTEN EXERCISE

1. Reduce to halves:

$\frac{2}{4}$ $\frac{4}{8}$ $\frac{8}{8}$ $\frac{12}{8}$ $\frac{16}{8}$ $\frac{24}{8}$ $\frac{32}{8}$ $\frac{40}{8}$

2. Reduce to fourths:

$\frac{1}{2}$ $\frac{2}{8}$ $\frac{4}{8}$ $\frac{16}{8}$ $\frac{10}{8}$ $\frac{20}{8}$ $\frac{18}{8}$ $\frac{22}{8}$

3. Reduce to eighths:

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{6}{4}$ $\frac{7}{2}$ $\frac{2}{16}$ $\frac{4}{16}$

4. Reduce to twelfths:

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{2}{24}$

5. Reduce to lowest terms:

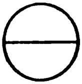
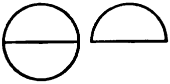
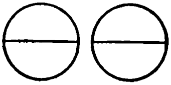
$\frac{2}{4}$ $\frac{4}{8}$ $\frac{10}{8}$ $\frac{8}{10}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{6}{8}$ $\frac{5}{10}$

6. Express $\frac{1}{3}$ in. and $\frac{1}{2}$ in. as sixths of an inch, and tell which is the greater.

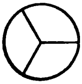
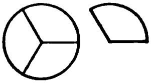
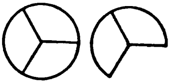
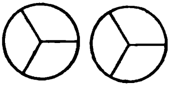
7. Express $\frac{1}{3}$ in. and $\frac{1}{4}$ in. as twelfths of an inch, and tell which is the greater.

ORAL EXERCISE

1. How many halves do you see in 1? in $1\frac{1}{2}$? in 2? Express $\frac{3}{2}$ as a mixed number. Express $\frac{4}{2}$ as a whole number.

2. How many thirds do you see in 1? in $1\frac{1}{3}$? in $1\frac{2}{3}$? in 2? Express $\frac{4}{3}$ and $\frac{5}{3}$ as mixed numbers; $\frac{6}{3}$ as a whole number.

WRITTEN EXERCISE

1. How many fourths are there in 1? in $1\frac{1}{4}$? in $1\frac{3}{4}$? in 2? Write the results in this way: $1 = \frac{4}{4}$, $1\frac{1}{4} = \frac{5}{4}$, and so on.

2. How many fifths are there in 1? in $1\frac{1}{5}$? in $1\frac{2}{5}$? in $1\frac{3}{5}$? in $1\frac{4}{5}$? in $1\frac{5}{5}$, or 2?

3. How many sixths are there in 1? in $1\frac{1}{6}$? in $1\frac{5}{6}$? in 2? in $2\frac{1}{6}$? in $2\frac{5}{6}$? in $2\frac{6}{6}$, or 3?

4. How many sevenths are there in 1? in $1\frac{1}{7}$? in $1\frac{2}{7}$? in $1\frac{3}{7}$? in $1\frac{4}{7}$? in $1\frac{5}{7}$? in $1\frac{6}{7}$? in 2?

5. How many eighths are there in 1? in 2? in $2\frac{1}{8}$? in $2\frac{3}{8}$? in $2\frac{5}{8}$? in $2\frac{7}{8}$? in 3?

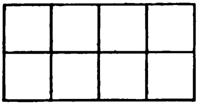
6. How many ninths are there in 1? in 2? in 3? in 4? in $1\frac{1}{9}$? in $2\frac{2}{9}$? in $2\frac{9}{9}$?

7. How many tenths are there in 1? in 2? in 3? in $3\frac{1}{10}$? in $3\frac{3}{10}$? in $3\frac{7}{10}$? in $3\frac{10}{10}$?

8. How many twelfths are there in 1? in 2? in 3? in 4? in $2\frac{1}{12}$? in $3\frac{5}{12}$? in $4\frac{7}{12}$? in $4\frac{11}{12}$?

ORAL EXERCISE

1. From the figure, $\frac{1}{2}$ equals how many eighths? how many fourths?



2. From the figure, $\frac{6}{8}$ equals how many fourths?

3. In what cases, in Exs. 1 and 2, did we reduce to lower terms, and in what cases to higher terms?

Reduce as indicated:

4. $\frac{1}{2}$ to sixteenths $\frac{1}{2}$ to sixths $\frac{4}{5}$ to tenths
5. $\frac{1}{4}$ to twelfths $\frac{1}{4}$ to eighths $\frac{3}{4}$ to twelfths
6. $\frac{3}{8}$ to sixteenths $\frac{2}{8}$ to fourths $\frac{6}{16}$ to eighths

WRITTEN EXERCISE

Reduce to any lower terms:

1. $\frac{4}{8}$ $\frac{6}{16}$ $\frac{12}{16}$ $\frac{8}{10}$ $\frac{8}{12}$ $\frac{14}{32}$ $\frac{16}{32}$ $\frac{24}{32}$

Reduce to sixteenths:

2. $\frac{3}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ $\frac{24}{32}$ $\frac{30}{32}$ $\frac{16}{64}$ $\frac{20}{64}$ $\frac{40}{64}$

Reduce to thirty-seconds:

3. $\frac{1}{4}$ $\frac{3}{4}$ $\frac{5}{8}$ $\frac{7}{8}$ $\frac{15}{16}$ $\frac{18}{64}$ $\frac{32}{64}$ $\frac{24}{64}$

Reduce to sixty-fourths:

4. $\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{16}$ $\frac{9}{16}$ $\frac{9}{32}$ $\frac{15}{32}$ $\frac{16}{32}$ $\frac{12}{128}$

Reduce to twelfths:

5. $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{6}$ $\frac{12}{24}$ $\frac{10}{24}$

Reduce to twenty-fourths:

6. $\frac{3}{8}$ $\frac{1}{2}$ $\frac{7}{12}$ $\frac{1}{3}$ $\frac{2}{48}$ $\frac{6}{48}$ $\frac{5}{12}$ $\frac{11}{12}$

ORAL EXERCISE

1. How many half spheres in 1 sphere? in $1\frac{1}{2}$ spheres?

2. How many halves in 2? in $2\frac{1}{2}$? in 3? in 5? in $5\frac{1}{2}$?

3. How many fourths in 1? in $1\frac{1}{4}$? in $1\frac{1}{2}$? in $1\frac{3}{4}$? in $5\frac{3}{4}$?

4. Express the following as thirds:

$2 \qquad 4 \qquad 5 \qquad 8 \qquad 1\frac{1}{3} \qquad 2\frac{1}{3} \qquad 3\frac{2}{3} \qquad 10\frac{1}{3}$

Express the following as fourths:

5. $6 \qquad 8 \qquad 2\frac{1}{4} \qquad 5 \qquad 3\frac{1}{4} \qquad 3\frac{3}{4} \qquad 8\frac{1}{4} \qquad 12$

6. $7 \qquad 9 \qquad 1\frac{3}{4} \qquad 2\frac{3}{4} \qquad 4\frac{1}{4} \qquad 4\frac{3}{4} \qquad 5\frac{1}{4} \qquad 5\frac{3}{4}$

7. Express the following as eighths:

$10 \qquad 7 \qquad 6\frac{1}{8} \qquad 6\frac{3}{8} \qquad 7\frac{1}{8} \qquad 7\frac{5}{8} \qquad 8\frac{1}{8} \qquad 8\frac{7}{8}$

Reduce $4\frac{5}{8}$ to eighths.

Since $1 = \frac{8}{8}$, we see that $4 = \frac{4 \times 8}{8} = \frac{32}{8}$.

Therefore, $4\frac{5}{8} = \frac{32}{8} + \frac{5}{8} = \frac{37}{8}$.

WRITTEN EXERCISE

Reduce as indicated:

1. 6 to eighths.
2. 9 to thirds.
3. $8\frac{1}{2}$ to halves.
4. $7\frac{4}{5}$ to fifths.
5. $8\frac{2}{3}$ to thirds.
6. $6\frac{5}{6}$ to sixths.
7. $7\frac{1}{9}$ to ninths.
8. $8\frac{3}{5}$ to fifths.
9. $7\frac{1}{3}$ to thirds.
10. $8\frac{1}{4}$ to fourths.
11. $5\frac{4}{9}$ to ninths.
12. $9\frac{3}{5}$ to fifths.
13. $9\frac{7}{8}$ to eighths.
14. $8\frac{5}{9}$ to ninths.
15. $6\frac{2}{9}$ to ninths.
16. 41 to tenths.
17. $7\frac{2}{5}$ to fifths.
18. $4\frac{2}{3}$ to thirds.
19. $9\frac{3}{4}$ to fourths.
20. $7\frac{1}{8}$ to eighths.
21. $3\frac{3}{8}$ to eighths.

59. Reduction to an Integer or a Mixed Number. Reduce $\frac{124}{5}$ to an integer or a mixed number.

Since $\frac{1}{5}$ means that 1 has been divided into 5 equal parts, $\frac{124}{5}$ means that 124×1 has been divided into 5 equal parts.

Therefore $\frac{124}{5} = 124 \div 5 = 24\frac{4}{5}$.

A fraction is an indicated division.

To reduce an improper fraction to an integer or a mixed number, divide the numerator by the denominator.

ORAL EXERCISE

1. Express $2\frac{1}{2}$ as halves; $\frac{5}{2}$ as a mixed number.

2. In $5 there are how many half dollars? Express 5 as halves. Express $\frac{10}{2}$ as an integer. Express $\frac{15}{5}$, $\frac{20}{4}$, $\frac{30}{3}$, and $\frac{40}{10}$ as integers.

3. Express the following as integers:

$\frac{12}{6}$ $\frac{15}{3}$ $\frac{16}{2}$ $\frac{81}{9}$ $\frac{25}{5}$ $\frac{125}{25}$ $\frac{64}{4}$ $\frac{48}{12}$

4. Express the following as mixed numbers:

$\frac{15}{7}$ $\frac{17}{8}$ $\frac{20}{3}$ $\frac{32}{10}$ $\frac{37}{7}$ $\frac{73}{12}$ $\frac{41}{5}$ $\frac{63}{8}$

WRITTEN EXERCISE

Reduce the following to integers or mixed numbers:

1. $\frac{33}{8}$ $\frac{125}{24}$ $\frac{632}{32}$ $\frac{127}{4}$ $\frac{81}{8}$ $\frac{107}{8}$
2. $\frac{47}{6}$ $\frac{298}{16}$ $\frac{190}{12}$ $\frac{87}{12}$ $\frac{92}{12}$ $\frac{116}{12}$
3. $\frac{42}{6}$ $\frac{236}{32}$ $\frac{425}{32}$ $\frac{93}{32}$ $\frac{76}{32}$ $\frac{160}{32}$
4. $\frac{78}{5}$ $\frac{349}{32}$ $\frac{276}{16}$ $\frac{198}{16}$ $\frac{725}{32}$ $\frac{525}{32}$
5. $\frac{65}{8}$ $\frac{427}{12}$ $\frac{629}{24}$ $\frac{332}{16}$ $\frac{342}{12}$ $\frac{358}{12}$
6. $\frac{69}{8}$ $\frac{625}{12}$ $\frac{221}{12}$ $\frac{625}{24}$ $\frac{896}{16}$ $\frac{257}{16}$
7. $\frac{29}{3}$ $\frac{175}{32}$ $\frac{118}{8}$ $\frac{125}{8}$ $\frac{441}{8}$ $\frac{549}{8}$

60. Similar Fractions. If several fractions have the same denominator, they are said to be ***similar fractions*** and to have a *common denominator.*

61. Least Common Denominator. If the common denominator of several fractions is the smallest possible, the fractions are said to have the *least common denominator* (L.C.D.).

For example, $\frac{4}{8}$ and $\frac{2}{8}$ have a common denominator, but they may be reduced to $\frac{2}{4}$ and $\frac{1}{4}$, and 4 is the least common denominator.

62. Reduction to L.C.D. In making some fine panels for a door a workman had to glue together three strips of wood. The first was $\frac{1}{4}$ in. thick, the second $\frac{3}{8}$ in., and the third $\frac{5}{12}$ in. How thick were the three together?

We shall find that to add $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{5}{12}$ we must reduce them to fractions having the least common denominator, and we must learn to do this before we can answer such a question as the one we have just read.

$$\frac{1}{4}=\frac{6\times1}{6\times4}=\frac{6}{24}$$
$$\frac{3}{8}=\frac{3\times3}{3\times8}=\frac{9}{24}$$
$$\frac{5}{12}=\frac{2\times5}{2\times12}=\frac{10}{24}$$

Since the required denominator must contain 4, it must contain all the factors of 4; and since it must contain 8 and 12, it must also contain all their factors.

We see that 4, 8, and 12 each contains 4, that 8 also contains 2, and that 12 contains 3. Therefore the least common denominator must contain the factors 4, 2, and 3, and is $4 \times 2 \times 3$, or 24.

Since $24 \div 4 = 6$, the terms of $\frac{1}{4}$ must be multiplied by 6, to reduce $\frac{1}{4}$ to 24ths, and we have $\frac{1}{4}=\frac{6}{24}$.

Since $24 \div 8 = 3$, the terms of $\frac{3}{8}$ must be multiplied by 3 to reduce $\frac{3}{8}$ to 24ths, and we have $\frac{3}{8}=\frac{9}{24}$.

Since $24 \div 12 = 2$, the terms of $\frac{5}{12}$ must be multiplied by 2 to reduce $\frac{5}{12}$ to 24ths, and we have $\frac{5}{12}=\frac{10}{24}$.

ORAL EXERCISE

Reduce the following to similar fractions:

1. $\frac{2}{3}, \frac{1}{2}$.
2. $\frac{3}{5}, \frac{2}{3}$.
3. $\frac{4}{16}, \frac{2}{8}$.
4. $\frac{2}{5}, \frac{7}{10}$.
5. $\frac{5}{15}, \frac{2}{30}$.
6. $\frac{1}{7}, \frac{2}{14}$.
7. $\frac{3}{5}, \frac{4}{10}$.
8. $\frac{3}{8}, \frac{3}{4}$.
9. $\frac{10}{20}, \frac{15}{30}$.
10. $\frac{6}{8}, \frac{9}{10}$.
11. $\frac{2}{3}, \frac{8}{30}$.
12. $\frac{1}{4}, \frac{8}{10}$.

Reduce to fractions having the least common denominator:

13. $\frac{1}{2}, \frac{2}{3}$.
14. $\frac{3}{5}, \frac{1}{2}$.
15. $\frac{8}{10}, \frac{3}{5}$.
16. $\frac{1}{4}, \frac{2}{3}$.
17. $\frac{7}{8}, \frac{1}{16}$.
18. $\frac{3}{16}, \frac{5}{32}$.
19. $\frac{7}{12}, \frac{1}{4}$.
20. $\frac{1}{2}, \frac{4}{5}$.
21. $\frac{2}{3}, \frac{3}{4}, \frac{1}{6}$.
22. $\frac{2}{9}, \frac{2}{3}, \frac{5}{6}$.
23. $\frac{3}{8}, \frac{3}{5}, \frac{3}{10}$.
24. $\frac{7}{8}, \frac{1}{5}, \frac{1}{40}$.

WRITTEN EXERCISE

Reduce to fractions having the least common denominator:

1. $\frac{1}{2}, \frac{5}{6}$.
2. $\frac{1}{4}, \frac{5}{6}$.
3. $\frac{3}{4}, \frac{1}{6}$.
4. $\frac{1}{8}, \frac{5}{6}$.
5. $\frac{5}{8}, \frac{3}{4}$.
6. $\frac{7}{8}, \frac{1}{3}$.
7. $\frac{5}{6}, \frac{2}{3}$.
8. $\frac{7}{8}, \frac{2}{3}$.
9. $\frac{3}{5}, \frac{2}{3}$.
10. $\frac{4}{5}, \frac{3}{8}$.
11. $\frac{5}{12}, \frac{7}{24}$.
12. $\frac{3}{16}, \frac{5}{24}$.
13. $\frac{7}{8}, \frac{5}{12}$.
14. $\frac{5}{8}, \frac{11}{24}$.
15. $\frac{7}{8}, \frac{13}{24}$.
16. $\frac{7}{8}, \frac{5}{32}$.
17. $\frac{5}{8}, \frac{11}{32}$.
18. $\frac{7}{16}, \frac{5}{32}$.
19. $\frac{15}{16}, \frac{31}{32}$.
20. $\frac{13}{16}, \frac{29}{32}$.
21. $\frac{5}{8}, \frac{7}{24}$.
22. $\frac{7}{8}, \frac{21}{24}$.
23. $\frac{5}{12}, \frac{3}{32}$.
24. $\frac{7}{12}, \frac{5}{32}$.
25. $\frac{11}{12}, \frac{15}{32}$.
26. $\frac{1}{24}, \frac{1}{32}$.
27. $\frac{5}{24}, \frac{3}{32}$.
28. $\frac{7}{24}, \frac{9}{32}$.
29. $\frac{7}{24}, \frac{9}{48}$.
30. $\frac{11}{24}, \frac{13}{48}$.
31. $\frac{7}{8}, \frac{9}{64}$.
32. $\frac{5}{16}, \frac{11}{64}$.
33. $\frac{11}{32}, \frac{15}{64}$.
34. $\frac{1}{12}, \frac{3}{64}$.
35. $\frac{5}{12}, \frac{7}{64}$.
36. $\frac{1}{24}, \frac{1}{64}$.
37. $\frac{7}{24}, \frac{25}{64}$.
38. $\frac{3}{8}, \frac{5}{16}, \frac{7}{24}$.
39. $\frac{7}{8}, \frac{5}{12}, \frac{11}{24}$.
40. $\frac{5}{8}, \frac{7}{12}, \frac{15}{32}$.

Reduce the fractional parts of the following mixed numbers to fractions having the least common denominator:

41. $4\frac{7}{12}, 6\frac{9}{16}$.
42. $5\frac{3}{16}, 7\frac{5}{24}$.
43. $9\frac{11}{24}, 7\frac{15}{32}$.

63. Adding Numbers having the Same Name. If we ask for the sum of 3 boys and 2 girls, the answer cannot be boys alone or girls alone. But we can say,

$$\begin{array}{rl} 3 \text{ boys} = & 3 \text{ children} \\ 2 \text{ girls} = & \underline{2 \text{ children}} \\ & 5 \text{ children, all together} \end{array}$$

In adding, we think of things as having the same name.

In the same way, if we wish to find the sum of $\frac{3}{4}$ and $\frac{1}{2}$, we must think of these as $\frac{3}{4}$ and $\frac{2}{4}$, the answer being $\frac{5}{4}$, or $1\frac{1}{4}$.

In adding fractions we think of them as having the same name, that is, as similar fractions.

Numerous illustrations of fractions have been given on pages 47–53. The same idea should be carried throughout, but it acquires more force if, from now on, the objective work is done by the pupil. Addition and subtraction of fractions should be taught exactly as addition and subtraction of integers were taught in the second grade — objectively, as suggested in the oral work on this page. Paper cutting, and diagrams of circles, rectangles, and lines should precede the use of figures. This work is best done by the teacher and pupils working together, the textbook furnishing the examples for practice.

ORAL EXERCISE

1. Express $\frac{1}{4}$ as eighths. To the result add $\frac{5}{8}$.

2. If you fasten a strip of molding $\frac{1}{4}$ in. thick to a strip of wood $\frac{5}{8}$ in. thick, how thick are the two together?

3. Express $\frac{4}{5}$ as tenths. To the result add $\frac{3}{10}$.

4. If you lay an arithmetic that is $\frac{4}{5}$ in. thick on a notebook that is $\frac{3}{10}$ in. thick, how thick are the two together?

5. Express $\frac{2}{3}$ as sixths. To the result add $\frac{1}{6}$.

6. If you sew a piece of cloth $\frac{2}{3}$ yd. wide to a strip $\frac{1}{6}$ yd. wide, how wide will the new piece be?

64. Addition of Fractions. As in § 63, we see that if we add 3 marbles and 5 pencils, we have as a sum neither marbles alone nor pencils alone, but just 8 things, of two different kinds. So if we add $\frac{3}{4}$ and $\frac{5}{6}$ we shall have as a sum neither fourths nor sixths, but just 8 parts of a unit, of two different sizes.

Therefore, to add fractions that are not similar and obtain a definite sum, we must first reduce them to similar fractions.

Suppose that we wish to find the sum of $\frac{3}{5}$, $\frac{9}{10}$, and $\frac{7}{12}$.

The L.C.D. is evidently $5 \times 2 \times 2 \times 3 = 60$. Dividing 60 by the three denominators, we have

$$60 \div 5 = 12, \text{ and } \frac{3}{5} = \frac{12 \times 3}{12 \times 5} = \frac{36}{60};$$

$$60 \div 10 = 6, \text{ and } \frac{9}{10} = \frac{6 \times 9}{6 \times 10} = \frac{54}{60};$$

and

$$60 \div 12 = 5, \text{ and } \frac{7}{12} = \frac{5 \times 7}{5 \times 12} = \frac{35}{60}.$$

Furthermore, $\frac{36}{60} + \frac{54}{60} + \frac{35}{60} = \frac{125}{60} = 2\frac{5}{60} = 2\frac{1}{12}$.

Therefore the sum of $\frac{3}{5}$, $\frac{9}{10}$, and $\frac{7}{12}$ is $2\frac{1}{12}$.

To add fractions, reduce them to fractions having the least common denominator, and add the numerators for the numerator of the sum, writing this numerator over the least common denominator. Then reduce to lowest terms.

ORAL EXERCISE

1. How many twelfths are $\frac{7}{12} + \frac{1}{12}$? How many thirds?

State the sum of each of the following pairs of fractions:

2. $\frac{1}{2} + \frac{1}{4}$ $\quad \frac{1}{2} + \frac{3}{4}$ $\quad \frac{1}{2} + \frac{1}{8}$ $\quad \frac{1}{2} + \frac{7}{8}$ $\quad \frac{1}{12} + \frac{5}{12}$

3. $\frac{1}{4} + \frac{3}{4}$ $\quad \frac{3}{8} + \frac{5}{8}$ $\quad \frac{2}{3} + \frac{5}{6}$ $\quad \frac{1}{8} + \frac{3}{8}$ $\quad \frac{1}{12} + \frac{11}{12}$

4. $\frac{1}{8} + \frac{5}{8}$ $\quad \frac{1}{5} + \frac{4}{5}$ $\quad \frac{1}{5} + \frac{2}{5}$ $\quad \frac{1}{4} + \frac{1}{8}$ $\quad \frac{5}{12} + \frac{7}{12}$

WRITTEN EXERCISE

Add the following fractions:

1. $\frac{1}{2}+\frac{1}{3}$.
2. $\frac{1}{2}+\frac{2}{3}$.
3. $\frac{1}{2}+\frac{1}{5}$.
4. $\frac{1}{2}+\frac{2}{5}$.
5. $\frac{1}{2}+\frac{3}{5}$.
6. $\frac{1}{2}+\frac{4}{5}$.
7. $\frac{1}{2}+\frac{3}{4}$.
8. $\frac{1}{2}+\frac{3}{8}$.
9. $\frac{1}{2}+\frac{7}{8}$.
10. $\frac{3}{4}+\frac{5}{8}$.
11. $\frac{3}{4}+\frac{7}{8}$.
12. $\frac{4}{5}+\frac{1}{8}$.
13. $\frac{3}{5}+\frac{3}{8}$.
14. $\frac{2}{5}+\frac{7}{8}$.
15. $\frac{4}{5}+\frac{7}{8}$.
16. $\frac{1}{2}+\frac{5}{6}$.
17. $\frac{3}{4}+\frac{5}{6}$.
18. $\frac{1}{6}+\frac{1}{8}$.
19. $\frac{1}{6}+\frac{3}{8}$.
20. $\frac{5}{6}+\frac{7}{8}$.

21. A piece of cardboard $\frac{1}{64}$ in. thick is laid on a book $\frac{5}{8}$ in. thick. How thick are the two together?

Add the following fractions:

22. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$.
23. $\frac{1}{2}+\frac{3}{4}+\frac{1}{8}$.
24. $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}$.
25. $\frac{1}{3}+\frac{1}{4}+\frac{1}{12}$.
26. $\frac{1}{3}+\frac{3}{4}+\frac{1}{12}$.
27. $\frac{2}{3}+\frac{3}{4}+\frac{7}{12}$.
28. $\frac{1}{3}+\frac{1}{8}+\frac{1}{12}$.
29. $\frac{2}{3}+\frac{1}{8}+\frac{1}{12}$.
30. $\frac{2}{3}+\frac{5}{8}+\frac{7}{12}$.

31. Add $\frac{1}{2}$ in., $\frac{1}{3}$ in., $\frac{1}{4}$ in., $\frac{1}{6}$ in., and $\frac{1}{12}$ in.

32. If you lay a notebook $\frac{1}{2}$ in. thick on a book $\frac{7}{8}$ in. thick, how thick are the two together?

33. Some plaster $\frac{3}{8}$ in. thick is coated with a finer plaster $\frac{3}{16}$ in. thick. How thick is the plaster then?

34. A plate of brass $\frac{1}{32}$ in. thick is laid on a plate of iron $\frac{1}{8}$ in. thick. What is the total thickness?

35. An iron rod $\frac{7}{8}$ in. in diameter is covered with a brass plating $\frac{1}{32}$ in. thick. What is the diameter of the rod then?

Add the following fractions:

36. $\frac{1}{4}, \frac{5}{24}$.
37. $\frac{3}{4}, \frac{11}{24}$.
38. $\frac{3}{8}, \frac{7}{24}$.
39. $\frac{5}{8}, \frac{11}{24}$.
40. $\frac{3}{16}, \frac{1}{24}$.
41. $\frac{5}{16}, \frac{5}{24}$.
42. $\frac{7}{16}, \frac{11}{24}$.
43. $\frac{9}{16}, \frac{13}{24}$.
44. $\frac{1}{2}, \frac{1}{48}$.
45. $\frac{3}{4}, \frac{1}{48}$.
46. $\frac{3}{4}, \frac{15}{28}$.
47. $\frac{3}{8}, \frac{13}{48}$.
48. $\frac{5}{8}, \frac{17}{48}$.
49. $\frac{7}{8}, \frac{23}{48}$.
50. $\frac{1}{16}, \frac{17}{48}$.
51. $\frac{7}{16}, \frac{31}{48}$.

65. Addition of Mixed Numbers. If Rachel needs $4\frac{2}{3}$ yd. of cloth for one piece of work, and $2\frac{2}{3}$ yd. of the same kind of cloth for another piece, how many yards should she buy?

We evidently have to add $4\frac{2}{3}$ yd. and $2\frac{2}{3}$ yd.

$$\boxed{\begin{array}{r} 4\frac{2}{3} \\ \underline{2\frac{2}{3}} \\ 7\frac{1}{3} \end{array}}$$

First adding the thirds we have

$$\frac{2}{3}+\frac{2}{3}=\frac{4}{3}=1\frac{1}{3}.$$

We write the $\frac{1}{3}$ in the fractions' column and add the 1 to the next column.

We then have $1+2+4=7$, and we write the 7 in the units' column.

The sum is $7\frac{1}{3}$, and so Rachel should buy $7\frac{1}{3}$ yd. of cloth.

WRITTEN EXERCISE

1. Robert is putting some picture molding around a room. The four sides of the room are $14\frac{1}{4}$ ft., 12 ft., $14\frac{1}{4}$ ft., and 12 ft. How much molding does he need?

2. One of Mary's books weighs $1\frac{1}{4}$ lb. and another weighs $\frac{3}{4}$ lb. How much do the two together weigh?

Add the following:

3.	**4.**	**5.**	**6.**	**7.**	**8.**
$12\frac{1}{2}$	$24\frac{1}{3}$	$12\frac{2}{3}$	$16\frac{2}{3}$	$28\frac{1}{3}$	$17\frac{2}{3}$
$14\frac{1}{2}$	$16\frac{1}{3}$	$3\frac{2}{3}$	$14\frac{2}{3}$	$14\frac{2}{3}$	$4\frac{2}{3}$
$\underline{16\frac{1}{2}}$	$\underline{17\frac{2}{3}}$	$\underline{4\frac{2}{3}}$	$\underline{15\frac{2}{3}}$	$\underline{15\frac{2}{3}}$	$\underline{8\frac{2}{3}}$

9.	**10.**	**11.**	**12.**	**13.**	**14.**
$41\frac{1}{2}$	$62\frac{1}{2}$	$39\frac{1}{2}$	$37\frac{1}{3}$	$57\frac{1}{3}$	$15\frac{2}{3}$
27	$17\frac{1}{2}$	$12\frac{1}{2}$	$29\frac{1}{3}$	$22\frac{2}{3}$	$4\frac{1}{3}$
$33\frac{1}{2}$	12	$33\frac{1}{2}$	16	$33\frac{2}{3}$	$17\frac{1}{3}$
$\underline{42}$	$\underline{15\frac{1}{2}}$	$\underline{40\frac{1}{2}}$	$\underline{4\frac{1}{3}}$	$\underline{4\frac{1}{3}}$	$\underline{9\frac{2}{3}}$

66. Addition of Mixed Numbers Continued. Sarah, Helen, and Emily are making dolls' dresses. Sarah uses $2\frac{1}{2}$ yd. of cloth, Helen $4\frac{2}{3}$ yd., and Emily $3\frac{1}{10}$ yd. How much do they use in all?

We see that we must add $2\frac{1}{2}$, $4\frac{2}{3}$, and $3\frac{1}{10}$.

The L.C.D. must contain the factor 2 because of the fraction $\frac{1}{2}$, the factor 3 because of the fraction $\frac{2}{3}$, and the factors 2 and 5 because of the fraction $\frac{1}{10}$. The L.C.D. must therefore contain the factors 2, 3, and 5, and so the L.C.D. is 30.

Reducing each fraction to a fraction with denominator 30, we have

$2\frac{15}{30} + 4\frac{20}{30} + 3\frac{3}{30} = 9\frac{38}{30} = 10\frac{8}{30} = 10\frac{4}{15}$,

and so the girls use $10\frac{4}{15}$ yd. of cloth.

$$\boxed{\begin{array}{rl} 2\frac{1}{2} = & 2\frac{15}{30} \\ 4\frac{2}{3} = & 4\frac{20}{30} \\ 3\frac{1}{10} = & \underline{3\frac{3}{30}} \\ & 9\frac{38}{30} = 10\frac{4}{15} \end{array}}$$

To add mixed numbers, add separately the integers and the fractions and find the sum of the results. Then reduce the fraction in this sum to lowest terms.

WRITTEN EXERCISE

1. A teacher's desk is $4\frac{1}{3}$ ft. long and $3\frac{1}{12}$ ft. wide. In planning some molding around the desk we wish to find the perimeter. What is the perimeter?

2. Some boys built a hut $7\frac{3}{4}$ ft. long and $5\frac{3}{8}$ ft. wide, and wish to find the perimeter. What is the perimeter?

3. The three sides of a triangular flower bed are $3\frac{2}{3}$ ft., $4\frac{1}{2}$ ft., and $4\frac{5}{16}$ ft. What is the perimeter?

4. If a boy weighs $56\frac{7}{8}$ lb., his books $1\frac{3}{4}$ lb., and his dog $20\frac{1}{2}$ lb., how much do all together weigh?

Add the following:

5. $1\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.
6. $2\frac{1}{3} + \frac{1}{6} + \frac{1}{2}$.
7. $3\frac{1}{2} + 2\frac{3}{4} + 1\frac{5}{8}$.
8. $1\frac{2}{3} + 2\frac{5}{6} + \frac{1}{2}$.
9. $3\frac{1}{4} + 4\frac{1}{8} + 3\frac{1}{16}$.
10. $2\frac{3}{4} + 1\frac{7}{8} + 4\frac{1}{16}$.

67. Subtraction of Fractions. Maud has $\frac{1}{2}$ yd. of lace, and in trimming a doll's waist she uses $\frac{3}{10}$ yd. How much lace has she left?

We see that we must subtract $\frac{3}{10}$ from $\frac{1}{2}$. The two denominators are 2 and 10, and 2 is a factor of 10. Therefore 10 is the L.C.D. of the fractions. Then we have

$$\frac{1}{2} = \frac{5}{10},$$

and $$\frac{5}{10} - \frac{3}{10} = \frac{2}{10} = \frac{1}{5}.$$

$$\boxed{\begin{array}{r} \frac{1}{2} = \frac{5}{10} \\ \frac{3}{10} = \underline{\frac{3}{10}} \\ \frac{2}{10} = \frac{1}{5} \end{array}}$$

Hence the difference is $\frac{1}{5}$, and so Maud has $\frac{1}{5}$ yd. of lace left.

To subtract fractions, reduce them to fractions having the least common denominator, and subtract the numerators for the numerator of the difference. Then reduce to lowest terms.

In practical work we rarely have to add or subtract fractions which have very large denominators, and the examples given in this book are no more difficult than the hardest one that we are likely to meet.

WRITTEN EXERCISE

1. If Kate has $\frac{7}{8}$ yd. of ribbon and uses $\frac{1}{2}$ yd. in trimming some sleeves, how much has she left?

2. If you live $\frac{7}{8}$ mi. from school, how far have you still to go when you have gone $\frac{3}{4}$ mi. of the distance?

Subtract the following:

3. $\frac{1}{2} - \frac{5}{12}$.
4. $\frac{7}{8} - \frac{1}{16}$.
5. $\frac{3}{8} - \frac{1}{4}$.
6. $\frac{2}{3} - \frac{5}{12}$.
7. $\frac{3}{4} - \frac{3}{16}$.
8. $\frac{5}{8} - \frac{1}{2}$.
9. $\frac{7}{8} - \frac{5}{24}$.
10. $\frac{3}{4} - \frac{7}{12}$.
11. $\frac{5}{12} - \frac{1}{4}$.
12. $\frac{1}{4} - \frac{1}{12}$.
13. $\frac{7}{16} - \frac{1}{8}$.
14. $\frac{3}{4} - \frac{1}{8}$.
15. $\frac{3}{4} - \frac{2}{3}$.
16. $\frac{1}{5} - \frac{1}{6}$.
17. $\frac{3}{5} - \frac{1}{2}$.
18. $\frac{5}{12} - \frac{3}{8}$.
19. $\frac{9}{10} - \frac{3}{5}$.
20. $\frac{5}{8} - \frac{1}{12}$.
21. $\frac{7}{8} - \frac{1}{4}$.
22. $\frac{7}{8} - \frac{2}{3}$.
23. $\frac{2}{3} - \frac{1}{2}$.
24. $\frac{7}{8} - \frac{1}{2}$.
25. $\frac{5}{8} - \frac{3}{32}$.
26. $\frac{9}{10} - \frac{1}{2}$.

68. Subtracting a Mixed Number. Albert has a board 12 in. wide from which he saws a strip $3\frac{2}{3}$ in. wide. How wide is the part of the board that is left?

We see that we must subtract $3\frac{2}{3}$ from 12.

We see that we cannot take $\frac{2}{3}$ from nothing, although we can evidently take $3\frac{2}{3}$ from 12.

We therefore think of 12 as $11\frac{3}{3}$, which we can do because $\frac{3}{3}=1$. We then have

$$\frac{3}{3}-\frac{2}{3}=\frac{1}{3},$$

and $$11-3=8.$$

$$\boxed{\begin{array}{r} 12 \\ 3\frac{2}{3} \\ \hline 8\frac{1}{3} \end{array}}$$

Therefore the difference between 12 and $3\frac{2}{3}$ is $8\frac{1}{3}$, and so the width of the board that Albert has left is $8\frac{1}{3}$ in.

We check the work by seeing that $8\frac{1}{3}+3\frac{2}{3}=12$.

WRITTEN EXERCISE

1. Fred is making a bird house. He has a piece of wood 18 in. long, and from this he saws a piece $6\frac{3}{4}$ in. long. How long is the piece which remains?

2. Alice has a piece of ribbon 22 in. long. She uses $14\frac{1}{2}$ in. in trimming a bag. How much ribbon has she left?

3. If you start out on an automobile trip of 38 mi., and stop to call on a friend who lives $7\frac{1}{4}$ mi. along the road, how much farther have you to go after leaving your friend?

Subtract the following:

4. $8-2\frac{2}{3}$.
5. $6-4\frac{3}{8}$.
6. $8-3\frac{2}{5}$.
7. $7-6\frac{7}{8}$.
8. $6-2\frac{3}{5}$.
9. $27-15\frac{3}{8}$.
10. $14-12\frac{3}{5}$.
11. $16-4\frac{5}{16}$.
12. $18-3\frac{7}{16}$.
13. $20-5\frac{9}{16}$.
14. 5 in. $-\ 3\frac{3}{4}$ in.
15. 7 in. $-\ 6\frac{3}{32}$ in.
16. 6 in. $-\ 1\frac{7}{32}$ in.
17. 8 in. $-\ 2\frac{11}{32}$ in.
18. 9 in. $-\ 3\frac{1}{64}$ in.

69. Subtracting from a Mixed Number. From a mixed number we may have to subtract a whole number, a fraction, or a mixed number.

1. From $17\frac{1}{3}$ to subtract 8.

We have $\frac{1}{3} - 0 = \frac{1}{3}$, and $17 - 8 = 9$. Therefore the result is $9\frac{1}{3}$.

$$\begin{array}{r} 17\frac{1}{3} \\ 8 \\ \hline 9\frac{1}{3} \end{array}$$

2. From $17\frac{1}{3}$ to subtract $\frac{5}{6}$.

We may think of $17\frac{2}{6}$ as $16\frac{8}{6}$, and $16\frac{8}{6} - \frac{5}{6} = 16\frac{3}{6} = 16\frac{1}{2}$.

$$\begin{array}{r} 17\frac{1}{3} = 17\frac{2}{6} \\ \frac{5}{6} \\ \hline 16\frac{1}{2} \end{array}$$

3. From $17\frac{1}{3}$ to subtract $8\frac{5}{6}$.

$$17\frac{1}{3} - 8\frac{5}{6} = 17\frac{2}{6} - 8\frac{5}{6}.$$

We may now think of $17\frac{2}{6}$ as $16\frac{8}{6}$, and $16\frac{8}{6} - 8\frac{5}{6} = 8\frac{3}{6} = 8\frac{1}{2}$.

WRITTEN EXERCISE

Subtract the following:

1. $91\frac{7}{8} - 19$.
2. $3\frac{1}{8} - \frac{3}{4}$.
3. $9\frac{1}{2} - 6\frac{2}{3}$.
4. $83\frac{5}{8} - 27$.
5. $5\frac{3}{8} - \frac{1}{2}$.
6. $8\frac{1}{4} - 5\frac{7}{8}$.
7. $120\frac{3}{4} - 69$.
8. $7\frac{1}{12} - \frac{3}{8}$.
9. $15\frac{3}{5} - 9\frac{3}{8}$.
10. $103\frac{5}{6} - 78$.
11. $8\frac{5}{12} - \frac{7}{8}$.
12. $16\frac{1}{3} - 7\frac{3}{4}$.
13. $200\frac{7}{12} - 96$.
14. $6\frac{2}{3} - \frac{4}{5}$.
15. $25\frac{1}{5} - 16\frac{7}{8}$.

16. If one book is $7\frac{1}{4}$ in. long and another is $6\frac{15}{16}$ in. long, what is the difference in their lengths?

17. If we have a piece of board $17\frac{1}{8}$ in. long and we need a piece only $15\frac{3}{4}$ in. long, how much shall we cut off?

18. If a lady has $14\frac{1}{4}$ yd. of cloth and uses $8\frac{3}{4}$ yd., how much cloth has she left?

19. How much must you add to $26\frac{7}{8}$ in. to make a yard?

20. How much must you take from 1 yd. to leave $16\frac{3}{8}$ in.?

WRITTEN EXERCISE

Add or subtract as indicated:

1. $2\frac{1}{2} + 3\frac{1}{4} + 4\frac{3}{4}$.

2. $2\frac{1}{4} + 9\frac{3}{4} - \frac{1}{8}$.

3. $3\frac{1}{2} + 5\frac{1}{4} + 5\frac{3}{4}$.

4. $3\frac{1}{4} + 7\frac{1}{8} - 2\frac{1}{3}$.

5. $5\frac{1}{2} + 6\frac{3}{4} + 4\frac{3}{4}$.

6. $5\frac{1}{4} + 6\frac{1}{8} - 3\frac{2}{3}$.

7. $8\frac{1}{2} + 3\frac{3}{8} + 9\frac{3}{4}$.

8. $8\frac{7}{8} + 5\frac{3}{8} - 2\frac{3}{4}$.

9. $4\frac{1}{2} + 4\frac{7}{8} + 8\frac{3}{4}$.

10. $7\frac{7}{8} + 3\frac{5}{8} - 4\frac{3}{4}$.

11. $5\frac{1}{2} + 7\frac{5}{8} + 11\frac{1}{3}$.

12. $8\frac{7}{8} + 5\frac{2}{3} - 7\frac{1}{4}$.

13. $7\frac{2}{3} + 8\frac{1}{2} + 12\frac{1}{4}$.

14. $15\frac{1}{2} - 2\frac{1}{4} - 1\frac{2}{3}$.

15. $5\frac{5}{6} + 12\frac{2}{3} + 21\frac{1}{4}$.

16. $22\frac{3}{4} - 3\frac{3}{8} - 2\frac{1}{2}$.

17. $7\frac{1}{6} + 18\frac{4}{5} + 23\frac{3}{8}$.

18. $17\frac{5}{6} - 2\frac{1}{12} - 3\frac{2}{3}$.

19. $8\frac{5}{6} + 21\frac{1}{12} + 20\frac{3}{4}$.

20. $23\frac{5}{6} - 3\frac{5}{12} - 2\frac{2}{3}$.

21. $7\frac{1}{6} + 18\frac{1}{12} + 22\frac{3}{4}$.

22. $33\frac{1}{6} - 2\frac{7}{12} - 3\frac{3}{4}$.

23. $9\frac{5}{6} + 15\frac{1}{12} + 13\frac{1}{3}$.

24. $41\frac{1}{6} - 3\frac{1}{12} - 2\frac{1}{3}$.

25. How much greater is $57\frac{1}{4}$ mi. than $29\frac{7}{8}$ mi.?

26. How much less is $36\frac{7}{8}$ mi. than $90\frac{1}{4}$ mi.?

27. If a room is $16\frac{1}{3}$ ft. long and $14\frac{1}{2}$ ft. wide, what is the perimeter of the room?

28. From a board 16 ft. long a man saws off a piece $2\frac{1}{4}$ ft. long and another piece $3\frac{1}{2}$ ft. long. How long is the remaining part?

29. A boy puts in his basket $3\frac{1}{2}$ lb. of meat, 2 lb. of sugar, and $1\frac{1}{4}$ lb. of figs. The basket weighs $2\frac{1}{3}$ lb. What is the weight of the basket and contents?

30. If a piece of ribbon $3\frac{3}{4}$ yd. long has another piece sewed to it so as to increase the total length by $3\frac{1}{2}$ yd., and then $5\frac{1}{4}$ yd. is cut off and used for trimming a hat, how many yards of ribbon are left?

WRITTEN EXERCISE

1. Frank is $4\frac{1}{8}$ ft. tall, and Rob lacks 4 in. of being as tall as Frank. How tall is Rob?

2. John is $11\frac{2}{3}$ yr. old, Mary is $\frac{1}{2}$ yr. older, and Tom is $1\frac{1}{4}$ yr. younger than Mary. How old is Tom?

3. If my finger nail is $\frac{7}{16}$ in. long, and the rest of my finger is $2\frac{3}{4}$ in. long, what is the total length of my finger?

4. If you weigh $56\frac{3}{4}$ lb., and your dog weighs $18\frac{1}{8}$ lb., and your books weigh $3\frac{1}{2}$ lb., what is the total weight?

5. Harry had a kite string $198\frac{2}{3}$ yd. long. His kite was up in a shower and the string shrank $1\frac{7}{8}$ yd. How long was it then?

6. A plate of glass $23\frac{1}{4}$ in. by $37\frac{1}{8}$ in. was set in a picture frame that covered it $\frac{3}{8}$ in. from each edge. What are the dimensions of the glass not covered by the frame?

7. In making a dress ruffle $3\frac{7}{8}$ in. wide, enough cloth must be allowed to turn in $\frac{7}{8}$ in. on one side and $1\frac{1}{4}$ in. on the other. What is the total width of cloth needed?

8. In running a gas pipe into a schoolroom the gas fitter found he needed three pieces of pipe of these lengths: 6 ft. $2\frac{1}{2}$ in., 5 ft. $3\frac{7}{8}$ in., and 10 ft. 2 in. He also needed a fourth piece half as long as the third one. What was the total length required?

9. A tenpenny nail is $2\frac{3}{4}$ in. long, and a fourpenny nail is $1\frac{3}{8}$ in. shorter; how long is a fourpenny nail? A sixpenny nail is $\frac{5}{8}$ in. longer than a fourpenny nail; how long is a sixpenny nail?

10. Five cans of sirup were measured very carefully and the first was found to contain $2\frac{1}{4}$ qt., the second $2\frac{3}{8}$ qt., the third $2\frac{7}{16}$ qt., the fourth $2\frac{5}{32}$ qt., and the fifth $2\frac{2}{3}$ qt. How much sirup was there in all?

From time to time teachers should read the note at the bottom of page 26 and remind their pupils of its importance.

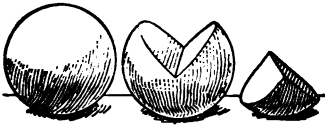
ORAL EXERCISE

1. How much is $\frac{1}{2}$ of \$4? $\frac{1}{2}$ of 4 fifths? $\frac{1}{2}$ of $\frac{4}{7}$?

2. How much is $\frac{1}{3}$ of 6 bu.? $\frac{1}{3}$ of 6 sevenths? $\frac{1}{3}$ of $\frac{6}{11}$?

3. How much is $\frac{1}{4}$ of $\frac{16}{17}$? $\frac{1}{4}$ of $\frac{8}{15}$? $\frac{1}{4}$ of $\frac{24}{25}$? $\frac{1}{3}$ of $\frac{6}{5}$?

4. In each of these cases in which you found a fraction of a fraction, how did you proceed?



5. In the above figure, *A* represents a whole sphere. What represents $\frac{3}{4}$ of a sphere? What represents $\frac{1}{3}$ of $\frac{3}{4}$ of a sphere?

State the values of the following:

6. $\frac{1}{2}$ of $\frac{2}{3}$ $\frac{1}{5}$ of $\frac{15}{16}$ $\frac{1}{6}$ of $\frac{12}{13}$ $\frac{1}{2}$ of $\frac{8}{9}$

7. $\frac{1}{8}$ of $\frac{16}{21}$ $\frac{1}{9}$ of $\frac{18}{25}$ $\frac{1}{10}$ of $\frac{20}{21}$ $\frac{1}{12}$ of $\frac{24}{25}$

8. Draw on the blackboard a rectangle $2\frac{1}{2}$ ft. long and 1 ft. high; mark it off into fifths; mark off each fifth into halves. Then how much is $\frac{1}{2}$ of $\frac{1}{5}$? How much is $\frac{1}{5}$ of $\frac{1}{2}$?

9. Draw another rectangle as in Ex. 8, marking off each fifth into thirds. How much is $\frac{1}{3}$ of $\frac{1}{5}$?

State the values of the following:

10. $\frac{1}{3}$ of $\frac{1}{5}$ $\frac{1}{5}$ of $\frac{1}{3}$ $\frac{1}{5}$ of $\frac{1}{4}$ $\frac{1}{4}$ of $\frac{1}{5}$

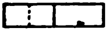
11. $\frac{1}{2}$ of $\frac{1}{5}$ $\frac{1}{2}$ of $\frac{1}{10}$ $\frac{1}{3}$ of $\frac{1}{10}$ $\frac{1}{2}$ of $\frac{1}{3}$

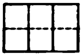
12. In Exs. 10 and 11 what did you do to the two denominators?

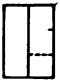
Pages 68 and 69 contain development work for the multiplication of fractions, which begins on page 70.

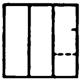
ORAL EXERCISE

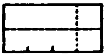
Draw these figures on the blackboard, and then answer the following questions:

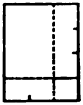

A


B


C


D


E


F

1. What part of *A* is $\frac{1}{2}$ of $\frac{1}{2}$ of *A*?

2. What part of *B* is $\frac{1}{2}$ of $\frac{1}{3}$ of *B*?

3. Show that $\frac{1}{2}$ of $\frac{1}{3}$ of *B* is the same as $\frac{1}{3}$ of $\frac{1}{2}$ of *B*.

4. What part of *C* is $\frac{1}{3}$ of $\frac{1}{2}$ of *C*?

5. What part of *D* is $\frac{1}{3}$ of $\frac{1}{3}$ of *D*?

6. Point to $\frac{1}{2}$ of *E*; to $\frac{1}{4}$ of $\frac{1}{2}$ of *E*. Also point to $\frac{1}{2}$ of $\frac{1}{4}$ of *E*. What part of *E* is $\frac{1}{2}$ of $\frac{1}{4}$ of *E*, or $\frac{1}{4}$ of $\frac{1}{2}$ of *E*?

7. Point to $\frac{1}{2}$ of $\frac{3}{4}$ of *E*; to $\frac{3}{4}$ of $\frac{1}{2}$ of *E*. What part of *E* is $\frac{3}{4}$ of $\frac{1}{2}$ of *E*?

8. Point to $\frac{1}{3}$ of *F*; to $\frac{1}{4}$ of $\frac{1}{3}$ of *F*. Also point to $\frac{1}{4}$ of *F*, and to $\frac{1}{3}$ of $\frac{1}{4}$ of *F*. Each result is what part of *F*?

WRITTEN EXERCISE

Draw rectangles to show the following:

1. $\frac{1}{5}$ of $\frac{1}{3}$, and $\frac{1}{3}$ of $\frac{1}{5}$.
2. $\frac{2}{3}$ of $\frac{1}{5}$, and $\frac{1}{5}$ of $\frac{2}{3}$.
3. $\frac{2}{5}$ of $\frac{2}{3}$, and $\frac{2}{3}$ of $\frac{2}{5}$.
4. $\frac{1}{8}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{1}{8}$.
5. $\frac{5}{8}$ of $\frac{1}{5}$, and $\frac{1}{5}$ of $\frac{5}{8}$.
6. $\frac{1}{10}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{1}{10}$.
7. $\frac{1}{2}$ of $\frac{1}{6}$, and $\frac{1}{6}$ of $\frac{1}{2}$.
8. $\frac{1}{2}$ of $\frac{5}{6}$, and $\frac{5}{6}$ of $\frac{1}{2}$.
9. $\frac{4}{5}$ of $\frac{1}{6}$, and $\frac{1}{6}$ of $\frac{4}{5}$.
10. $\frac{3}{8}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{3}{8}$.
11. $\frac{2}{5}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{2}{5}$.
12. $\frac{3}{10}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{3}{10}$.

ORAL EXERCISE

1. How much is 2 times 3 in.? 2 times $\frac{3}{7}$? 5 times $\frac{2}{11}$? How did you multiply these fractions by an integer?

2. How much is 4 times $\frac{1}{4}$? 4 times $\frac{3}{4}$? 2 times $\frac{3}{4}$? 5 times $\frac{7}{40}$? 7 times $\frac{4}{49}$? How did you multiply these fractions by an integer?

70. Multiplication of a Fraction by a Whole Number. From the answers to the above questions we see that the following is true:

To multiply a fraction by a whole number, multiply the numerator by the multiplier.

Before actually multiplying, indicate the multiplication and cancel common factors if possible.

For example, to multiply $\frac{11}{63}$ by 18. Since we have 11 sixty-thirds, if we have 18 times as much we shall have

$$\frac{\overset{2}{\cancel{18}} \times 11}{\underset{7}{\cancel{63}}}, \text{ or } \frac{22}{7}, \text{ or } 3\frac{1}{7}.$$

Multiply the following, using cancellation whenever possible:

3. $\frac{5}{9}$ by 9.
4. $\frac{3}{4}$ by 2.
5. $\frac{3}{8}$ by 16.
6. $\frac{5}{8}$ by 40.
7. $\frac{5}{8}$ by 80.
8. $\frac{3}{8}$ by 8.
9. $\frac{4}{5}$ by 10.
10. $\frac{7}{8}$ by 24.
11. $\frac{4}{5}$ by 5.
12. $\frac{7}{8}$ by 4.
13. $\frac{5}{8}$ by 24.
14. $\frac{5}{8}$ by 48.
15. $\frac{9}{10}$ by 5.
16. $\frac{3}{10}$ by 5.
17. $\frac{3}{16}$ by 48.
18. $\frac{7}{12}$ by 24.

WRITTEN EXERCISE

Multiply the following, using cancellation whenever possible:

1. $\frac{4}{25}$ by 35.
2. $\frac{5}{48}$ by 84.
3. $\frac{7}{72}$ by 16.
4. $\frac{7}{16}$ by 24.
5. $\frac{7}{32}$ by 28.
6. $\frac{5}{12}$ by 132.
7. $\frac{11}{64}$ by 36.
8. $\frac{7}{64}$ by 96.
9. $\frac{7}{36}$ by 27.
10. $\frac{5}{48}$ by 64.
11. $\frac{9}{32}$ by 48.
12. $\frac{11}{64}$ by 192.

ORAL EXERCISE

1. How much is $\frac{1}{2}$ of 6? $\frac{1}{2}$ of 60? $\frac{1}{2}$ of 600?

2. How much is $\frac{1}{3}$ of 6? $\frac{2}{3}$ of 6? $\frac{1}{3}$ of 60? $\frac{2}{3}$ of 60?

3. How much is $\frac{1}{4}$ of 8? $\frac{3}{4}$ of 8? $\frac{1}{4}$ of 80? $\frac{3}{4}$ of 80?

4. How much is $\frac{1}{5}$ of 25? $\frac{2}{5}$ of 25? $\frac{3}{5}$ of 25? $\frac{4}{5}$ of 25?

Find the following parts of numbers:

5. $\frac{1}{8}$ of 24 $\frac{3}{8}$ of 24 $\frac{5}{8}$ of 24 $\frac{7}{8}$ of 24
6. $\frac{1}{6}$ of 30 $\frac{5}{6}$ of 30 $\frac{1}{6}$ of 48 $\frac{5}{6}$ of 48
7. $\frac{1}{9}$ of 72 $\frac{2}{9}$ of 72 $\frac{4}{9}$ of 72 $\frac{5}{9}$ of 72

8. How do you find a fractional part of a number?

WRITTEN EXERCISE

1. Find $\frac{1}{8}$ of 736. Then find $\frac{3}{8}$ of 736.

2. Find $\frac{1}{12}$ of 1728. Then find $\frac{5}{12}$ of 1728.

Find the following parts of numbers:

3. $\frac{3}{8}$ of 1768 $\frac{5}{8}$ of 1768 $\frac{5}{8}$ of 2576
4. $\frac{7}{8}$ of 2576 $\frac{1}{6}$ of 1740 $\frac{5}{6}$ of 1752
5. $\frac{5}{6}$ of 1764 $\frac{5}{6}$ of 1776 $\frac{1}{16}$ of 3376
6. $\frac{3}{8}$ of 3376 $\frac{5}{8}$ of 3392 $\frac{7}{16}$ of 3408
7. $\frac{5}{8}$ of 6752 $\frac{5}{8}$ of 6784 $\frac{5}{32}$ of 6816

8. A man has to travel 117 mi. After traveling $\frac{5}{9}$ of the distance, how many miles has he still to travel?

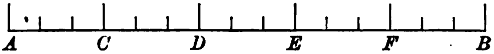
9. A man owns 176 head of cattle. If he sells $\frac{3}{8}$ of them, how many does he sell? How many has he left?

10. A man owns 336 acres of land. If he sells $\frac{3}{8}$ of it, how many acres does he sell? How many acres has he left?

71. Multiplication by a Fraction. To take a fractional part of a number is called *multiplying by a fraction.*

We write "$\frac{1}{2}$ of 4" or "$\frac{1}{2} \times 4$," the × being read "of."

72. Multiplication of a Fraction by a Fraction. Suppose that we have to multiply $\frac{4}{5}$ by $\frac{2}{3}$.



In this figure let the line *AB* be divided into five equal parts, and let each of the five parts be divided into three equal parts. Then each part evidently is $\frac{1}{15}$ of *AB*.

That is, $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$ of the whole line.

Then $\frac{1}{3}$ of $\frac{4}{5}$ must be 4 times as much, or $\frac{4}{15}$,

and $\frac{2}{3}$ of $\frac{4}{5}$ must be twice $\frac{4}{15}$, or $\frac{8}{15}$.

Therefore, *to multiply a fraction by a fraction, find the product of the numerators for the required numerator and the product of the denominators for the required denominator.*

Always cancel if possible. For example,

$$\frac{11}{12} \text{ of } \frac{15}{22} = \frac{\cancel{11} \times \overset{5}{\cancel{15}}}{\underset{4}{\cancel{12}} \times \underset{2}{\cancel{22}}} = \frac{5}{8}.$$

WRITTEN EXERCISE

Multiply the following:

1. $\frac{3}{4} \times \frac{8}{9}$.
2. $\frac{2}{3} \times \frac{9}{10}$.
3. $\frac{2}{3} \times \frac{15}{16}$.
4. $\frac{2}{3} \times \frac{11}{24}$.
5. $\frac{2}{3} \times \frac{25}{32}$.
6. $\frac{3}{4} \times \frac{5}{24}$.
7. $\frac{1}{8} \times \frac{4}{5}$.
8. $\frac{3}{8} \times \frac{7}{8}$.
9. $\frac{7}{8} \times \frac{4}{7}$.
10. $\frac{5}{8} \times \frac{3}{10}$.
11. $\frac{3}{16} \times \frac{8}{9}$.
12. $\frac{7}{16} \times \frac{8}{21}$.
13. $\frac{1}{32} \times \frac{4}{5}$.
14. $\frac{1}{48} \times \frac{24}{25}$.
15. $\frac{7}{48} \times \frac{16}{21}$.
16. $\frac{1}{2} \times \frac{4}{5}$.
17. $\frac{1}{5} \times \frac{5}{8}$.
18. $\frac{2}{5} \times \frac{5}{8}$.
19. $\frac{4}{5} \times \frac{5}{8}$.
20. $\frac{7}{32} \times \frac{16}{21}$.

Find the products of:

21. $\frac{3}{5} \times \frac{5}{48}$.
22. $\frac{4}{5} \times \frac{25}{48}$.
23. $\frac{5}{9} \times \frac{81}{100}$.
24. $\frac{5}{8} \times \frac{24}{25}$.
25. $\frac{3}{8} \times \frac{16}{21}$.
26. $\frac{7}{8} \times \frac{32}{35}$.
27. $\frac{9}{16} \times \frac{4}{9}$.
28. $\frac{7}{16} \times \frac{5}{7}$.
29. $\frac{5}{16} \times \frac{32}{45}$.
30. $\frac{13}{16} \times \frac{2}{3}$.
31. $\frac{15}{16} \times \frac{2}{5}$.
32. $\frac{7}{12} \times \frac{4}{9}$.
33. $\frac{5}{12} \times \frac{3}{5}$.
34. $\frac{11}{12} \times \frac{4}{5}$.
35. $\frac{11}{16} \times \frac{4}{11}$.
36. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$.
37. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$.
38. $\frac{2}{3} \times \frac{2}{3} \times \frac{3}{4}$.
39. $\frac{3}{5} \times \frac{3}{4} \times \frac{1}{8}$.
40. $\frac{3}{4} \times \frac{3}{8} \times \frac{4}{5}$.
41. $\frac{5}{8} \times \frac{7}{8} \times \frac{3}{8}$.

A few examples with denominators larger than 10 are given only for drill.

42. If an express train runs 1 mi. in $\frac{3}{4}$ min., how long will it take the train to run $\frac{7}{8}$ mi. at the same rate?

43. If a train runs $\frac{1}{3}$ of a certain distance in 1 hr., what part of the distance can it run in $\frac{3}{4}$ hr.?

44. If the distance to a certain house is $\frac{3}{8}$ mi., what fraction of a mile has a boy gone when he has covered $\frac{5}{16}$ of the distance?

45. If it takes a train $\frac{3}{4}$ hr. to reach a certain station, what fraction of an hour will it take to cover $\frac{3}{5}$ of the distance?

46. If you live $\frac{4}{5}$ mi. from school, and Jack lives $\frac{1}{8}$ as far, how far does he live from school?

47. If $\frac{3}{4}$ yd. of lace is needed to trim two neckbands, how much lace will be needed to trim one neckband?

48. If an automobile travels a mile in $\frac{49}{60}$ min. in a certain race, how long would it take to travel $\frac{5}{8}$ mi. at the same rate?

49. If Kate lives $\frac{7}{8}$ mi. from school, and Mary lives $\frac{1}{3}$ as far, how far from school does Mary live? If Sue lives $\frac{1}{2}$ as far as Mary, how far from school does Sue live?

50. A milliner used $\frac{7}{8}$ yd. of velvet for the upper brim and the crown of a hat, and $\frac{2}{5}$ as much for the lower brim. How much velvet did she use for the hat?

ORAL EXERCISE

1. How much is 4 times 5? 4 times $\frac{2}{9}$? 4 times $5\frac{2}{9}$?
2. How much is 2×8? $\frac{3}{4}$ of 8? $2\frac{3}{4}$ times 8?
3. How much is $\frac{10}{3}$ of $\frac{6}{5}$? Then how much is $3\frac{1}{3} \times 1\frac{1}{5}$?
4. How much is $\frac{15}{4}$ of $\frac{4}{3}$? Then how much is $3\frac{3}{4} \times 1\frac{1}{3}$?

73. Multiplication of Mixed Numbers. From these answers we see the truth of the following statements:

To multiply a mixed number by an integer, multiply the whole number and the fractional part separately and add the products.

For example, $6 \times 3\frac{2}{5} = 18\frac{12}{5} = 20\frac{2}{5}$.

To multiply an integer by a mixed number, multiply by the whole number and the fractional part separately and add the products.

For example, $3\frac{2}{5} \times 6 = 3 \times 6 + \frac{2}{5}$ of $6 = 18\frac{12}{5} = 20\frac{2}{5}$.

To multiply a mixed number by a mixed number, reduce each to an improper fraction and multiply the results.

For example, $5\frac{2}{5} \times 3\frac{3}{4} = \frac{27}{\cancel{5}}$ of $\frac{\overset{3}{\cancel{15}}}{4} = \frac{81}{4} = 20\frac{1}{4}$.

WRITTEN EXERCISE

Multiply the following:

1. $17 \times 2\frac{19}{34}$.
2. $27 \times 41\frac{5}{9}$.
3. $49 \times 81\frac{6}{7}$.
4. $81 \times 62\frac{5}{9}$.
5. $48 \times 37\frac{3}{10}$.
6. $26\frac{3}{4} \times 32$.
7. $37\frac{4}{5} \times 45$.
8. $29\frac{4}{9} \times 36$.
9. $49\frac{7}{8} \times 68$.
10. $25\frac{2}{3} \times 18$.
11. $47\frac{1}{10} \times 65$.
12. $26\frac{1}{2} \times 63$.
13. $35\frac{7}{8} \times 36$.
14. $6\frac{7}{8} \times 7\frac{7}{8}$.
15. $9\frac{1}{3} \times 15\frac{7}{8}$.
16. $16\frac{3}{4} \times 14\frac{2}{3}$.
17. $15\frac{5}{9} \times 14\frac{3}{7}$.
18. $14\frac{3}{4} \times 16\frac{4}{5}$.
19. $18\frac{4}{7} \times 23\frac{7}{8}$.
20. $18\frac{1}{8} \times 27\frac{3}{5}$.
21. $15\frac{3}{4} \times 28\frac{4}{9}$.

ORAL EXERCISE

1. If you have $\frac{3}{8}$ yd. of cloth and need $\frac{1}{3}$ of it for a piece of work, what part of a yard do you need?

2. If the class has $\frac{7}{8}$ gal. of milk and $\frac{4}{7}$ of it is used in one lesson, what part of a gallon is used?

Think of $\frac{4}{7} \times \frac{7}{8}$, and cancel 4 and 7 mentally.

Usually $\frac{1}{3}$ of $\frac{3}{8}$ is written $\frac{1}{3} \times \frac{3}{8}$, × being read "of" after a fraction, but "times" after a whole number or a mixed number.

3. If the distance to a certain house is $\frac{5}{16}$ mi., how far has a boy gone when he has covered $\frac{4}{5}$ of this distance?

Find the values of the following:

4. $\frac{1}{2}$ of $\frac{1}{2}$.
5. $\frac{1}{2}$ of $\frac{1}{4}$.
6. $\frac{1}{4}$ of $\frac{1}{2}$.
7. $\frac{1}{2}$ of $\frac{1}{3}$.
8. $\frac{1}{2}$ of $\frac{2}{3}$.
9. $\frac{2}{3}$ of $\frac{1}{2}$.
10. $\frac{1}{3}$ of $\frac{1}{4}$.
11. $\frac{1}{3}$ of $\frac{3}{4}$.

WRITTEN EXERCISE

Add the following:

1. $4\frac{1}{3}$, $3\frac{2}{3}$, $5\frac{1}{3}$.
2. $2\frac{1}{8}$, $1\frac{1}{8}$, $6\frac{3}{8}$.
3. $4\frac{7}{8}$, $3\frac{2}{3}$, $5\frac{11}{24}$.
4. $3\frac{1}{16}$, $5\frac{3}{32}$, $7\frac{11}{64}$.
5. $2\frac{1}{10}$, $16\frac{3}{20}$, $27\frac{7}{30}$.
6. $8\frac{7}{16}$, $15\frac{15}{32}$, $73\frac{51}{64}$.

7. The sides of a triangle are $3\frac{3}{4}$ in., $2\frac{7}{8}$ in., and $3\frac{1}{2}$ in., respectively. What is the perimeter?

Subtract the following:

8. $28\frac{5}{8} - 7\frac{25}{32}$.
9. $32\frac{7}{8} - 9\frac{31}{32}$.
10. $12\frac{1}{3} - 8\frac{1}{4}$.
11. $15\frac{1}{4} - 7\frac{1}{3}$.
12. $31\frac{5}{6} - 7\frac{7}{12}$.
13. $42\frac{5}{6} - 8\frac{5}{12}$.

14. From the sum of $16\frac{1}{2}$ ft. and $4\frac{1}{2}$ ft. subtract $12\frac{3}{4}$ ft.

Find the values of the following:

15. $\frac{1}{2}$ of $\frac{3}{4}$.
16. $\frac{3}{4}$ of $\frac{1}{2}$.
17. $\frac{5}{8}$ of $\frac{4}{5}$.
18. $\frac{3}{8}$ of $\frac{8}{21}$.
19. $\frac{13}{16}$ of $\frac{2}{3}$.
20. $\frac{15}{16}$ of $\frac{2}{5}$.
21. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.
22. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.

23. A piece of tin $\frac{1}{40}$ in. thick is folded once. How thick are both parts together?

24. A piece of paper $\frac{1}{60}$ in. thick is folded once, and then folded again, making four leaves. How thick are the four leaves together?

25. If the height of the upright piece in a sundial is $\frac{5}{32}$ of the diameter of the dial and the diameter of the dial is 8 in., what is the height of the upright piece?

26. If the length of a paddle of a water wheel is $\frac{3}{7}$ of the diameter and the diameter is 21 in., what is the length of the paddle?

27. A certain recipe calls for 8 oz. of sugar. To make $2\frac{1}{2}$ times the recipe, how many ounces of sugar must we use?

28. A certain recipe calls for 2 cups of cream. To make $3\frac{1}{2}$ times the recipe, how much cream must we use?

29. If an automobile goes a mile in 59 sec. (or $\frac{59}{60}$ min.) in a certain race, how long will it take to go $\frac{3}{4}$ mi. at the same rate?

30. A man has a city lot that contains $\frac{12}{25}$ A. If he sells $\frac{5}{8}$ of it, how much does he sell? How much has he left?

31. If it takes $2\frac{7}{8}$ yd. of cloth to make a boy's suit, how many yards will it take to make a dozen suits?

32. A tin cup is found to hold $\frac{15}{16}$ pt. When it is $\frac{2}{3}$ full the cup contains what part of a pint?

33. If a glass jar holds $\frac{15}{16}$ qt., what part of a quart will it contain when $\frac{4}{5}$ full? What part of a quart will it contain when $\frac{2}{5}$ full? when $\frac{3}{5}$ full?

34. In making the plans for a house an architect wishes to draw a line that shall be $\frac{2}{5}$ as long as one that is $\frac{15}{16}$ in. long. How long shall he draw it?

74. Cash Checks. In large stores the clerks are required to fill out cash checks like the following:

Sold by No. 127		Amount rec'd, $15.00		
½ yd.	Silk	.57		29
4⅜ "	Summer silk	.38	1	67
11½ "	Silk	.99	11	39
	Total		13	35
	Change due		1	65
			15	00

WRITTEN EXERCISE

Make out cash checks for the following sales:

1. 14½ yd. gingham @ 25¢, 9 yd. cotton @ 12½¢, 10½ yd. madras @ 35¢. Amount received, $10.

2. 6½ yd. cheviot @ $1.10, 9½ yd. velveteen @ 98¢, 5⅝ yd. silk @ $1.25. Amount received, $25.

3. 11½ yd. India linen @ 35¢, 6⅝ yd. dimity @ 25¢, 12⅛ yd. linen suiting @ 75¢. Amount received, $20.

4. 9½ yd. India linen @ 38¢, 5⅛ yd. cheviot @ $1.25, 15 yd. cotton @ 12½¢. Amount received, $15.

5. 7 yd. muslin @ 25¢, 12½ yd. lining @ 11¢, 6⅝ yd. India linen @ 45¢, ½ yd. velvet @ $3. Amount received, $10.

6. 14½ yd. muslin @ 24¢, 4⅛ yd. velvet @ $2.40, 16¾ yd. lining @ 12¢, 7⅜ yd. silk @ $1.60, 5¾ yd. suiting @ 80¢, 6½ yd. ribbon @ 30¢. Amount received, $50.

The teacher should ask the pupils to make problems similar to those given above, using the local prices of common materials.

75. Cancellation. We have seen, on page 50, that we may divide both terms of a fraction by the same number without changing the value of the fraction; that is, we may *cancel* any factor that is found in both terms.

Teachers should make it clear that this is a case of *division*; that we cannot *subtract* the same number from both terms of the fraction.

In order to divide both terms of a fraction by the same number we must know more about different divisors.

76. Common Divisor. A factor that is common to two or more numbers is called a *common divisor.*

For example, 5 is a common divisor of 15 and 25. It might be called a common factor, but common divisor is the usual name.

77. Greatest Common Divisor. The greatest factor common to two or more numbers is called their *greatest common divisor* (G.C.D.). For example, although 3 is a common divisor of 12, 18, and 48, the number 6 is the *greatest* common divisor.

78. Finding the G.C.D. The greatest common divisor of such numbers as we have met or shall meet in fractions is easily found. For example, consider 12 and 30. We see that

$$12 = 2 \times 3 \times 2, \qquad 30 = 2 \times 3 \times 5,$$

and that 2 and 3 are the only common factors. Therefore 2 × 3, or 6, is the greatest common divisor.

79. Prime Number. A number that has no factors except itself and 1 is called a *prime number.* For example, 7, 11, and 17 are prime numbers.

80. Composite Number. A number not prime is called *composite.* For example, 25, 39, and 51 are composite numbers.

We use the terms "prime" and "composite" only in connection with whole numbers.

81. Even Number. A number that contains the factor 2 is called an *even number*. For example, 20, 38, and 1112 are even numbers.

82. Odd Number. A number that is not even is called an *odd number*. For example, 21, 39, and 127 are odd numbers.

83. Prime Factor. A factor that is a prime number is called a *prime factor*. For example, the prime factors of 68 are 2, 2, and 17, and the prime factors of 25 are 5 and 5.

84. Mutually-Prime Numbers. Numbers that have no common factors are said to be *prime to each other*. For example, 12 and 35.

ORAL EXERCISE

1. What do you mean by the factors of a number?

2. Name two factors of 35; of 77; of 26; of 34.

3. Name three factors of 30; of 70; of 66; of 110.

4. Name a factor that is common to 35 and 45.

5. Name two factors common to 30 and 48.

6. Find a common divisor of 20 and 22; of 16 and 36.

7. Find the greatest common divisor of 75 and 100.

8. Which of the following numbers have no factors except 1 and themselves: 4, 5, 6, 7, 10, 11, 12, 13?

9. In reducing $\frac{12}{16}$ to lowest terms is it sufficient to cancel the common divisor 2? What kind of common divisor must be canceled in reducing a fraction to lowest terms?

10. Find the odd numbers and the prime numbers from 1 to 30.

11. Find the prime factors of 20; of 48; of 60; of 77; of 50.

12. Find the greatest common divisor of 8, 12, 20, and 24.

13. Find the greatest common divisor of 9, 15, 21, and 60.

85. Divisible Numbers. When we speak of one number being *divisible* by another we mean that the quotient is a whole number.

86. Divisibility by 2. *A number is divisible by 2 if the units are so divisible.* For example, 64, or 60 + 4, must be divisible by 2 if 4, the number of units, is divisible by 2, because any multiple of 10 is divisible by 2.

87. Divisibility by 5. *A number is divisible by 5 if it ends in 0 or 5.* For example, 180 and 275 are divisible by 5.

88. Divisibility by 3. *A number is divisible by 3 if the sum of its digits is so divisible.* For example, the sum of the digits of 411 is 6, and 6 is divisible by 3. Hence 411 is also divisible by 3.

ORAL EXERCISE

1. Is 2 a factor of 14? of 27? of 50? of 45? of 600? How do you tell whether 2 is a factor of a number?

2. Is 2 a factor of 10? of any number of 10's? Then is 2 a factor of any number if it is a factor of the units?

3. Is 5 a factor of 10? of any number of 10's? Then is 5 a factor of any number if it is a factor of the units?

WRITTEN EXERCISE

1. From this list select the numbers divisible by 2:

660 4907 6255 3027 1356 37,268

2. From this list select the numbers divisible by 3:

4867 7075 9636 8124 3672 12,475

3. From this list select the numbers divisible by 5:

1235 5778 9100 8328 3445 31,035

89. Other Tests of Divisibility. There are various tests of divisibility besides those on page 80. They are easily illustrated or explained, and may be given or not as the teacher prefers. The more important are the following:

A number is divisible by 4 if the number represented by its two right-hand figures is so divisible.

A number is divisible by 6 if it is even and if the sum of its digits is divisible by 3.

A number is divisible by 8 if the number represented by its three right-hand figures is so divisible.

A number is divisible by 9 if the sum of its digits is so divisible.

A number is divisible by 11 if the difference between the sums of the digits in its even and odd places is so divisible.

For example, 430,507 is divisible by 11, for $7+5+3=15$, $0+0+4=4$, and $15-4=11$.

ORAL EXERCISE

1. State the prime factors of 15, 18, 27, 35, 42, and 75.
2. State one factor of 395, 123, 777, 692, 1275, and 1263.
3. Is 5 a factor of 20? of 78? of 45? of 72? of 800?
4. State all the odd numbers from 10 to 30.
5. State all the prime numbers from 1 to 40.
6. Select the numbers divisible by 2, 3, and 5 respectively:
   4263 7065 2022 3708 5445 13,005
7. Select the numbers divisible by 4, 6, and 8 respectively:
   3104 3204 3304 7275 3116 54,012
8. Select the numbers divisible by 9 and 11 respectively:
   5004 1001 9009 6264 2244 12,342

ORAL EXERCISE

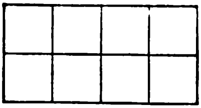
State the answers to the following:

1. 15 ft. ÷ 5 $\frac{15}{16} \div 5$ $\frac{15}{32} \div 5$ $\frac{15}{64} \div 5$

2. 25 pt. ÷ 5 $\frac{25}{24} \div 5$ $\frac{25}{8} \div 5$ $\frac{25}{32} \div 5$

3. 14 in. ÷ 7 $\frac{14}{15} \div 7$ $\frac{14}{9} \div 7$ $\frac{14}{3} \div 7$

4. Dividing the numerator of a fraction does what to the fraction itself? Give an example.



5. If you divide $\frac{1}{2}$ of this rectangle into two equal parts, each is what part of the rectangle? How much is $\frac{1}{2} \div 2$?

6. If you divide $\frac{1}{4}$ of the rectangle into two equal parts, each is what part of the rectangle? How much is $\frac{1}{4} \div 2$? What have you done to the denominator?

7. If you divide $\frac{3}{4}$ of the rectangle into two equal parts, how many eighths of the rectangle do you then have? How much is $\frac{3}{4} \div 2$? What have you done to the denominator?

8. Multiplying the denominator of a fraction does what to the fraction itself?

Divide the following fractions by dividing the numerators:

9. $\frac{4}{5} \div 2$ $\frac{15}{16} \div 3$ $\frac{15}{32} \div 5$ $\frac{25}{64} \div 5$ $\frac{21}{32} \div 7$

10. $\frac{6}{7} \div 3$ $\frac{12}{25} \div 6$ $\frac{16}{25} \div 8$ $\frac{27}{32} \div 9$ $\frac{33}{64} \div 11$

Divide the following fractions by multiplying the denominators:

11. $\frac{1}{3} \div 5$ $\frac{2}{3} \div 7$ $\frac{3}{5} \div 4$ $\frac{3}{8} \div 7$ $\frac{3}{4} \div 5$

12. $\frac{4}{5} \div 9$ $\frac{7}{8} \div 3$ $\frac{3}{8} \div 3$ $\frac{5}{8} \div 9$ $\frac{3}{5} \div 7$

Divide the following:

13. $\frac{4}{15} \div 2$ $\frac{3}{5} \div 2$ $\frac{7}{8} \div 4$ $\frac{5}{8} \div 2$ $\frac{7}{8} \div 2$

14. $\frac{15}{16} \div 10$ $\frac{25}{32} \div 5$ $\frac{5}{12} \div 4$ $\frac{15}{8} \div 5$ $\frac{7}{8} \div 5$

90. How to Divide a Fraction by a Whole Number. From the answers to the questions on page 82 we have the following:

Dividing the numerator by a whole number divides the fraction by that number.

Multiplying the denominator by a whole number divides the fraction by that number.

If we have a mixed number to be divided, we may reduce the mixed number to an improper fraction.

Thus $2\frac{1}{3} \div 3 = \frac{7}{3} \div 3 = \frac{7}{9}$.

Sometimes, however, it is easier to divide the mixed numbers. For example, to divide a number like $244\frac{1}{2}$ by 3 it is easier not to reduce to an improper fraction.

Thus $244\frac{1}{2} \div 3 = 81 + \frac{1}{3} \text{ of } 1\frac{1}{2} = 81 + \frac{1}{2} = 81\frac{1}{2}$.

WRITTEN EXERCISE

Divide the following:

1. $\frac{1}{2} \div 2 \qquad \frac{1}{3} \div 2 \qquad \frac{1}{4} \div 2 \qquad \frac{3}{4} \div 2 \qquad \frac{3}{8} \div 2$
2. $\frac{1}{2} \div 3 \qquad \frac{2}{3} \div 3 \qquad \frac{1}{4} \div 3 \qquad \frac{3}{4} \div 3 \qquad \frac{3}{8} \div 3$
3. $\frac{2}{5} \div 3 \qquad \frac{4}{5} \div 3 \qquad \frac{1}{8} \div 2 \qquad \frac{6}{5} \div 3 \qquad \frac{8}{5} \div 4$
4. $\frac{5}{8} \div 5 \qquad \frac{5}{8} \div 2 \qquad \frac{5}{8} \div 3 \qquad \frac{5}{8} \div 4 \qquad \frac{7}{8} \div 7$
5. $\frac{6}{7} \div 2 \qquad \frac{6}{7} \div 3 \qquad \frac{6}{7} \div 5 \qquad \frac{6}{7} \div 6 \qquad \frac{5}{8} \div 6$
6. $\frac{5}{8} \div 5 \qquad 3\frac{3}{4} \div 5 \qquad 7\frac{1}{2} \div 3 \qquad 4\frac{1}{5} \div 7 \qquad 4\frac{2}{3} \div 7$

Divide the whole part and the fractional part separately, and add the results:

7. $12\frac{3}{4} \div 3 \qquad 21\frac{1}{2} \div 3 \qquad 36\frac{3}{5} \div 3 \qquad 24\frac{4}{5} \div 4 \qquad 35\frac{15}{16} \div 5$
8. $23\frac{1}{3} \div 7 \qquad 25\frac{1}{2} \div 8 \qquad 28\frac{1}{8} \div 9 \qquad 28\frac{3}{4} \div 5 \qquad 134\frac{1}{2} \div 5$

ORAL EXERCISE

1. How many quarters of a dollar in $1?

2. How many thirds of an inch in 1 in.? Then $1 \div \frac{1}{3}$ is how many? What can you say about the values of $1 \div \frac{1}{3}$ and 3×1?

3. How many thirds in 1? in 2? What can you say about the values of $2 \div \frac{1}{3}$ and 3×2?

4. How many eighths of an inch in 1 in.? What can you say about the values of $1 \div \frac{1}{8}$ and 8×1?

5. How many fifths of an inch in 1 in.? Then $1 \div \frac{1}{5}$ is how many? What can you say about the values of $1 \div \frac{1}{5}$ and 5×1?

6. Tell the values of

$1 \div \frac{1}{8}$ $1 \div \frac{1}{9}$ $1 \div \frac{1}{7}$ $1 \div \frac{1}{10}$ $1 \div \frac{1}{20}$

7. How do you divide by a unit fraction?

8. If a sheet of veneer is $\frac{1}{16}$ in. thick, how many sheets pressed together will have a thickness of $\frac{3}{16}$ in.?

9. If a sheet of felt is $\frac{1}{16}$ in. thick, how many sheets are there in a pile $1\frac{1}{16}$ in. thick?

10. If a bicycle is traveling at the rate of a mile in $\frac{1}{8}$ hr., how many miles will it travel in $3\frac{1}{8}$ hr.?

11. How many weights of $\frac{1}{16}$ lb. (or 1 oz.) each will it take to weigh $2\frac{1}{16}$ lb.?

12. How many pints ($\frac{1}{8}$ gal.) are there in 3 gal.? How many pints are there in $2\frac{1}{2}$ gal.?

13. How many inches ($\frac{1}{12}$ ft.) are there in 3 ft.? How many inches are there in 10 ft.?

14. If each book is $\frac{1}{2}$ in. thick, how many books are there in a pile that is $3\frac{1}{2}$ in. high?

91. Dividing by a Unit Fraction. From the work given on page 84 it is clear that division by a unit fraction is simple.

To divide a number by a unit fraction, multiply the number by the denominator of the fraction.

Thus $1 \div \frac{1}{8} = 1 \times 8 = 8,$

$3 \div \frac{1}{8} = 3 \times 8 = 24,$

and $\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \times 8 = 6.$

WRITTEN EXERCISE

Divide the following:

1. $25 \div \frac{1}{3}$.
2. $32 \div \frac{1}{4}$.
3. $37 \div \frac{1}{5}$.
4. $43 \div \frac{1}{6}$.
5. $51 \div \frac{1}{2}$.
6. $63 \div \frac{1}{8}$.
7. $27 \div \frac{1}{2}$.
8. $36 \div \frac{1}{12}$.
9. $51 \div \frac{1}{16}$.
10. $81 \div \frac{1}{8}$.
11. $16 \div \frac{1}{4}$.
12. $23 \div \frac{1}{4}$.
13. $32 \div \frac{1}{6}$.
14. $54 \div \frac{1}{4}$.
15. $29 \div \frac{1}{3}$.
16. $48 \div \frac{1}{5}$.
17. $63 \div \frac{1}{8}$.
18. $27 \div \frac{1}{6}$.
19. $17 \div \frac{1}{12}$.
20. $97 \div \frac{1}{12}$.
21. $39 \div \frac{1}{20}$.
22. $3\frac{1}{2} \div \frac{1}{16}$.
23. $7\frac{3}{8} \div \frac{1}{32}$.
24. $9\frac{7}{16} \div \frac{1}{64}$.
25. $\frac{2}{3} \div \frac{1}{2}$.
26. $\frac{4}{5} \div \frac{1}{10}$.
27. $\frac{3}{8} \div \frac{1}{32}$.
28. $\frac{3}{4} \div \frac{1}{8}$.
29. $\frac{5}{6} \div \frac{1}{12}$.
30. $2\frac{1}{2} \div \frac{1}{2}$.

31. If a sheet of veneer is $\frac{1}{16}$ in. thick, how many sheets pressed together will have a thickness of $1\frac{1}{8}$ in.?

32. If a sheet of blotting paper is $\frac{1}{32}$ in. thick, how many sheets are there in a pile that is $15\frac{5}{8}$ in. thick?

33. How many weights of $\frac{1}{16}$ lb. (or 1 oz.) each will it take to weigh $17\frac{3}{4}$ lb.?

34. If a wagon wheel makes $\frac{1}{12}$ of a revolution in going 1 foot, how many feet will it go in making $17\frac{3}{4}$ revolutions?

ORAL EXERCISE

1. I have a room 18 ft. wide. I wish to find how many strips of carpet $\frac{3}{4}$ yd. wide will be needed to run lengthwise of the room. First I must find how many yards there are in 18 ft.

2. If I have to divide 6 yd. by $\frac{3}{4}$ yd., I may think of the 6 yd. as fourths. Express 6 as fourths.

3. If now I have to divide $\frac{24}{4}$ by $\frac{3}{4}$, I may think of the results in the cases of 24¢ ÷ 3¢, 24 ft. ÷ 3 ft., and 24 in. ÷ 3 in. What are the results?

4. Then how much is $\frac{24}{4} \div \frac{3}{4}$? How many strips of carpet shall I need to carpet the room mentioned in Ex. 1?

5. Divide $\frac{6}{5}$ by $\frac{2}{5}$. Multiply $\frac{6}{5}$ by $\frac{5}{2}$. Compare the results.

6. Show that $\frac{4}{5} \div \frac{2}{5}$ is the same as $\frac{5}{2} \times \frac{4}{5}$.

7. Study the following:

$$15 \div \frac{3}{5} = \frac{5}{\cancel{3}} \times \overset{5}{\cancel{15}} = 25.$$

8. Study the following:

$$\frac{3}{8} \div \frac{5}{16} = \frac{\overset{2}{\cancel{16}}}{5} \times \frac{3}{\cancel{8}} = \frac{6}{5} = 1\tfrac{1}{5}.$$

WRITTEN EXERCISE

1. How many paper pads, each $\frac{5}{16}$ in. thick, will it take to make a package 5 in. thick?

2. How many thicknesses of board, each $\frac{3}{4}$ in. thick, will it take to make a pile $9\frac{3}{4}$ in. high?

Divide the following:

3. $12 \div \frac{2}{3}$.
4. $14 \div \frac{2}{3}$.
5. $9 \div \frac{3}{4}$.
6. $5 \div \frac{3}{4}$.
7. $4 \div \frac{4}{5}$.
8. $\frac{5}{8} \div \frac{4}{5}$.
9. $\frac{7}{8} \div \frac{5}{8}$.
10. $5\frac{1}{8} \div \frac{1}{2}$.

92. Dividing by a Fraction. An inverted fraction is called the ***reciprocal*** of the fraction. That is, the reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$.

From page 86 we have found how to divide by a fraction. We shall now look a little more carefully at the reason. We found on page 85 that when we have to divide by $\frac{1}{3}$ we may multiply by its reciprocal, 3. We also know that when we multiply the divisor by 2 we divide the quotient by 2.

That is, $6 \div \frac{1}{3} = 3 \times 6,$

and $6 \div \frac{2}{3} = \text{half as much} = \frac{3 \times 6}{2} = 9.$

That is, $6 \div \frac{2}{3} = \frac{3}{2} \text{ of } 6.$

Therefore, *to divide by a common fraction, multiply by the reciprocal of that fraction.*

That is, $15 \div \frac{3}{5} = \frac{5}{\cancel{3}} \times \overset{5}{\cancel{15}} = 25.$

Similarly, $\frac{3}{8} \div \frac{5}{6} = \frac{\overset{3}{\cancel{6}}}{5} \times \frac{3}{\underset{4}{\cancel{8}}} = \frac{9}{20},$

and $4\frac{2}{3} \div 2\frac{4}{5} = \frac{14}{3} \div \frac{14}{5} = \frac{5}{\cancel{14}} \times \frac{\cancel{14}}{3} = \frac{5}{3} = 1\frac{2}{3}.$

WRITTEN EXERCISE

Divide the following:

1. $\frac{7}{8} \div \frac{2}{3}$.
2. $\frac{5}{6} \div \frac{3}{4}$.
3. $\frac{9}{10} \div \frac{4}{5}$.
4. $\frac{5}{12} \div \frac{7}{8}$.
5. $\frac{3}{8} \div \frac{5}{12}$.
6. $\frac{7}{16} \div \frac{3}{32}$.
7. $\frac{5}{64} \div \frac{7}{32}$.
8. $\frac{9}{24} \div \frac{5}{16}$.
9. $\frac{3}{40} \div \frac{6}{25}$.
10. $\frac{7}{48} \div \frac{7}{12}$.
11. $\frac{3}{4} \div 1\frac{1}{16}$.
12. $2\frac{7}{16} \div 3\frac{1}{8}$.
13. $9\frac{7}{12} \div 5\frac{3}{4}$.
14. $5\frac{1}{4} \div 18\frac{3}{8}$.
15. $3\frac{9}{20} \div 2\frac{3}{10}$.
16. $4\frac{5}{9} \div 13\frac{2}{3}$.
17. $6\frac{3}{10} \div 1\frac{4}{5}$.
18. $4\frac{9}{10} \div 12\frac{5}{6}$.

ORAL EXERCISE

Divide the following:

1. $\frac{1}{2} \div 2$ $\frac{1}{2} \div \frac{1}{2}$ $\frac{1}{2} \div \frac{1}{4}$ $\frac{1}{2} \div \frac{1}{8}$ $\frac{1}{2} \div \frac{1}{3}$

2. $\frac{1}{2} \div \frac{3}{4}$ $\frac{1}{2} \div \frac{3}{8}$ $\frac{1}{2} \div \frac{5}{8}$ $\frac{1}{2} \div \frac{7}{8}$ $\frac{1}{2} \div \frac{1}{16}$

3. $\frac{1}{4} \div \frac{1}{8}$ $\frac{3}{4} \div \frac{1}{8}$ $\frac{3}{4} \div \frac{5}{8}$ $\frac{3}{4} \div \frac{7}{8}$ $\frac{3}{4} \div \frac{9}{8}$

4. Divide $\frac{1}{3}$ by 2; 2 by $\frac{1}{3}$; 2 by 3; 3 by 2.

5. Divide $\frac{3}{4}$ by 3; 3 by $\frac{3}{4}$; 3 by $\frac{3}{5}$; 3 by $\frac{3}{8}$.

6. Divide $\frac{4}{5}$ by 2; 2 by $\frac{4}{5}$; 2 by $\frac{2}{3}$; $\frac{2}{3}$ by 2.

7. Divide $\frac{3}{8}$ by $\frac{5}{8}$; $\frac{5}{8}$ by $\frac{3}{8}$; $\frac{3}{4}$ by $\frac{5}{4}$; $\frac{5}{4}$ by $\frac{3}{4}$.

8. How many half-yard pieces can be made by cutting 5 yd. of ribbon? by cutting 7 yd. of ribbon?

9. If your recipe for making cake calls for $\frac{1}{2}$ lb. of sugar, 6 lb. of sugar is enough for how many times as much cake?

WRITTEN EXERCISE

Divide the following:

1. $\frac{2}{3} \div \frac{3}{4}$.
2. $\frac{3}{4} \div \frac{2}{3}$.
3. $\frac{2}{3} \div \frac{4}{5}$.
4. $\frac{4}{5} \div \frac{2}{3}$.
5. $\frac{3}{8} \div \frac{3}{4}$.
6. $\frac{3}{4} \div \frac{3}{8}$.
7. $\frac{5}{8} \div \frac{4}{5}$.
8. $\frac{4}{5} \div \frac{5}{8}$.
9. $\frac{7}{8} \div \frac{3}{4}$.
10. $\frac{3}{4} \div \frac{7}{8}$.
11. $\frac{4}{5} \div \frac{7}{8}$.
12. $\frac{7}{8} \div \frac{4}{5}$.
13. $\frac{5}{9} \div \frac{2}{3}$.
14. $\frac{2}{3} \div \frac{5}{9}$.
15. $\frac{5}{9} \div \frac{4}{5}$.
16. $\frac{4}{5} \div \frac{5}{9}$.
17. $\frac{3}{5} \div \frac{2}{3}$.
18. $\frac{2}{3} \div \frac{3}{5}$.
19. $\frac{5}{12} \div \frac{2}{3}$.
20. $\frac{2}{3} \div \frac{5}{12}$.
21. $5 \div \frac{5}{8}$.
22. $5\frac{1}{4} \div \frac{5}{8}$.
23. $3\frac{3}{4} \div \frac{7}{8}$.
24. $8\frac{1}{3} \div \frac{5}{6}$.
25. $3\frac{5}{8} \div \frac{3}{4}$.
26. $3\frac{5}{8} \div \frac{3}{5}$.
27. $4\frac{7}{8} \div \frac{3}{8}$.
28. $5\frac{4}{9} \div 2\frac{1}{3}$.

29. How many mats, each $\frac{3}{4}$ yd. long, can be made from a strip of carpet 9 yd. long, making no allowance for turning in or hemming?

30. How many towels, each $\frac{5}{8}$ yd. long, can be made from a strip of toweling $11\frac{1}{4}$ yd. long, making no allowance for hemming?

WRITTEN EXERCISE

Divide the following:

1. $21 \div 5\frac{1}{4}$.

2. $77 \div 9\frac{1}{3}$.

3. $69 \div 5\frac{3}{4}$.

4. $35 \div 5\frac{5}{8}$.

5. $21 \div 4\frac{2}{3}$.

6. $14 \div 2\frac{5}{8}$.

7. $61\frac{1}{4} \div 8\frac{3}{4}$.

8. $71 \div 8\frac{7}{8}$.

9. $17\frac{7}{8} \div 4\frac{7}{8}$.

10. $33 \div 7\frac{4}{5}$.

11. $15\frac{2}{3} \div 6\frac{1}{3}$.

12. $17\frac{1}{8} \div 2\frac{1}{2}$.

13. $12\frac{1}{2} \div 3\frac{1}{2}$.

14. $15\frac{1}{2} \div 3\frac{1}{3}$.

15. $78 \div 8\frac{17}{32}$.

16. $16\frac{4}{5} \div 4\frac{1}{5}$.

17. $79\frac{4}{5} \div 8\frac{1}{7}$.

18. $324 \div 7\frac{1}{5}$.

19. $25\frac{3}{5} \div 5\frac{1}{3}$.

20. $52\frac{2}{5} \div 8\frac{3}{16}$.

21. $61\frac{7}{8} \div 4\frac{13}{16}$.

22. $77 \div 6\frac{5}{12}$.

23. $27\frac{1}{2} \div 33\frac{1}{3}$.

24. $40\frac{3}{4} \div 6\frac{1}{3}$.

25. $28\frac{11}{16} \div 2\frac{1}{8}$.

26. $29\frac{1}{5} \div 58\frac{2}{5}$.

27. $20\frac{1}{8} \div 15\frac{1}{3}$.

28. $31\frac{5}{8} \div 15\frac{13}{16}$.

29. $13\frac{9}{10} \div 4\frac{2}{5}$.

30. $151\frac{7}{8} \div 15\frac{3}{16}$.

31. $876\frac{1}{2} \div 129\frac{5}{12}$.

32. $202\frac{1}{8} \div 14\frac{7}{16}$.

33. $171\frac{7}{8} \div 17\frac{3}{16}$.

34. How many strips of cloth, each $5\frac{3}{8}$ yd. long, can we get from a strip $37\frac{5}{8}$ yd. long?

35. How many city lots, each $31\frac{3}{4}$ ft. front, can a man get from a piece of land having $222\frac{1}{4}$ ft. front?

36. How many yards of cloth at $33\frac{1}{3}$¢ a yard can be bought for $7? for $19? for $27?

37. How many books, each $\frac{7}{8}$ in. thick, will it take to make a pile $21\frac{7}{8}$ in. high?

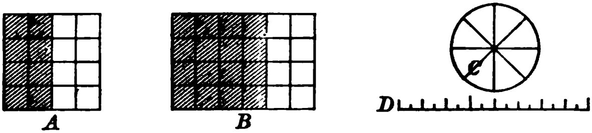
38. How many strips of wood, each $\frac{9}{16}$ in. thick, will it take to make a pile $19\frac{1}{8}$ in. high?

39. How many sheets of bookbinding board, each $\frac{3}{32}$ in. thick, will it take to make a pile $9\frac{3}{16}$ in. high?

40. If the product of two numbers is 46, and one of the numbers is $2\frac{7}{8}$, what is the other?

ORAL EXERCISE

1. In line *D*, point to $\frac{3}{16}$ of the line; also to $\frac{6}{16}$. We see that $\frac{6}{16}$ is how many times $\frac{3}{16}$? How does multiplying the numerator by 2 change the value of the fraction? How does dividing the numerator by 2 change the value of the fraction?



These figures can be used more easily if drawn on the board.

2. Point to $\frac{1}{2}$ of square *A*; also to $\frac{1}{4}$. We see that $\frac{1}{2}$ is how many times $\frac{1}{4}$? How does multiplying the denominator by 2 change the value of the fraction? How does dividing the denominator by 2 change the value of the fraction?

3. In rectangle *B*, show that $\frac{2}{3} = \frac{4}{6}$. Then does it change the value of a fraction to multiply or divide both terms by the same number? Show this with the circle *C*.

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93. Review of Important Principles. From the above exercise we again see the truth of these important principles:

Multiplying the numerator of a fraction by any number multiplies the fraction by that number.

Dividing the numerator of a fraction by any number divides the fraction by that number.

Multiplying the denominator of a fraction by any number divides the fraction by that number.

Dividing the denominator of a fraction by any number multiplies the fraction by that number.

Dividing both terms of a fraction by the same number does not change the value of the fraction.

ORAL EXERCISE

1. How many cents in \$$\frac{1}{8}$? Then 16 times \$$\frac{1}{8}$ is the same as 16 times how many cents?

2. How many cents in \$$\frac{1}{3}$? Then 9 times \$$\frac{1}{3}$ is the same as 9 times how many cents?

3. How many cents in \$$\frac{1}{6}$? Then 24 times \$$\frac{1}{6}$ is the same as 24 times how many cents?

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94. Use of Aliquot Parts in Multiplication. It is easier to multiply \$$\frac{1}{8}$, \$$\frac{1}{6}$, and \$$\frac{1}{3}$ than it is to multiply $12\frac{1}{2}$¢, $16\frac{2}{3}$¢, and $33\frac{1}{3}$¢. Such parts of a dollar or of any other unit are often called, as we found in § 28, aliquot parts.

At $33\frac{1}{3}$¢ each, 9 neckties cost 9 times \$$\frac{1}{3}$, or \$3.

At $16\frac{2}{3}$¢ each, 12 bottles of ink cost 12 times \$$\frac{1}{6}$, or \$2.

At $12\frac{1}{2}$¢ each, 9 balls cost 9 times \$$\frac{1}{8}$, or \$$1\frac{1}{8}$, or \$$1.12\frac{1}{2}$.

While goods are seldom marked $16\frac{2}{3}$¢ or \$$\frac{1}{6}$, they are often marked 6 for \$1, which is the same thing.

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4. At $12\frac{1}{2}$¢ a yard, how much will 16 yd. of cloth cost?

5. At $16\frac{2}{3}$¢ a yard, how much will 18 yd. of cloth cost?

6. At $33\frac{1}{3}$¢ a yard, how much will 21 yd. of cloth cost?

7. At 8 doz. marbles for \$1, how much will 6 doz. cost?

8. At $16\frac{2}{3}$¢ each, find the cost of 9 water pitchers.

9. At the rate of 3 for \$1, how much must the school pay for 15 arithmetics?

10. At $12\frac{1}{2}$¢ each, find the cost of 10 glasses; of 4 lamp chimneys; of 20 cans of soup; of 32 yd. of calico.

11. How many jackets can be cut from $17\frac{1}{2}$ yd. of cloth, allowing $2\frac{1}{2}$ yd. to a jacket?

ORAL EXERCISE

1. How much is $8 \times \$\frac{1}{8}$? $8 \times \$1\frac{1}{8}$? $8 \times \$1.12\frac{1}{2}$?
2. How much is $6 \times \$\frac{1}{6}$? $6 \times \$1\frac{1}{6}$? $6 \times \$1.16\frac{2}{3}$?
3. How much is $3 \times \$\frac{1}{3}$? $3 \times \$1\frac{1}{3}$? $3 \times \$1.33\frac{1}{3}$?
4. At $\$1.33\frac{1}{3}$ a yard, how much will 15 yd. of lace cost?
5. At $\$1.16\frac{2}{3}$ a yard, how much will 9 yd. of silk cost?
6. At $\$1.12\frac{1}{2}$ a yard, how much will 6 yd. of velvet cost?
7. Find the cost of 12 books at $\$1.12\frac{1}{2}$ each; at $\$1.16\frac{2}{3}$ each.
8. At $\$1.33\frac{1}{3}$ each, find the cost of 3 chairs; of 18 chairs.
9. At $\$1.16\frac{2}{3}$ each, find the cost of 6 maps; of 12 maps; of 30 maps; of 54 maps.
10. At $\$1.12\frac{1}{2}$ each, find the cost of 8 books; of 40 books; of 32 books; of 20 books.

At $\$1.12\frac{1}{2}$ each, 7 books cost $\$7 + \$\frac{7}{8}$, or $\$7.87\frac{1}{2}$ (\$7.88).
At $\$1.16\frac{2}{3}$ each, 5 books cost $\$5 + \$\frac{5}{6}$, or $\$5.83\frac{1}{3}$ (\$5.83).
At $\$1.33\frac{1}{3}$ each, 8 lamps cost $\$8 + \$\frac{8}{3}$, or $\$10\frac{2}{3}$ (\$10.67).

WRITTEN EXERCISE

1. At $12\frac{1}{2}$¢ each, find the cost of 147 balls.
2. At $16\frac{2}{3}$¢ each, find the cost of 113 notebooks.
3. At $33\frac{1}{3}$¢ each, find the cost of 101 handkerchiefs.
4. At $\$1.33\frac{1}{3}$ a volume, find the cost of 165 books.
5. At $\$1.16\frac{2}{3}$ a yard, find the cost of 142 yd. of lace.
6. At $\$1.12\frac{1}{2}$ each, find the cost of 112 yd. of velvet.
7. At $\$2.16\frac{2}{3}$ each, find the cost of 72 umbrellas.
8. What is the difference in cost of 144 books at $\$1.16\frac{2}{3}$ a volume, and the same number at $\$1.12\frac{1}{2}$ a volume?

WRITTEN REVIEW DRILL

Add the following:

1. $\frac{1}{2} + \frac{1}{3} \qquad 47\frac{1}{2} + 93\frac{1}{3} \qquad 72\frac{1}{4} + 83\frac{1}{6} \qquad 29\frac{3}{4} + 47\frac{1}{5}$

2. $\frac{1}{5} + \frac{1}{3} \qquad 62\frac{1}{5} + 35\frac{1}{3} \qquad 85\frac{4}{5} + 16\frac{2}{3} \qquad 87\frac{5}{6} + 19\frac{2}{3}$

3. $\frac{1}{2} + \frac{1}{6} \qquad 67\frac{1}{2} + 29\frac{1}{6} \qquad 89\frac{3}{8} + 23\frac{1}{2} \qquad 67\frac{4}{5} + 49\frac{1}{2}$

4. $\frac{4}{5} + \frac{3}{4} \qquad 39\frac{4}{5} + 28\frac{3}{4} \qquad 87\frac{3}{5} + 62\frac{7}{10} \qquad 37\frac{4}{5} + 18\frac{1}{10}$

Subtract the following:

5. $\frac{4}{5} - \frac{1}{2} \qquad 79\frac{4}{5} - 22\frac{1}{2} \qquad 86\frac{2}{3} - 37\frac{1}{2} \qquad 48\frac{3}{4} - 23\frac{7}{8}$

6. $\frac{3}{4} - \frac{2}{5} \qquad 92\frac{3}{4} - 37\frac{2}{5} \qquad 30\frac{2}{3} - 19\frac{7}{8} \qquad 82\frac{1}{2} - 68\frac{7}{8}$

7. $\frac{2}{5} - \frac{1}{8} \qquad 72\frac{2}{5} - 56\frac{1}{8} \qquad 41\frac{3}{8} - 16\frac{5}{6} \qquad 272 - 46\frac{2}{3}$

Multiply the following:

8. $27 \times 14\frac{2}{3} \qquad 2\frac{1}{2} \times 92\frac{1}{2} \qquad 2\frac{1}{2} \times 3\frac{1}{4} \qquad 74\frac{2}{5} \times 65$

9. $18 \times 96\frac{3}{4} \qquad 5\frac{1}{2} \times 68 \qquad 4\frac{7}{8} \times 2\frac{1}{2} \qquad 40\frac{1}{3} \times 96$

10. $85 \times 32\frac{2}{5} \qquad 1\frac{3}{4} \times 96 \qquad 6\frac{3}{8} \times 4\frac{1}{2} \qquad 21\frac{7}{8} \times 35\frac{1}{2}$

11. $10 \times 12\frac{1}{2} \qquad 6\frac{1}{3} \times 7\frac{1}{5} \qquad 5\frac{1}{3} \times 3\frac{1}{2} \qquad 27\frac{1}{5} \times 4\frac{1}{4}$

12. $65 \times 9\frac{4}{5} \qquad 29 \times 35\frac{1}{4} \qquad 5\frac{1}{4} \times 6\frac{2}{3} \qquad 42\frac{1}{2} \times 408\frac{2}{3}$

Divide the following:

13. $\frac{2}{3} \div \frac{3}{8} \qquad \frac{3}{8} \div \frac{2}{3} \qquad \frac{7}{8} \div \frac{4}{5} \qquad 2\frac{4}{5} \div \frac{7}{8}$

14. $\frac{5}{8} \div \frac{3}{4} \qquad \frac{3}{4} \div \frac{5}{8} \qquad \frac{3}{4} \div \frac{7}{8} \qquad 3\frac{3}{4} \div 5\frac{1}{2}$

15. $\frac{5}{8} \div 2\frac{1}{5} \qquad \frac{5}{8} \div \frac{2}{3} \qquad \frac{2}{3} \div 4\frac{2}{3} \qquad 7\frac{3}{8} \div 1\frac{3}{4}$

16. $\frac{2}{3} \div 3\frac{1}{3} \qquad \frac{1}{3} \div \frac{2}{3} \qquad \frac{4}{5} \div 3\frac{1}{2} \qquad 5\frac{5}{8} \div 1\frac{1}{8}$

17. $\frac{1}{5} \div \frac{1}{3} \qquad \frac{1}{3} \div 1\frac{1}{5} \qquad \frac{9}{10} \div 2\frac{1}{3} \qquad 2\frac{4}{5} \div 3\frac{1}{2}$

Find the value of:

18. $\frac{2}{5}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of 1 cu. ft. $\qquad$ $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of 1 cu. in.

19. $\frac{1}{2}$ of 5 × 8 sq. ft. $\qquad$ $\frac{3}{5}$ of 15 × 48 sq. ft.

20. $\frac{7}{8}$ of 4 × 16 sq. in. $\qquad$ $\frac{3}{7}$ of 13 × 14 sq. in.

ORAL EXERCISE

1. What part of 4 is 2? What part of 10 is 8? How do you find what part one number is of another?

2. What part of 7 is 5? of 9 is 4? of 7 is 6? of 16 is 12? of 25 is $12\frac{1}{2}$? of 100 is $33\frac{1}{3}$? of 100 is $16\frac{2}{3}$?

95. Fractional Parts. From these answers we have:

To find what part the second of two numbers is of the first, divide the second by the first.

For example, what part of 51 is 17? 17 is $\frac{17}{51}$, or $\frac{1}{3}$, of 51.

WRITTEN EXERCISE

Find what part the second of these numbers is of the first:

1. 14, 8.
2. 18, 14.
3. $\frac{7}{8}$, $\frac{2}{3}$.
4. $\frac{22}{7}$, $\frac{6}{11}$.
5. $3\frac{1}{3}$, $1\frac{2}{3}$.
6. 17, $8\frac{1}{2}$.
7. 82, $9\frac{1}{9}$.
8. $41\frac{3}{5}$, $5\frac{1}{5}$.
9. $66\frac{2}{3}$, $16\frac{2}{3}$.
10. 243, 27.
11. 100, $37\frac{1}{2}$.
12. $45\frac{3}{5}$, $7\frac{3}{5}$.
13. $16\frac{5}{8}$, $2\frac{3}{8}$.
14. $42\frac{1}{2}$, $14\frac{1}{6}$.
15. $62\frac{1}{8}$, $7\frac{7}{8}$.
16. $66\frac{2}{3}$, $33\frac{1}{3}$.
17. 225, $37\frac{1}{2}$.
18. $22\frac{1}{2}$, $16\frac{2}{3}$.
19. $40\frac{1}{8}$, $21\frac{2}{5}$.
20. $37\frac{1}{2}$, $33\frac{1}{3}$.
21. 421, $60\frac{1}{7}$.
22. $33\frac{1}{3}$, $16\frac{2}{3}$.
23. 427, $47\frac{4}{9}$.
24. 325, $29\frac{6}{11}$.

25. Frank is 12 yr. old and his brother is 18 yr. old. Frank's age is what part of his brother's?

26. The age of Frank's brother, who is 18 yr. old, is what part of his uncle's, who is 54 yr. old?

27. If Frank is 12 yr. old and his brother is 18 yr. old, Frank's age was what part of his brother's 6 yr. ago? It will be what part of his brother's age 6 yr. from now?

ORAL EXERCISE

1. 10 is $\frac{1}{2}$ of what number? $\frac{1}{3}$ of what number?

2. 12 is $\frac{1}{3}$ of what number? If 12 is $\frac{2}{3}$ of some number, what is $\frac{1}{3}$ of the number? What is $\frac{3}{3}$ of the number?

3. If 15 is $\frac{3}{7}$ of some number, what is $\frac{1}{7}$ of the number? What is the number?

96. Given a Part of a Number to find the Number. If $33\frac{1}{3}$ is $\frac{2}{3}$ of some number, what is the number?

Because $\frac{2}{3}$ of some number $= 33\frac{1}{3} = \frac{100}{3}$,
therefore $\frac{1}{3}$ of the number $= \frac{1}{2}$ of $\frac{100}{3} = \frac{50}{3}$,
and $\frac{3}{3}$ of the number $= 3$ times $\frac{50}{3} = 50$.

Therefore $33\frac{1}{3}$ is $\frac{2}{3}$ of 50.

WRITTEN EXERCISE

Find the number of which:

1. 23 is $\frac{1}{3}$.
2. 38 is $\frac{2}{3}$.
3. 63 is $\frac{7}{8}$.
4. 75 is $\frac{3}{8}$.
5. 45 is $\frac{9}{10}$.
6. 150 is $\frac{3}{5}$.
7. 222 is $\frac{6}{7}$.
8. 125 is $\frac{25}{32}$.
9. 144 is $\frac{9}{16}$.
10. \$150 is $\frac{3}{5}$.
11. \$2.25 is $\frac{5}{8}$.
12. \$10.25 is $\frac{5}{9}$.

13. If there are 85 girls in a school, and they are $\frac{5}{9}$ of the total number of pupils, how many pupils are there?

14. In my library there are 164 books on history, and these are $\frac{2}{17}$ of all my books. How many books have I?

15. If a man spends \$1687.50 this year, and this is $\frac{3}{4}$ of his income, how much is his income?

16. A man left to his son $\frac{3}{5}$ of his estate, this share amounting to \$3210. How much was the estate worth?

97. Cancellation Extended. Since a fraction is an indicated division, *equal factors may be canceled from both dividend and divisor without changing the quotient.*

1. Divide $2 \times 7 \times 15$ by $21 \times 5 \times 6$.

$$\frac{\cancel{2}\times\cancel{7}\times\overset{\cancel{5}}{\cancel{15}}}{\underset{\cancel{3}}{\cancel{21}}\times\cancel{5}\times\underset{3}{\cancel{6}}}=\frac{1}{3}$$

Writing this as a fraction, 2 is canceled from 2 and 6; 7 is canceled from 7 and 21; 3 is canceled from 3 (left from the 21) and 15; 5 is canceled from 5 (left from the 15) and 5. Therefore the quotient is $\frac{1}{3}$, for evidently 1 is a factor of the numerator.

2. What is the product of $\frac{1}{15}$, $45\frac{1}{2}$, $33\frac{1}{3}$, and $16\frac{7}{8}$?

$$\frac{1\times 91\times\overset{25}{\cancel{100}}\times\overset{\overset{\overset{3}{\cancel{9}}}{\cancel{27}}}{\cancel{135}}}{\underset{\cancel{3}}{\cancel{15}}\times 2\times\cancel{3}\times\underset{2}{\cancel{8}}}=\frac{6825}{4}=1706\tfrac{1}{4}$$

Reducing to improper fractions, $45\frac{1}{2}=\frac{91}{2}$, $33\frac{1}{3}=\frac{100}{3}$, and $16\frac{7}{8}=\frac{135}{8}$. Proceeding as before, the product is $1706\frac{1}{4}$.

WRITTEN EXERCISE

Divide, using cancellation:

1. $7 \times 21 \times 77 \times 6$ by $49 \times 33 \times 42$.
2. $20 \times 25 \times 36 \times 48$ by $27 \times 40 \times 15 \times 12$.
3. $32 \times 27 \times 44 \times 39$ by $55 \times 72 \times 26 \times 8 \times 3$.
4. $125 \times 108 \times 51 \times 8$ by $40 \times 45 \times 85 \times 12 \times 3$.

Multiply, using cancellation:

5. $13\frac{1}{3} \times 32\frac{1}{5} \times 16\frac{7}{8}$.
6. $27\frac{1}{5} \times 24\frac{1}{2} \times 37\frac{1}{2}$.
7. $37\frac{1}{2} \times 22\frac{1}{4} \times 12\frac{4}{5}$.
8. $15\frac{2}{3} \times 16\frac{7}{8} \times 33\frac{1}{3}$.
9. $42\frac{2}{3} \times 66\frac{2}{3} \times 74\frac{1}{4}$.
10. $7\frac{3}{5} \times 2\frac{1}{7} \times 5\frac{1}{4} \times 21\frac{1}{3}$.
11. $6\frac{3}{4} \times 7\frac{3}{4} \times 9\frac{1}{3} \times 12\frac{1}{3}$.
12. $9\frac{3}{5} \times 6\frac{2}{3} \times 3\frac{1}{2} \times 15\frac{2}{3}$.

Multiply, using cancellation when possible:

13. $18 \times \frac{3}{4}$.
14. $27 \times \frac{5}{12}$.
15. $32 \times \frac{7}{8}$.
16. $48 \times \frac{3}{8}$.
17. $56 \times \frac{7}{12}$.
18. $\frac{3}{8}$ of 26.
19. $\frac{7}{8}$ of 34.
20. $\frac{3}{5}$ of 35.
21. $\frac{3}{8}$ of 52.
22. $\frac{5}{6}$ of 33.
23. $\frac{3}{8}$ of $\frac{4}{9}$.
24. $\frac{7}{8}$ of $\frac{3}{4}$.
25. $\frac{7}{8}$ of $\frac{4}{21}$.
26. $\frac{5}{6}$ of $\frac{3}{10}$.
27. $\frac{6}{7}$ of $\frac{7}{12}$.

Using cancellation when possible, find:

28. $\frac{2}{3}$ of $\frac{3}{8}$.
29. $\frac{5}{8}$ of $\frac{4}{9}$.
30. $\frac{3}{5}$ of $\frac{15}{16}$.
31. $\frac{4}{5}$ of $\frac{15}{16}$.
32. $\frac{5}{8}$ of $\frac{4}{15}$.
33. $\frac{3}{7}$ of $\frac{14}{9}$.
34. $\frac{6}{7}$ of $\frac{14}{9}$.
35. $\frac{5}{6}$ of $\frac{3}{16}$.
36. $\frac{5}{6}$ of $\frac{9}{20}$.
37. $\frac{5}{8}$ of $\frac{4}{15}$.
38. $\frac{1}{5}$ of $1\frac{2}{3}$.
39. $\frac{3}{5}$ of $6\frac{2}{3}$.
40. $\frac{7}{8}$ of $4\frac{4}{5}$.
41. $\frac{5}{6}$ of $4\frac{3}{8}$.
42. $\frac{5}{8}$ of $9\frac{3}{5}$.

Find the products of:

43. $\frac{2}{3}$ of $\frac{3}{8}$ of $\frac{4}{5}$.
44. $\frac{7}{8}$ of $\frac{4}{5}$ of $\frac{15}{14}$.
45. $\frac{5}{8}$ of $\frac{2}{5}$ of $\frac{4}{9}$ of $\frac{3}{5}$.
46. $\frac{3}{8}$ of $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{6}$.
47. $\frac{5}{8}$ of $1\frac{3}{5} \times 2\frac{1}{2} \times \frac{3}{5}$.
48. $\frac{7}{8}$ of $1\frac{1}{7} \times 3\frac{2}{3} \times \frac{3}{22}$.
49. $1\frac{2}{3} \times 3\frac{1}{5} \times 2\frac{1}{2} \times 5\frac{1}{4}$.
50. $1\frac{7}{8} \times 3\frac{1}{2} \times 2\frac{2}{3} \times 3\frac{1}{4}$.
51. $1\frac{2}{3} \times 2\frac{2}{5} \times 3\frac{1}{2} \times 4\frac{3}{5}$.
52. $2\frac{2}{5} \times 5\frac{1}{2} \times 10 \times 3\frac{3}{8}$.

Using cancellation when possible, find:

53. $14 \div \frac{7}{8}$.
54. $25 \div \frac{15}{16}$.
55. $\frac{3}{8} \div \frac{4}{5}$.
56. $\frac{5}{9} \div \frac{15}{16}$.
57. $\frac{3}{4} \div \frac{9}{10}$.
58. $70\frac{2}{3} \div 8$.
59. $\frac{5}{8} \div \frac{3}{16}$.
60. $\frac{4}{5} \div \frac{2}{3}$.
61. $\frac{3}{5} \div \frac{9}{10}$.
62. $\frac{7}{8} \div \frac{5}{16}$.
63. $\frac{15}{16} \div \frac{7}{8}$.
64. $\frac{3}{8} \div \frac{9}{10}$.
65. $\frac{5}{8} \div 25$.
66. $25 \div \frac{5}{8}$.
67. $3\frac{3}{5} \div 9$.
68. $9 \div 7\frac{1}{5}$.
69. $18 \div 4\frac{1}{2}$.
70. $12\frac{1}{2} \div 3\frac{1}{8}$.
71. $16\frac{2}{3} \div 4\frac{1}{6}$.
72. $37\frac{1}{2} \div 12\frac{1}{2}$.
73. $87\frac{1}{2} \div 37\frac{1}{2}$.

LITTLE EXAMINATIONS

I. 1. Reduce $62\frac{1}{2}$ to halves.
2. Reduce $\frac{27}{3}$ to units.
3. $\frac{2}{3}+\frac{1}{2}+\frac{1}{4}$.
4. $325\frac{1}{2}-246\frac{7}{8}$.
5. $\frac{3}{8}$ of 3536.
6. $\frac{3}{4}\times\frac{4}{5}$.
7. $3\frac{1}{4}\div\frac{3}{4}$.
8. 8 × $1.25.

II. 1. Reduce $7\frac{3}{4}$ to fourths.
2. Reduce $\frac{75}{15}$ to units.
3. $5\frac{1}{2}+\frac{3}{4}+3\frac{1}{4}$.
4. $526\frac{1}{3}-243\frac{1}{2}$.
5. $\frac{3}{4}$ of $124.48.
6. $\frac{4}{5}$ of $12\frac{1}{2}$.
7. $2\frac{1}{2}\div\frac{3}{4}$.
8. 9 × 1.33\frac{1}{3}$.

III. 1. Reduce $9\frac{7}{8}$ to eighths.
2. Reduce $\frac{125}{25}$ to units.
3. $16\frac{2}{3}+33\frac{1}{3}+175$.
4. $426\frac{1}{4}-342\frac{7}{8}$.
5. $\frac{2}{3}$ of $67.29.
6. $\frac{3}{4}$ of $16\frac{1}{2}$.
7. $5\frac{1}{4}\div 2\frac{1}{2}$.
8. $75\times 33\frac{1}{3}$.

IV. 1. Reduce $7\frac{2}{3}$ to thirds.
2. Reduce $\frac{19}{4}$ to units.
3. $7\frac{1}{2}+9\frac{3}{8}+4\frac{1}{4}$.
4. $3\frac{7}{8}+6\frac{3}{4}+3\frac{1}{2}$.
5. $507\frac{1}{8}-239\frac{3}{4}$.
6. $\frac{5}{8}$ of $17.60.
7. $\frac{4}{5}$ of 1475.
8. $2\frac{3}{4}\times 96$.
9. $8\frac{3}{4}\div 3\frac{1}{2}$.
10. 7 × 1.12\frac{1}{2}$.

V. 1. Reduce $8\frac{4}{5}$ to fifths.
2. Reduce $\frac{17}{3}$ to units.
3. $8\frac{3}{4}+9\frac{3}{4}+6\frac{7}{8}$.
4. $4\frac{2}{3}+7\frac{3}{4}-2\frac{1}{2}$.
5. $721\frac{3}{4}-249\frac{7}{8}$.
6. $\frac{7}{8}$ of $72.96.
7. $\frac{3}{16}$ of 3360.
8. $3\frac{4}{5}\times 750$.
9. $9\frac{7}{8}\div 2\frac{1}{2}$.
10. 6 × 7.33\frac{1}{3}$.

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. Write in words the numbers 1,020,346 and MCDXCII.

2. Write in figures the number seven million, seven.

3. Add $4276 + 39{,}542 + 278 + 9463 + 48{,}278 + 9368$.

4. Add $\$48.72 + \$63.96 + \$8.75 + \$128.50 + \$2.35$.

5. From 401,324 subtract 96,849.

6. Find the difference between $742.25 and $96.79.

7. Multiply 43,298 by 5276.

8. Multiply $175.75 by 46.

9. Divide the sum of 27,960 and 39,290 by 250.

10. Divide 472,932 by 302, and 945,864 by 604.

11. Reduce to lowest terms: $\frac{4}{8}$, $\frac{4}{16}$, $\frac{4}{32}$, $\frac{8}{32}$, $\frac{16}{32}$, $\frac{32}{32}$.

12. How many fourths are there in 3? in $3\frac{1}{4}$? in $5\frac{3}{4}$?

13. Reduce $15\frac{3}{4}$ to fourths; to eighths.

14. Reduce to integers or mixed numbers: $\frac{25}{2}$, $\frac{37}{4}$, $\frac{31}{8}$.

15. Find the value of $3\frac{1}{2} + \frac{1}{4} + 2\frac{3}{8} + 4\frac{3}{4} + 1\frac{7}{8}$.

16. From $7\frac{1}{4}$ in. take the sum of $1\frac{1}{2}$ in. and $2\frac{7}{8}$ in.

17. Find $\frac{3}{4}$ of $7\frac{1}{2}$ in., and $\frac{5}{8}$ of $16\frac{1}{2}$ ft.

18. How many strips of molding $8\frac{3}{4}$ in. long can be cut from a strip 3 ft. long, and how much will be left over?

ORAL EXERCISE

1. How much will $2\frac{1}{2}$ lb. of coffee cost at 24¢ a pound?

2. How much will $\frac{3}{4}$ lb. of pepper cost at 24¢ a pound?

3. How much will $2\frac{1}{2}$ lb. of butter cost at 30¢ a pound?

4. How much will 5 oz. of cinnamon cost at 32¢ a pound?

5. How much will 20 lb. of loaf sugar cost at $6\frac{1}{4}$¢ a pound?

6. A grocer buys 240 cans of corn and sells half of them. How many cans of corn does he sell?

7. A merchant buys 200 yd. of cloth and sells a quarter of it. How many yards of cloth does he sell?

8. Find the cost of $1\frac{1}{2}$ lb. of cocoa at 22¢ per half-pound can.

WRITTEN EXERCISE

1. At $1.38 a yard, how much will 37 yd. of carpet cost?

2. What is the cost of $6\frac{1}{4}$ lb. of mackerel @ 16¢, $3\frac{1}{2}$ lb. of codfish @ 12¢, and 4 cans of salmon at the rate of 2 for a quarter?

3. I bought $\frac{1}{2}$ gal. of olive oil at $3.30 a gallon, but $\frac{1}{4}$ of what I bought was lost by leaking. What rate per gallon do I pay for what is left?

4. In buying 48 cans of tomatoes, how much is saved by buying two cases of 2 doz. cans each, at $2.89 a case, over buying at the rate of 3 cans for 40¢?

5. I bought 3 gal. of best maple sirup @ 1.12\frac{1}{2}$ a gallon and $2\frac{1}{2}$ gal. New Orleans molasses @ 62¢ a gallon, and gave the grocer a $5 bill. How much change did I get?

Teachers should send the pupils to local stores to ascertain prices, and these prices should be used in the above and similar problems.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. How do you subtract a number of three figures from a greater number when the units' figure to be subtracted is greater in value than the units' figure of the minuend, and the tens' figure to be subtracted is greater than the tens' figure of the minuend? Give your reasons.

2. How do you multiply by a number of three figures, the last two figures being zero? Give your reasons.

3. How do you divide by a number of three figures, the last two figures being zero? Give your reasons.

4. What do you mean by a power of a number? by the square of a number? by the cube of a number? Illustrate.

5. What is cancellation? What are its advantages? On what principle does cancellation depend?

6. If you know the cost of some land and the cost per acre, how do you find the number of acres?

7. If you know the weight of a cubic foot of water and the number of cubic feet of water in a cistern, how do you find the weight of the water in the cistern?

8. If you have the difference of two numbers and one of the two numbers given, how do you find the other number?

9. If you have the quotient of two fractions given and know the dividend, how do you find the divisor?

10. If you have the product of two fractions given and one of the fractions, how do you find the other fraction?

11. If you know the cost of a certain number of acres of land, how do you find the cost of a certain other number of acres?

III. INDUSTRIAL PROBLEMS

BRAINS PAY

1. Joseph's father has told him he could grow some tomatoes on shares, the father furnishing the land and the plants, and Joseph doing the work. Joseph was given the use of a rectangular plot of land 4 rd. by 5 rd. There being 160 sq. rd. to an acre, what part of an acre does Joseph have to cultivate?

2. Joseph reads that it is possible to average a crop worth $140 an acre for plants started in the most scientific way in a hotbed, but he found that this was the result of more nearly perfect conditions than he met in his garden. He got $\frac{4}{7}$ of this figure. How much did that amount to on his plot of ground?

3. Joseph was given half of the money received. How much was his share worth? How much would it have been worth if he had raised tomatoes at the rate of 6 T. to the acre, selling the tomatoes at $10 a ton? How much did he gain by using his brains?

GETTING A ROPE FOR A SWING

1. Do you know how heavy a rope is needed for a swing or a trapeze? Would it be an inch in diameter or a half inch? Well, it depends on whether it is a manila rope or a hemp rope, and on whether or not it is new. But we can find out nearly enough for our purposes by some rules that have been worked out. The weight needed to break a rope is called the breaking strength. For new manila rope we find the breaking strength in pounds by squaring the diameter (that is, by multiplying this number by itself) and then multiplying the result by 7200. For example, the breaking strength of new manila rope $\frac{3}{4}$ in. thick is found thus:

$$\tfrac{3}{4} \times \tfrac{3}{4} \times 7200 \text{ lb.} = 4050 \text{ lb.}$$

Find the breaking strength of $\frac{3}{16}$-inch and $\frac{3}{8}$-inch ropes.

2. But we do not wish to have the swing break with us, and all ropes are not equally well made, so we divide the answer by 6, a number called the factor of safety. If the breaking strength is 4050 lb., the safe load is $\frac{1}{6}$ of 4050 lb., or 675 lb. Find the safe loads for ropes of the diameters given in Ex. 1.

3. If you decide to use a $\frac{1}{2}$-inch new manila rope, what will be the breaking strength and the safe load? Is this sure to be enough for a good swing?

Although a person in a swing is supported by two ropes, he often swings by one rope only, so we must have this rope strong enough to hold if father or uncle want to swing in this way.

4. If we want the swing to be perfectly safe for two grown persons, will it be all right to use a $\frac{5}{8}$-inch new manila rope?

5. Hemp rope is three fourths as strong as manila rope. Find the safe loads for hemp ropes of diameters $\frac{3}{8}$ in. and $\frac{3}{4}$ in.

The figures on this page are, of course, approximate, for ropes differ in quality, but these figures are based on the best scientific investigations.

WHAT DO PEACHES COST?

1. Let us find the cost of peaches. If a peach grower has 15 acres in his orchard, and it rents for $6 an acre each year, how much does this amount to for 15 acres?

2. He has to have the trees pruned. If he has 367 hr. of work done at 20¢ an hour, how much must he pay for pruning?

3. The peach grower pays $1\frac{1}{4}$¢ a gallon for the spray and must use 100 gal. to an acre. How much does this cost him?

4. He must also have all the imperfect peaches picked off while they are young. He pays for 113 hr. of work at 15¢ an hour. How much does this add to his expense account?

5. The peaches have to be picked and packed carefully. Picking and packing require 964 hr. of work at 15¢ an hour. Find this part of the cost.

6. The other expenses amount to $34.50. Add this to the grower's expense account. How much does this make?

7. The peaches average 142 bu. to the acre. Find how many bushels the grower gets, and how much they actually cost him.

CHAPTER III

DENOMINATE NUMBERS

ORAL EXERCISE

1. How many ounces in a pound? An ounce is what part of a pound?

2. State the number of gills in a pint, the number of pints in a quart, and the number of quarts in a gallon.

3. State the number of inches in a foot and the number of feet in a yard. An inch is what part of a foot?

4. State the number of quarts in a peck and the number of pecks in a bushel. A peck is what part of a bushel?

5. How many seconds in a minute? minutes in an hour? hours in a day? days in a week? days in a common year? days in a leap year? What months have 30 days?

Express the following as indicated:

6. 6 pt. as quarts.
7. 14 da. as weeks.
8. $1\frac{1}{2}$ lb. as ounces.
9. 48 hr. as days.
10. 8 qt. as gallons.
11. 8 pk. as bushels.
12. 15 pt. as quarts.
13. 32 oz. as pounds.
14. 3 yd. as feet.
15. 12 ft. as yards.
16. 4 wk. as days.
17. $2\frac{1}{2}$ qt. as pints.

18. How many pints in a quart? in $1\frac{1}{2}$ qt.? in 2 qt.? in a gallon? in 2 gal.? in $2\frac{1}{2}$ gal.?

19. How many quarts in a peck? in 2 pk.? in a bushel? in 2 bu.? in $2\frac{1}{2}$ bu.? in 10 bu.? in 100 bu.?

WRITTEN EXERCISE

1. If John picked 12 qt. of berries to-day and half as many yesterday, how many quarts did he pick yesterday? How many quarts did he pick in the two days?

2. If Charles gathered 21 qt. of chestnuts and Will gathered $\frac{2}{3}$ as many, how many quarts did Will gather? How many quarts did they both gather?

3. If Mary lives $\frac{1}{2}$ mi. from school and Louise lives half as far, how far from school does Louise live?

4. If school opens at 9 A.M. and closes at 30 min. past 3 P.M., and there is $1\frac{1}{4}$ hr. recess at noon, how many hours are we in school to-day?

5. There being $5\frac{1}{2}$ yd. in a rod, how many yards in $17\frac{2}{3}$ rd.? how many yards in 32 rd.?

6. How much will $8\frac{3}{4}$ doz. pencils cost at the rate of 25¢ $(\$\frac{1}{4})$ for a half dozen?

7. At the rate of $3\frac{3}{4}$ lb. a week, how much sugar will a family use in the $4\frac{3}{7}$ weeks of January?

8. At the rate of $14\frac{7}{8}$ lb. of meat a week, how much meat will a family use in the $4\frac{2}{7}$ weeks of April?

9. How far will a commercial salesman travel in 6 da. at an average of $42\frac{1}{2}$ mi. a day?

10. An ocean steamer averaged $16\frac{7}{8}$ mi. an hour for $23\frac{1}{2}$ hr. What was the total distance?

11. Some blotting paper is $\frac{3}{64}$ in. thick. What will be the height of a pile of $2\frac{3}{4}$ doz. sheets?

12. If an arithmetic is $1\frac{1}{8}$ in. thick and a reader is $\frac{15}{16}$ in. thick, what is the height of a pile of 7 arithmetics and 9 readers?

13. At 28¢ a pound, how much will $4\frac{1}{2}$ lb. of steak cost?

In this and the following examples estimate the approximate answer and write it down. After the problem is solved compare the approximate answer with the real one. This is one of the best checks against absurd results. Thus 5 × 28¢ is a reasonable approximation for $4\frac{1}{2}$ × 28¢.

14. At 24¢ a pound, how much will $6\frac{3}{4}$ lb. of meat cost?

15. At 16¢ a pound, how much will $5\frac{1}{4}$ lb. of cheese cost?

16. At 20¢ a yard, how much will $18\frac{3}{4}$ yd. of calico cost?

17. At $6.50 a ton, how much will $5\frac{3}{4}$ tons of coal cost?

18. If 6 yd. of silk cost $6.75, what is the price per yard?

19. How many bushels of potatoes will 32 acres produce, if an average acre produces 125 bu.?

20. How many bushels of oats will 81 acres produce, if an average acre produces $33\frac{1}{3}$ bu.?

21. How much will 250 doz. yards of millinery braid cost at $62\frac{1}{2}$¢ a dozen yards? at 75¢ a dozen yards?

22. There are 44,000 yd. in 25 mi. How many yards are there in one mile?

23. If 25 tons of coal cost $150, what will be the cost of 11 tons? of 15 tons? of 32 tons? of 75 tons?

24. How long will it take an express train to go $362\frac{1}{2}$ mi. at the rate of 50 mi. an hour?

25. A dealer pays $124 for some gingham at 25¢ a yard. How many yards of gingham does he buy?

26. A dealer pays $160 for some percaline at 25¢ a yard. How many yards of percaline does he buy?

27. If you buy $3\frac{1}{2}$ doz. oranges at 40¢ a dozen, and 2 doz. bananas at 25¢ a dozen, how much change should you receive from $2?

98. Tables for Reference. While most of these tables have been learned and used in the Primary Book, they are here inserted for reference and for such review as may be necessary.

99. Table of Length. The following is the table of length:

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

The *hand* (4 in.) is used in measuring the height of horses at the shoulder. Sailors use the *fathom* (6 ft.) and the *cable length* (120 fathoms) for measuring depths, and the *knot* (nautical mile = $1\frac{3}{20}$ common, or statute, miles = $6080\frac{1}{4}$ ft.) for distances at sea.

1. Required to find the number of inches in 3 ft.

$$3 \text{ ft.} = 3 \times 12 \text{ in.} = 36 \text{ in.}$$

2. Required to find the number of feet in 27 rd.

$$27 \text{ rd.} = 27 \times 16\tfrac{1}{2} \text{ ft.} = 445\tfrac{1}{2} \text{ ft.}$$

ORAL EXERCISE

Find the number of inches in:

1. 5 ft.
2. 9 ft.
3. 10 ft.
4. $1\frac{1}{2}$ ft.
5. 1 yd.
6. 10 yd.
7. $\frac{1}{2}$ yd.
8. $\frac{1}{4}$ yd.

Find the number of feet in:

9. 2 yd.
10. 4 yd.
11. 7 yd.
12. 10 yd.
13. 1 rd.
14. 10 rd.
15. $5\frac{1}{2}$ yd.
16. $\frac{1}{2}$ yd.

17. If you buy $\frac{1}{4}$ yd. of velvet, how many inches do you buy?

18. If a horse is 16 hands high, what is his height in inches? in feet and inches?

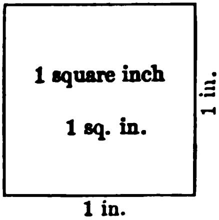
100. Table of Square Measure. The following is the table of square measure:

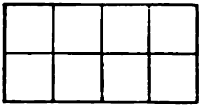
144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Carpenters, architects, and mechanics often write 8″ for 8 in., and 5′ for 5 ft. They also use sq.″ and sq.′ for square inches and square feet. In this book both of these forms are used.

If a square is 1 in. on a side, it is 1 in. square, and its area is 1 sq. in.

If a rectangle is 4 in. long and 2 in. wide, its area is 4 × 2 sq. in., or 8 sq. in.





The pupils may express this in either of these two forms:

4 × 2 sq. in. = 8 sq. in.,
or 2 × 4 sq. in. = 8 sq. in.

In practical work we simply multiply 4 by 2, saying: "There are *as many* square inches as 2 × 4, or 8 sq. in."

WRITTEN EXERCISE

1. Find the number of square inches in the top of a desk whose area is 3 sq. ft.

2. The school platform has an area of 3 sq. yd. What is its area in square feet?

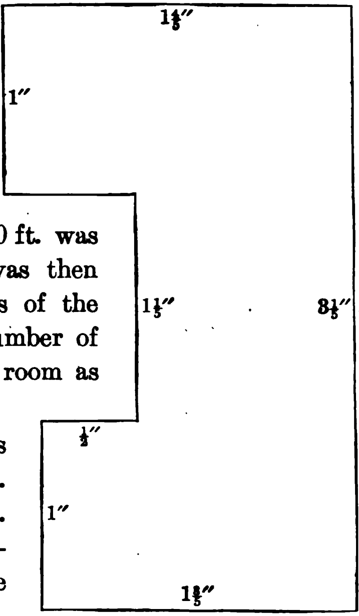
3. A flower bed has an area of 2 sq. rd. What is its area in square yards? in square feet?

4. The school grounds contain half an acre. What is the area in square rods?

WRITTEN EXERCISE

1. A farm is plotted as shown below. The scale is 1 in. to 100 rd. Find the number of square rods and also the number of acres in the farm.

As stated in the Primary Book, to draw to the scale of 1 in. to 100 rd., we represent 100 rd. by 1 in. To draw to the scale $\frac{1}{12}$, we represent 12 in. by 1 in.



2. A rectangular meadow is plotted to the scale of 1 in. to 50 rd. The plot is 2 in. by $3\frac{1}{5}$ in. Find the number of acres in the meadow.

3. A plan for a schoolroom drawn to the scale of 1 in. to 50 ft. was 1 in. long and $\frac{7}{10}$ in. wide. It was then decided to reduce the dimensions of the room one fifth. What was the number of square feet of floor space in the room as finally built?

4. A plan for a playhouse is drawn to the scale of 1 in. to $2\frac{1}{2}$ ft. The plan is $6\frac{2}{5}$ in. long and 4 in. wide, and the playhouse is rectangular. Compute the area of the floor of the playhouse.

5. The class has considered the question of a model kitchen in an apartment house. The pupils drew a plan to the scale $\frac{1}{60}$, representing the kitchen as 3 in. long and 2 in. wide. What was the floor space of the proposed kitchen?

6. In Ex. 5, what would be the floor space of the proposed kitchen if the scale were $\frac{1}{48}$?

WRITTEN EXERCISE

1. Measure this page of the arithmetic. Draw a plan of the page to the scale $\frac{1}{4}$.

2. Draw a plan of the same page to the scale $\frac{1}{2}$.

3. Measure the inside of the cover of this arithmetic. It is a little larger than the page. Draw a plan to the scale $\frac{1}{5}$.

4. Measure the top of your desk. Draw a plan to the scale of 1 in. to 1 ft.

5. Draw a plan of the top of your desk to the scale $\frac{1}{6}$.

6. Measure the length and width of the schoolroom. Draw a plan and write below it the scale that you have used.

7. Measure the size of the paper upon which you make your drawings. Draw a plan to the scale $\frac{1}{8}$.

8. Measure the size of any other book you are studying. Draw a plan of a page to the scale $\frac{1}{5}$.

9. A drawing has been made of a box cover. The cover is 14 in. long and the drawing is $3\frac{1}{2}$ in. long. What scale was used?

10. A drawing of a floor 24 ft. long and 20 ft. wide is 6 in. long and 5 in. wide. What scale was used?

11. A drawing is made of a rectangle. The drawing has an area of 6 sq. in. If a scale twice as large were used, what would be the area of the drawing? Draw a plan of the two rectangles, so as to show that the answer is correct.

12. A drawing is made of a lamp screen $18\frac{3}{4}$ in. high, and the drawing is $3\frac{1}{8}$ in. high. What scale was used?

Teachers should have measurements made of desks, doors, blackboards, and other objects in the schoolroom, and scale drawings made from these measurements. The pupils should gradually come to know what the working drawings of the carpenter, the builder, and the manufacturer mean.

101. Table of Cubic Measure. The following is the table of cubic measure:

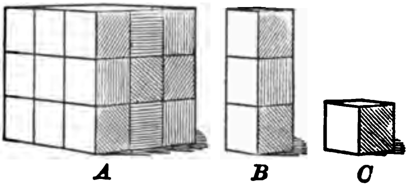
1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

A *perch* of stone or masonry is usually considered as 1 rd. long, 1 ft. high, and 1½ ft. thick, and it contains 24¾ cu. ft. It varies, however, in different parts of the country. A cubic yard of earth is considered a *load*.

If a block is 1 in. on an edge, and is in the form of a cube, it is a 1-inch cube, and the volume is 1 cu. in., as in *C*.

If a block is 1 in. square at the base and 3 in. high, the volume is 3 cu. in., as in *B*.

If a block is 3 in. square at the base, the base contains 3 × 3 sq. in., or 9 sq. in. If the block is 3 in. high, the volume is 3 × 9 cu. in., or 27 cu. in., as in *A*.



If a block is 4 in. long, 3 in. wide, and 2 in. high, the volume is

2 × 3 × 4 cu. in. = 24 cu. in.

WRITTEN EXERCISE

Find the volumes of boxes of these dimensions:

1. 3 in., 4 in., 5 in.
2. 2 in., 5 in., 9 in.
3. 4 in., 7 in., 12 in.
4. 8 in., 12 in., 16 in.
5. 3½ in., 8 in., 9½ in.
6. 4 ft., 12½ ft., 14 ft.
7. 2½ ft., 3½ ft., 4 ft.
8. 2¾ ft., 3½ ft., 8 ft.
9. 32 in., 25 in., 10 in.
10. 48 in., 27 in., 10 in.
11. How many cubic feet in 8 perches of 24¾ cu. ft. each?

102. Table of Weight. The following is the table of weight:

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

The ton of 2000 lb. is sometimes called the *short ton*, there being a *long ton* of 2240 lb. which is used in some transactions in mining products.

Goldsmiths still use an old table of *troy weight*, and apothecaries and physicians use the table of *apothecaries' weight* in prescriptions of medicine, but neither is of enough importance to justify teaching it.

The avoirdupois pound contains 7000 grains, the troy pound (12 troy ounces) 5760 grains.

A *carat* weight, used in weighing diamonds, is (since 1913) 200 milligrams, or 3.08647 grains. The word "carat" is also used in speaking of the purity of gold and means $\frac{1}{24}$; 16 carats fine means $\frac{16}{24}$ pure gold.

ORAL EXERCISE

Find the number of ounces in:

1. $\frac{1}{2}$ lb. 2. $\frac{1}{4}$ lb. 3. $\frac{3}{4}$ lb. 4. $1\frac{1}{2}$ lb. 5. 2 lb.

Find the number of pounds in:

6. $\frac{1}{2}$ T. 7. $\frac{1}{4}$ T. 8. $\frac{3}{4}$ T. 9. $1\frac{1}{2}$ T. 10. 2 T.

WRITTEN EXERCISE

1. How many pounds in $2\frac{1}{2}$ T.? in $1\frac{3}{4}$ T.? in $3\frac{3}{4}$ T.?
2. How many ounces in $2\frac{1}{2}$ lb.? in $2\frac{3}{4}$ lb.? in $3\frac{1}{4}$ lb.?
3. How many ounces in 7 lb.? in $7\frac{1}{4}$ lb.? in $9\frac{3}{4}$ lb.?
4. How many grains are there in $\frac{1}{16}$ lb., or 1 oz.?
5. How many pounds in $2\frac{3}{4}$ T.? in $3\frac{1}{2}$ T.? in $5\frac{1}{4}$ T.?
6. How many pounds in 70 oz.? in 84 oz.? in 112 oz.?
7. How many tons in 2500 lb.? in 3000 lb.? in 4500 lb.?
8. How many ounces in $3\frac{3}{4}$ lb.? in $4\frac{1}{4}$ lb.? in $5\frac{1}{2}$ lb.?

103. Table of Liquid Measure. The following is the table of liquid measure:

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

A gallon contains 231 cu. in. Barrels vary in size, although in estimating the capacity of tanks and cisterns, 31½ gal. are considered a barrel (bbl.), and 2 bbl. a hogshead.

There is also a table of *apothecaries' liquid measure*, in which 16 fluid ounces make 1 pint.

ORAL EXERCISE

1. If you need ½ pt. of cream for a recipe, how many gills do you need? What part of a quart do you need?

2. If you need 3 pt. of milk for some cooking, how many quarts do you need?

3. A quart is what part of a gallon? Does your answer show you why this measure is called a quart?

4. How many pints are there in a gallon? how many gills?

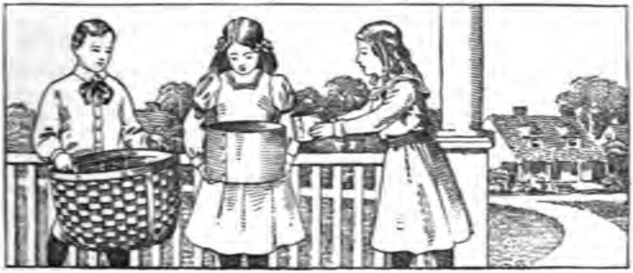
5. Name the various measures shown in the picture.

6. Name something that is sold by the pint; something that is sold by the quart; something that is sold by the gallon.

104. Table of Dry Measure. The following is the table of dry measure:

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

A bushel contains nearly $2150\frac{1}{2}$ cu. in., or about $1\frac{1}{4}$ cu. ft. The *dry quart* contains $67\frac{1}{5}$ cu. in. and the *liquid quart* contains $57\frac{3}{4}$ cu. in.



ORAL EXERCISE

1. In the picture, point to the quart measure, the peck measure, and the bushel measure.

2. If you feed a horse 8 qt. of oats a day, how many days will 1 bu. of oats last?

3. If you have 2 bu. of shelled corn, how many days will this last your chickens if you feed them 1 qt. of corn a day?

4. How many quarts in 2 pk.? in 4 pk.?

5. How many pecks in 2 bu.? in 4 bu.? in 8 bu.?

6. How many quarts in 1 bu.? in 2 bu.? in 10 bu.?

7. How many pints in 3 qt.? in 8 qt.? in 1 pk.? in 2 pk.?

8. Name something that is sold by the bushel; something that is sold by the peck.

105. Table of Time. The following is the table of time:

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) = 1 year (yr.)

Thirty days has September,
April, June, and November.

The other months have 31 days, except February, which has 28 days in common years and 29 days in leap years. The *common year* has 365 days, or 52 weeks and 1 day; the *leap year* has 366 days. A *century* is 100 years.

For 45 minutes after 7 o'clock we usually write 7.45, 7 : 45, or 7 45, using A.M. and P.M. respectively to denote morning and afternoon.

Years whose numbers are divisible by 4 but not by 100, and those divisible by 400, are leap years. Thus 1916, 2000, but not 2100, are leap years.

106. Table of Value. The table of value is as follows:

10 mills = 1 cent (ct. or ¢)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)

The mill is not coined. The $10 gold piece is sometimes called an eagle.

WRITTEN EXERCISE

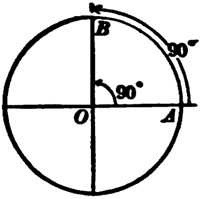
1. If you go to school $9\frac{1}{2}$ mo. a year, and we allow 4 wk. to the month, how many weeks do you go to school each year?

2. If you are in school $5\frac{1}{2}$ hr. a day for 5 da. each week, you are in school how many hours a week?

3. If you are in school $5\frac{1}{2}$ hr. a day, how many minutes a day are you in school?

4. If you are saving 5-cent pieces to buy a 75-cent box of note paper, how many 5-cent pieces will you have to save?

107. Circular Measure. The circumference of a circle is measured in *degrees*.



There are 360 degrees (written 360°) in a circumference and 90° in a right angle.

Because the angle at *O*, the center, increases just as fast as the part of the circumference between its arms increases, an angle is also measured in degrees.

108. Perpendicular Lines. A line that makes a right angle with another line is said to be *perpendicular* to it.

109. Arc. A portion of a circumference is called an *arc*.

110. Arc and Angle Measures. In measuring arcs,

360° = a circumference

That is, we divide the circumference, as stated above, into 360°.

In measuring angles we divide four right angles into 360°. That is,

360° = 4 right angles

In measuring both angles and arcs,

60 seconds (60″) = 1 minute (1′)
60 minutes = 1 degree (1°)

ORAL EXERCISE

1. How many degrees in $\frac{1}{2}$ of a circumference?

2. If one line is perpendicular to another, what kind of angle do the two lines form? How many degrees in the angle?

3. Is an angle acute, right, or obtuse if it contains 100°? 60°? 90°? 120°? 1°?

4. A machinist wishes to put together two pieces of steel making an angle $\frac{1}{3}$ of a right angle. How many degrees is this?

111. Table of Dozens. In some kinds of business this table is used:

12 units = 1 dozen (doz.)
12 dozen, or 144 units = 1 gross (gro.)

Formerly the term "great gross" was used for 12 gross, or 1728, and the word "score" was often used for "twenty."

ORAL EXERCISE

1. How many eggs are 3 doz. eggs? 4 doz. eggs?
2. How many oranges are $2\frac{1}{2}$ doz. oranges? $3\frac{1}{2}$ doz. oranges?
3. How many bananas are $1\frac{1}{4}$ doz. bananas?
4. How many pens in 2 gross? in 6 gross? in 8 gross?
5. How many dozen in $\frac{2}{3}$ of a gross? in $3\frac{1}{2}$ gross?
6. How much will a gross of padlocks cost at $5 a dozen?
7. How much will a gross of shears cost at $6 a dozen?
8. How much will $\frac{1}{2}$ gross of knives cost at $4 a dozen?

WRITTEN EXERCISE

1. How many are 7 gross? $9\frac{2}{3}$ gross? 16 gross?
2. How much will 9 gross of tablets cost at 25¢ a dozen?
3. At $7.20 a gross, how much will a dozen lamp chimneys cost? How much will $2\frac{1}{2}$ doz. cost?
4. A dealer bought 25 gross of blackboard erasers of which $\frac{1}{10}$ were damaged. How many were in good condition?
5. Which is cheaper, to buy a gross of articles for $125 or to buy 12 doz. at $11 a dozen? how much cheaper?
6. How much will $2\frac{1}{2}$ doz. knives cost at $100 a gross?
7. How much will 15 doz. saws cost at $200 a gross?

112. Counting Sheets of Paper. Formerly, sheets of paper were counted thus:

24 sheets = 1 quire
20 quires, or 480 sheets = 1 ream

Now, for convenience in counting, 500 sheets are often called a *ream*, and the word "quire" is used only for folded note paper, other paper being usually sold by the pound.

ORAL EXERCISE

1. How many sheets of fine writing paper in 2 quires? in 5 quires? in $3\frac{1}{2}$ quires? in 10 quires? in $\frac{1}{8}$ of a quire?

2. If writing paper costs 15¢ a quire, how much will 4 quires cost? How many sheets will there be?

Notice that 4 times 24 is the same as 4 times (25 − 1), or 100 − 4.

3. At 20¢ a quire, how much will 7 quires of paper, with $1 worth of envelopes, cost?

4. If a dealer buys paper at 19¢ a quire and sells it at 25¢ a quire, how much will he make on 10 quires?

WRITTEN EXERCISE

1. How many sheets in 17 quires? in $\frac{1}{3}$ of a quire?

2. In a dozen reams of 500 sheets each how many more sheets of paper are there than in a dozen reams of 480 sheets each?

3. At $16\frac{2}{3}$¢ a quire, how much will $7\frac{1}{2}$ quires of paper cost?

Remember the use of aliquot parts.

4. At $12\frac{1}{2}$¢ a quire, how much will $9\frac{1}{2}$ quires of paper cost? $6\frac{1}{2}$ quires? a dozen quires?

5. At 24¢ a quire for paper, and 1¢ each for envelopes, how much will it cost for paper, envelopes, and postage for eight letters from where you live to some other place in this country? Allow 1 sheet of paper to a letter.

113. Denominate Number. A concrete number that expresses measure is called a *denominate number.* For example, 3 ft., $5, and 3 bu. 2 pk. are denominate numbers, but 3 books and 5 chickens are not denominate numbers.

114. Compound Number. A denominate number in which two or more units are expressed is called a *compound number.* For example, 2 ft. 3 in. and 4 mi. 3 rd. 2 ft. are compound numbers.

115. Reduction. We may express 2 ft. 3 in. as 27 in., or as $2\frac{1}{4}$ ft. This changing of the form of a denominate number to another form without changing the value is called *reduction.*

116. Reduction Descending. Reduction from a higher to a lower denomination is called *reduction descending.* For example, 3 ft. may be reduced to 36 in.

117. Reduction Ascending. Reduction from a lower to a higher denomination is called *reduction ascending.* For example, 24 oz. may be reduced to 1 lb. 8 oz.

Simple cases of reduction, familiar from the work in primary arithmetic, have already been considered in the preceding pages.

Definitions like the above have little value, the subject of compound numbers having lost most of its former importance. We speak of "2 ft. 3 in." instead of "$2\frac{1}{4}$ ft.," but we rarely use the pound and ounce *together*, usually speaking of "2 lb. 4 oz." as "$2\frac{1}{4}$ pounds." But little time is now given to work in denominate numbers, particularly to work involving more than two denominations.

ORAL EXERCISE

1. Express 10 bu. as pecks; as quarts; as pints.
2. Express 64 qt. as pecks; as bushels; as pints.
3. Express 40 pt. as quarts; as gallons; as gills.
4. Express 20 ft. as inches; 21 ft. as yards; 21 yd. as feet.
5. Express 36 in. as yards; 36 ft. as yards; 30 yd. as feet.

118. Examples in Reduction Descending. 1. A strip of carpet is $7\frac{1}{4}$ yd. long. Is it long enough to reach the length of a hall that is 22 ft. long?

Here it is convenient to reduce $7\frac{1}{4}$ yd. to feet, or else to reduce 22 ft. to yards, so as to make the comparison.

$$\begin{array}{r} 7\frac{1}{4}\ \text{(yd.)} \\ 3\phantom{\text{(yd.)}} \\ \hline 21\frac{3}{4}\ \text{(ft.)} \end{array}$$

Because 1 yd. = 3 ft.,

we see that $7\frac{1}{4}$ yd. $= 7\frac{1}{4} \times 3$ ft. $= 21\frac{3}{4}$ ft.

While we think of multiplying 3 ft. by $7\frac{1}{4}$, we actually multiply $7\frac{1}{4}$ by 3, because this is shorter.

We therefore see that the carpet is too short by $\frac{1}{4}$ ft., or 3 in.

2. Reduce 7 rd. to inches.

$$\begin{array}{r} 16\frac{1}{2}\ \text{(rd.)} \\ 7\phantom{\text{(rd.)}} \\ \hline 115\frac{1}{2}\ \text{(ft.)} \\ 12\phantom{\text{(ft.)}} \\ \hline 1386\ \text{(in.)} \end{array}$$

Because 1 rd. $= 16\frac{1}{2}$ ft.,

we see that 7 rd. $= 7 \times 16\frac{1}{2}$ ft. $= 115\frac{1}{2}$ ft.

Because 1 ft. = 12 in.,

we see that $115\frac{1}{2}$ ft. $= 115\frac{1}{2} \times 12$ in. $= 1386$ in.

In this case we find it easier to multiply $16\frac{1}{2}$ by 7 than to multiply 7 by $16\frac{1}{2}$.

Such an example is rare. We seldom find a practical case more difficult than Ex. 1. It is very unusual to find a case in which we have to reduce, say, 17 yd. 2 ft. 7 in. to inches.

WRITTEN EXERCISE

Reduce the following:

1. 8 ft. 2 in. to inches.
2. 2 rd. 3 ft. to feet.
3. 5 rd. 9 ft. to feet.
4. 2 lb. 2 oz. to ounces.
5. 2 gal. 3 qt. to quarts.
6. 10 cd. to cubic feet.
7. 3 rd. 5 ft. to feet.
8. 2 A. to square rods.
9. 5′ 6″ to inches.
10. 5 yd. 1 ft. to feet.
11. $2\frac{3}{4}$ A. to square rods.
12. $23\frac{1}{3}$ cu. yd. to cubic feet.

13. A carpenter saws a length of 7 ft. 11 in. from a board. How many inches does he saw off?

14. A lady is using 2 yd. 8 in. of cloth. How many inches of cloth is she using?

15. A dealer is putting 7 lb. 12 oz. of candy in boxes. How many ounce boxes can he fill with the candy?

16. A man puts 16½ gal. of gasoline in a tank. How many quarts of gasoline does he put in?

17. A milk dealer sells 2 gal. 2 qt. of cream. How many pints of cream does he sell?

18. A farmer has 16½ bu. of oats in his bin. How many pecks of oats has he? How many quarts?

19. A farmer has built 64 rd. 8 ft. of fence. How many feet of fence has he built?

20. If a pile of wood contains 17 cords, how many cubic feet of wood does it contain?

21. How many square rods are there in 1¼ acres?

22. A rug is 3 ft. 8 in. long and 1 ft. 7 in. wide. How many inches in the perimeter of the rug?

23. A strip of carpet is 19 yd. 2 ft. long. How many feet in the length of the strip?

24. In excavating a cellar 600 cu. yd. of earth are removed. How many cubic feet of earth are removed?

25. A piece of land is 12 rd. long and 7 rd. wide. How many square rods in the piece? How many square feet?

26. A man buys a load of coal weighing 1 T. 750 lb. How many pounds of coal does he buy?

As suggested on page 26, teachers should frequently call the attention of their pupils to the necessity of estimating an answer in advance of the solution.

119. Examples in Reduction Ascending. 1. If the milk dealer sells 200 pt. of cream in a week, how many gallons does he sell?

Here we have to reduce 200 pt. to gallons.

Because $1 \text{ pt.} = \frac{1}{2} \text{ qt.}$,

we see that $200 \text{ pt.} = 200 \times \frac{1}{2} \text{ qt.} = 100 \text{ qt.}$

Because $1 \text{ qt.} = \frac{1}{4} \text{ gal.}$,

we see that $100 \text{ qt.} = 100 \times \frac{1}{4} \text{ gal.} = 25 \text{ gal.}$

$$\begin{array}{r@{\,}l} 2\overline{)200} & \text{(pt.)} \\ 4\overline{)100} & \text{(qt.)} \\ 25 & \text{(gal.)} \end{array}$$

2. Reduce 175 in. to feet and inches.

Because $1 \text{ in.} = \frac{1}{12} \text{ ft.}$,

we see that $175 \text{ in.} = 175 \times \frac{1}{12} \text{ ft.}$

$= 14\frac{7}{12} \text{ ft.} = 14 \text{ ft. } 7 \text{ in.}$

$$\begin{array}{r@{\,}l} 12\overline{)175} & \text{(in.)} \\ 14 & \text{(ft.)} \\ 7 & \text{(in.) rem.} \end{array}$$

It should be understood that these reductions are rather more difficult than those usually found in practical work.

WRITTEN EXERCISE

Reduce the following:

1. 96 in. to feet.
2. 48 oz. to pounds.
3. 176 oz. to pounds.
4. 3600 min. to hours.
5. 27 pk. to bushels.
6. 92 in. to feet.
7. 99 in. to yards.
8. 7200 sec. to hours.
9. 275 ft. to rods and feet.
10. 16,000 lb. to tons.
11. 527 pt. to bushels and pints; to quarts and pints.
12. 500 sq. in. to square feet and square inches.
13. 475 yd. to feet; to a fraction of a mile.
14. $\frac{3}{4}$ in. to a fraction of a foot; to a fraction of a yard.
15. $266\frac{2}{3}$ rd. to yards; to a fraction of a mile.
16. $6\frac{1}{2}$ in. to a fraction of a foot; to a fraction of a yard.
17. A woman uses 81 in. of ribbon. Express this as yards and inches; as yards and a fraction.

18. A carpenter uses 42 in. of molding. Express this as feet and inches; as feet and a fraction; as yards and inches.

19. A milk dealer sells 302 qt. of milk. Express this as gallons and quarts; as gallons and a fraction.

20. A milk dealer sells 78 pt. of cream. Express this as quarts and pints; as quarts and a fraction; as gallons and pints; as gallons and a fraction.

21. A dealer sells 137 qt. of gasoline. Express this as gallons and quarts; as gallons and a fraction.

22. A boy is running a half-mile race. After he has run 660 yd. what fraction of the distance has he run?

23. Express 775 in. as feet; as yards.

24. Express 32 in. as feet; 32 oz. as pounds; 32 ft. as yards; 32 pt. as quarts; 32 qt. as gallons.

25. Express 96 oz. as pounds; 96 in. as yards and a fraction; 96 ft. as yards; 96 min. as hours and minutes.

26. If you need $9\frac{1}{2}$ yd. of cloth for a garment, and have 72 in. already, how many yards will you have to buy?

27. If you cut 86 sq. in. from a piece of cloth containing $2\frac{1}{2}$ sq. ft., how many square feet and square inches are left?

28. If you cut off $\frac{5}{8}$ of a piece of tin containing 3 sq. ft., how many square inches do you cut off?

29. From a sheet of tin 3 ft. long and $2\frac{1}{2}$ ft. wide a piece is cut containing 500 sq. in. How many square feet and square inches are left?

30. From an iron plate 2 ft. 6 in. by 7 ft. 6 in. a piece is cut containing 108 sq. in. How many square feet are left?

31. If from a piece of glass 4 ft. 6 in. long by 3 ft. wide a strip 1 in. wide is cut from each side, how many square inches are left?

120. Addition of Compound Numbers. The general principles of adding compound numbers are similar to those of adding ordinary whole numbers. The only difference is that in compound numbers the number of units of a lower denomination required to make one unit of the next higher varies with the denomination we are using.

In our school shop we wished to make a picture frame 2 ft. 8 in. long and 1 ft. 6 in. wide. How much molding do we need?

There are four sides to the frame, these being 2 ft. 8 in., 2 ft. 8 in., 1 ft. 6 in., and 1 ft. 6 in.

We first add the inches and find the sum to be 28 in., or 2 ft. 4 in. We write the 4 under the inches, and add the 2 with the feet.

Practically we would see that 6 in. + 6 in. = 1 ft., and 8 in. + 8 in. = 1 ft. 4 in.

Adding the feet, we have 8 ft.

Hence the result is 8 ft. 4 in.

ft.	in.
2	8
2	8
1	6
1	6
8	4

In practical life we rarely meet cases more difficult than those given in the following examples.

WRITTEN EXERCISE

Add the following:

1.		2.		3.		4.	
ft.	in.	sq. ft.	sq. in.	yd.	in.	lb.	oz.
5	8	94	30	11	17	148	8
9	10	82	85	16	20	862	13
22	6	71	63	35	19	421	9

5.		6.		7.		8.	
ft.	in.	sq. ft.	sq. in.	rd.	ft.	lb.	oz.
28	11	38	48	26	13	385	13
17	9	56	84	47	9	421	12
35	5	93	57	56	11	635	9

121. Subtraction of Compound Numbers. The general principles of subtracting compound numbers are quite similar to those of subtracting ordinary whole numbers.

From a board 9 ft. 2 in. long Robert wishes to cut a piece 3 ft. 8 in. long for a table top which he is making. How much will he have left?

ft.	in.
9	2
3	8
5	6

Since we cannot subtract 8 in. from 2 in., we take 1 ft., which equals 12 in., of the 9 ft. and add it to the 2 in. We then have 14 in. – 8 in. = 6 in.

We now have 8 ft. left of the 9 ft.

Then 8 ft. – 3 ft. = 5 ft.

Therefore Robert has left a board 5 ft. 6 in. long.

In practical work we so rarely have to subtract numbers of more than two denominations that in this book, which gives only the essentials of arithmetic, all other cases are excluded.

WRITTEN EXERCISE

Subtract the following:

1.

ft.	in.
62	8
15	10

2.

yd.	ft.
32	2
18	1

3.

yd.	in.
16	3
9	5

4.

gal.	qt.
32	1
18	3

5.

ft.	in.
91	4½
38	8¼

6.

bu.	qt.
31	23
17	17

7.

rd.	ft.
37	2
19	15

8.

A.	sq. rd.
64	75
43	119

9.

bu.	pk.
61	1
37	2

10.

qt.	pt.
71	0
39	1

11.

lb.	oz.
35	8
27	13

12.

sq. ft.	sq. in.
96	72
38	107

122. Multiplication of Compound Numbers. The general principles of multiplying compound numbers are similar to those of multiplying ordinary whole numbers.

1. A wholesale grocer is packing English walnuts in fancy boxes, 4 lb. 9 oz. to the box. He sends them out in cases of a half dozen boxes. How many pounds of walnuts in six of these boxes?

lb.	oz.
4	9
	6
27	6

Here we have to multiply 4 lb. 9 oz. by 6.

We might multiply as with other numbers and obtain 24 lb. 54 oz., and reduce this to 27 lb. 6 oz. But it is easier to say, "6 × 9 oz. = 54 oz. = 3 lb. 6 oz.; 6 × 4 lb. = 24 lb., which added to 3 lb. = 27 lb."

2. Multiply 243 ft. 7 in. by 27.

ft.	in.
243	7
	27
6561	189
6576	9

Multiplying as with other numbers, the result is 6561 ft. 189 in., which is reduced to 6576 ft. 9 in.

In this case it is not easy to reduce while we are multiplying, as we did in Ex. 1. We therefore multiply as with whole numbers and reduce the product to a simpler form.

The tendency at present is to avoid the multiplication of compound numbers except in very simple cases, chiefly those involving feet and inches. Instead of multiplying 5 lb. 4 oz. by 8, we multiply $5\frac{1}{4}$ lb. by 8, and similarly in other cases.

ORAL EXERCISE

Multiply the following:

1. 3 ft. 4 in. by 2.
2. 7 ft. 2 in. by 4.
3. 8 ft. 1 in. by 9.
4. 9 yd. 6 in. by 3.
5. 8 yd. 3 in. by 8.
6. 6 yd. 2 in. by 7.
7. 8 lb. 2 oz. by 6.
8. 7 rd. 1 ft. by 8.
9. 11 lb. 3 oz. by 4.
10. 9 gal. 1 qt. by 3.
11. 7 qt. 1 pt. by 8.
12. 15 lb. 4 oz. by 4.
13. 17 lb. $1\frac{1}{2}$ oz. by 10.
14. 20 rd. 6 ft. by 12.

WRITTEN EXERCISE

Multiply the following:

1. 4 ft. 6 in. by 5.
2. 4 yd. 2 in. by 6.
3. 9 ft. 3 in. by 4.
4. 4 lb. 4 oz. by 9.
5. 6 yd. 3 in. by 8.
6. 7 lb. 15 oz. by 4.
7. 8 lb. 13 oz. by 6.
8. 7 lb. 11 oz. by 9.
9. 6 ft. 8 in. by 7.
10. 7 ft. 11 in. by 4.
11. 5 ft. $10\frac{1}{2}$ in. by 2.
12. 4 yd. 27 in. by 3.
13. 6 yd. 33 in. by 9.
14. 27 lb. 3 oz. by 4.
15. 4 mi. 3 ft. by 8.
16. 73 ft. 2 in. by 5.
17. 39 lb. 5 oz. by 7.
18. 30 yd. 1 in. by 8.
19. 3 gal. 2 qt. by 2.
20. 8 gal. 2 qt. by 9.
21. 6 gal. 3 qt. by 7.
22. 4 mi. 118 rd. by 3.
23. 6 mi. 109 rd. by 7.
24. 5 mi. 1250 ft. by 3.

25. If each side of a five-sided figure (pentagon) is 1 ft. $6\frac{1}{2}$ in., what is the perimeter?

26. If each side of a six-sided figure (hexagon) is 1 ft. $7\frac{7}{8}$ in., what is the perimeter?

27. If a cubic foot of water weighs 62 lb. 8 oz., what is the weight of the water in a tank that contains 48 cu. ft.?

28. If the height of each story above the first, or ground, story in a city office building is 11 ft. 9 in., and the height of the first story is 15 ft. 8 in., and there are nine stories in all, what is the height of the building?

29. A man travels in an automobile 6 da. a week for 3 wk. The first week he averages 8 gal. 2 qt. of gasoline a day, the second week 7 gal. 3 qt., and the third week 8 gal. 1 qt. How much gasoline does he use in the three weeks?

123. Division of Compound Numbers. If you have a board 7 ft. 6 in. long, and cut it into two equal parts to make a top for a workshop bench, how long is each part?

$$\begin{array}{r|rr} & \text{ft.} & \text{in.} \\ 2 & 7 & 6 \\ \hline & 3 & 9 \end{array}$$

Here we have to divide 7 ft. 6 in. by 2.

We see that 7 ft. ÷ 2 = 3 ft. with remainder 1 ft., or 12 in.

Then we have 12 in. + 6 in., or 18 in., still to be divided.

18 in. ÷ 2 = 9 in.

Therefore the result is 3 ft. 9 in.

The teacher will recognize the fact that we seldom have to divide compound numbers, and that when a case arises it usually involves only two denominations.

It is very rare to find a practical case in which we need to divide one compound number by another. If, for example, we have to divide 41 ft. 3 in. by 2 ft. 9 in., we simply reduce both to feet (41¼ ft. ÷ 2¾ ft. = 15) or to inches (495 in. ÷ 33 in. = 15). In this book these unreal cases have no place.

WRITTEN EXERCISE

1. The perimeter of an equilateral (equal-sided) triangle is 11 ft. 4½ in. Find the length of each side.

2. The perimeter of a square is 6 ft. 10 in. Find the length of each side.

3. A steamship changed its longitude 59° 37′ in 7 da. How many degrees of longitude did it sail per day?

4. If you are in school 5 hr. 30 min. a day, and recite half the time, how many hours and minutes do you recite?

Divide as follows:

5. 75 ft. 9 in. ÷ 3.
6. 91 ft. 8 in. ÷ 5.
7. 7 lb. 8 oz. ÷ 10.
8. 6 lb. 3 oz. ÷ 11.
9. 2 mi. 120 rd. ÷ 38.
10. 20 hr. 12 min. ÷ 6.
11. 17 wk. 6 da. ÷ 12½.
12. 36 hr. 59 min. ÷ 7.
13. 143 ft. 4 in. ÷ 8.
14. 39 lb. 12 oz. ÷ 12.

15. At the rate of a mile in 4 min., how far will an automobile go in 3 hr.? in $3\frac{1}{2}$ hr.?

16. How far will a horse go in 3 hr., traveling at the rate of a mile in 9 min.?

17. If a wheel is $10\frac{18}{25}$ ft. in circumference, how many times will it turn in going 25 mi.?

18. A square plot of ground has a perimeter of 510 ft. 8 in. Find the length of each side.

19. A cistern capable of holding 464 cu. ft. of water is $\frac{1}{3}$ full. Express the amount of water in cubic feet and cubic inches.

20. At 75 lb. to the acre, how many acres of land can be enriched with $1687\frac{1}{2}$ lb. of fertilizer?

21. A woman buys $3\frac{1}{4}$ lb. of coffee for $1.04. How much does she pay per pound?

22. If a man walks at the rate of a mile in $19\frac{1}{2}$ min., how many miles can he walk in 39 min.?

23. If a cubic foot of gold weighs 1187 lb. 8 oz., and if gold is 19 times as heavy as water, how much does a cubic foot of water weigh?

24. If it takes a man 8 hr. 36 min. to walk a certain distance, how long will it take an automobile to travel the same distance if it goes 12 times as fast?

25. A man can row across a lake $1\frac{3}{4}$ mi. wide in 28 min. At this rate, how long will it take him to row 1 mi.?

26. A man can row 1 mi. in 20 min. At this rate, how long will it take him to row across a lake $1\frac{3}{4}$ mi. wide?

27. On a trip of 3 mi. an automobile took 2 min. 34 sec. for the first mile, 2 min. 17 sec. for the second, and 1 min. 56 sec. for the third. What was the average rate per mile?

WRITTEN REVIEW

1. How many feet are there in $\frac{1}{8}$ mi.? in $\frac{1}{4}$ mi.?

2. How many yards are there in $\frac{3}{8}$ mi.? in $\frac{3}{4}$ mi.?

3. How many yards are there in 5 rd.? in $7\frac{1}{2}$ rd.?

4. How many yards are there in 79 ft.? in 113 in.?

5. If a river is $7\frac{1}{2}$ fathoms deep, what is its depth in feet?

6. Calling $1\frac{3}{20}$ mi. a knot, how many miles in 160 knots?

7. How many square inches in $7\frac{1}{2}$ sq. ft.?

8. How many square feet in 2 sq. yd. 7 sq. ft.?

9. How many cubic feet in 49 cu. yd. 7 cu. ft.?

10. A hand is 4 in. If a horse is 15 hands high, what is its height in inches? What is its height in feet?

11. Taking $24\frac{3}{4}$ cu. ft. as a perch, how many perches in 99 cu. ft. of stone?

12. Taking, as is often done, 25 cu. ft. as a perch, how many cubic feet in $17\frac{1}{2}$ perches of stone?

13. A gallon contains 231 cu. in. How many gallons of water will it take to fill an aquarium containing $1\frac{1}{2}$ cu. ft.?

14. Allowing $31\frac{1}{2}$ gal. to the barrel, how many gallons of water will it take to fill a tank that holds $16\frac{1}{2}$ bbl.?

15. Allowing $2150\frac{1}{2}$ cu. in. to the bushel, how many cubic inches in a bin that holds 16 bu.?

16. Allowing $67\frac{1}{5}$ cu. in. to the dry quart, how many cubic inches in a peck? How many cubic inches in a pint?

17. Allowing $2150\frac{1}{2}$ cu. in. to the bushel, how many cubic feet in a bin that holds 3456 bu.?

18. Allowing $67\frac{1}{5}$ cu. in. to the dry quart, how many quarts of oats will it take to fill a box that contains $604\frac{4}{5}$ cu. in.?

19. Allowing $1\frac{1}{4}$ cu. ft. to a bushel, how many bushels of wheat can be put into a bin that will hold 900 cu. ft.?

20. A dry quart contains $67\frac{1}{5}$ cu. in. How many quarts in a measure containing 336 cu. in.?

21. A railway train goes $63\frac{9}{10}$ mi. in an hour and a half. What is its rate per hour?

22. If a man walks at the rate of a mile in $19\frac{1}{3}$ min., how many miles can he walk in 1 hr. $37\frac{1}{2}$ min.?

23. A woman buys $15\frac{1}{2}$ yd. of carpet for $7.44. What does she pay per yard?

24. If a cubic foot of water weighs 62 lb. 8 oz., what is the weight of the water in a cistern that contains 464 cu. ft.?

25. How many pounds do 5 cu. yd. 18 cu. ft. of earth weigh, if a cubic yard weighs 1 T. 825 lb.?

26. A man can row across a lake, $2\frac{3}{4}$ mi., in 42 min. At this rate, how long will it take him to row 1 mi.?

27. A certain horse is 14 hands high. Express this in inches; in feet and inches; in feet and a fraction.

28. A cubic foot of water weighs nearly 1000 oz. Express this weight in pounds and ounces, and in pounds and a fraction of a pound.

29. On a trip of $2\frac{1}{2}$ mi. an automobile took 2 min. 16 sec. for the first mile, 2 min. 10 sec. for the second, and 1 min. 6 sec. for the last $\frac{1}{2}$ mi. What was the average rate per mile?

30. If a cubic foot of ice weighs 57 lb. 8 oz., what will be the weight in tons of the mass of ice 1 ft. thick that covers a pond whose surface is 100,000 sq. yd.?

31. If you are in school 5 hr. a day, how many minutes are you in school during the 5 da. of the week? how many seconds?

Add the following:

32.		**33.**		**34.**		**35.**	
ft.	in.	ft.	in.	ft.	in.	ft.	in.
127	4	329	7	635	7	246	11
286	3	206	8	82	6	321	9
491	7	129	11	39	10	480	7
302	5	43	5	41	11	39	10

36.		**37.**		**38.**		**39.**	
yd.	in.	gal.	qt.	bu.	pk.	bu.	pk.
278	10	27	1	427	1	171	1
63	13	92	3	63	1	26	3
49	17	167	1	29	3	48	1
326	5	43	2	28	2	39	2

Subtract the following:

40. $9\frac{1}{2}$ lb. – 9 oz.

41. $3\frac{2}{3}$ ft. – 8 in.

42. $8\frac{2}{3}$ rd. – 5 yd.

43. $7\frac{1}{2}$ bu. – 3 pk.

44. 480 ft. 2 in. – 197 ft. 10 in.

45. 293 lb. 5 oz. – 169 lb. 12 oz.

46. 402 yd. 5 in. – 327 yd. 12 in.

47. 572 bu. 1 pk. – 296 bu. 3 pk.

Multiply the following:

48. 102 lb. 1 oz. by 42.

49. 249 lb. 2 oz. by 32.

50. 248 ft. 2 in. by 75.

51. 63 lb. 2 oz. by 27.

52. 78 lb. 9 oz. by 16; by 32.

53. 329 ft. 7 in. by 96; by 48.

Divide the following:

54. 87 lb. 8 oz. by 4.

55. 648 gal. by 4 gal.

56. 302 lb. 8 oz. by 2.

57. 125 ft. 10 in. by 5.

58. 95 lb. 1 oz. by 3.

59. 981 gal. by 9 gal.

60. 429 lb. 4 oz. by 4; by 8.

61. 425 ft. 5 in. by 25; by $12\frac{1}{2}$.

LITTLE EXAMINATIONS

I. 1. $72\frac{1}{2}$ ft. = (?) in.
2. 75 in. = (?) ft.
3. $2\frac{1}{2}$ lb. = (?) oz.
4. 36 qt. = (?) pk.
5. $8\frac{1}{4}$ ft. − 9 in.
6. $2\frac{1}{2} \times 3\frac{1}{2}$ ft.
7. 3 × 7 ft. 9 in.
8. $72\frac{1}{2}$ ft. ÷ 3.

II. 1. $39\frac{3}{4}$ ft. = (?) in.
2. 96 in. = (?) yd.
3. $6\frac{1}{2}$ pk. = (?) qt.
4. 36 sq. ft. = (?) sq. yd.
5. $2\frac{1}{2}$ yd. − 8 in.
6. $3\frac{7}{8} \times 6\frac{3}{4}$ ft.
7. 8 × 4 bu. 2 qt.
8. $43\frac{3}{4}$ ft. ÷ 6.

III. 1. $16\frac{1}{2}$ ft. = (?) yd.
2. 1280 A. = (?) sq. mi.
3. $3\frac{3}{4}$ T. = (?) lb.
4. 7 sq. ft. = (?) sq. in.
5. $3\frac{1}{2}$ lb. + 9 oz.
6. $5\frac{3}{4} \times 7\frac{1}{2}$ lb.
7. 6 × 8 lb. 6 oz.
8. $7\frac{1}{2}$ yd. ÷ 9.

IV. 1. 273 ft. = (?) yd.
2. $3\frac{1}{2}$ sq. mi. = (?) A.
3. $62\frac{1}{2}$ gal. = (?) qt.
4. 35 bu. = (?) pk.
5. $2\frac{1}{2}$ cu. ft. = (?) cu. in.
6. $6\frac{1}{2}$ hr. + 15 min.
7. $6\frac{3}{4}$ bu. − 3 pk.
8. $6\frac{3}{8} \times 8\frac{1}{2}$ bu.
9. 9 × 4 yd. 6 in.
10. 6 lb. 3 oz. ÷ 7.

V. 1. $32\frac{1}{2}$ yd. = (?) in.
2. 256 cu. ft. = (?) cd.
3. $7\frac{1}{2}$ gal. = (?) pt.
4. $2\frac{3}{4}$ hr. = (?) min.
5. 96 gi. = (?) gal.
6. $2\frac{1}{2}$ yd. + 7 ft.
7. 16 ft. − 10 in.
8. $7\frac{3}{4} \times 8\frac{1}{2}$ mi.
9. 7 × 6 gal. 3 qt.
10. 7 rd. 3 ft. ÷ 5.

REVIEW AND DRILL

I. DRILL SECTION

ORAL EXERCISE

Add the following:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
42	48	79	58	38	44	78
63	13	32	62	26	29	37

In adding 49 and 65 think of 49 + 60 + 5 = 109 + 5 = 114. In all such cases it is easier and more rapid to begin at the left to add.

Make change from $1, having made purchases as follows:

8. 24¢ 58¢ 74¢ 38¢ 87¢ 49¢ 59¢

9. 35¢ 62¢ 85¢ 76¢ 92¢ 63¢ 82¢

10. Multiply by 25: 24, 48, 56, 16, 64, 32, 84.

11. Multiply by 50: 16¢, 20¢, 34¢, 42¢, 66¢, 28¢, 84¢.

12. Add $\frac{1}{2}$ to 2; to $1\frac{1}{2}$; to $2\frac{1}{2}$; to $1\frac{1}{4}$; to $1\frac{3}{4}$; to $\frac{1}{3}$.

13. Multiply by $2\frac{1}{2}$: 6, 10, 12, 24, 16, 18, 20.

14. Express the following in the next lower denominations:

5 yd. 8 hr. 5 wk. 9 gal. 25 qt. 6 lb. 2 A.

15. Express the following in the next higher denominations:

48 mo. 64 oz. 32 qt. 21 da. 56 wk. 48 pt. 33 ft.

16. Give the statement from the table of denominate numbers that is suggested by each of the following numbers:

160 320 27 128 8 16 9

WRITTEN EXERCISE

Copy, add, and check the following in eight minutes or less:

1.	2.	3.	4.
$359.68	$496.48	$396.04	$982.69
298.76	127.92	72.63	604.05
400.92	600.00	129.67	81.97
640.32	503.52	871.64	631.83
701.24	872.08	280.93	400.00

Multiply the first number by each of the two numbers below it:

5.	6.	7.	8.	9.
28,764	84,216	12,987	18,625	16,284
345	974	317	315	708
678	287	526	475	354

10. In Exs. 5 and 6, divide the first number by each of the two numbers below it.

11. In Exs. 7–9, divide the first number by each of the two numbers below it.

Multiply the following:

12. 27 × \$42.30.

13. 40 × \$78.20.

14. $6\frac{1}{2}$ × \$47.50.

15. 112 × \$1.12$\frac{1}{2}$.

16. 87 × 32$\frac{2}{3}$.

17. 72 × \$2.16$\frac{2}{3}$.

18. 165 × \$1.33$\frac{1}{3}$.

19. 48 × \$1.66$\frac{2}{3}$.

20. 56 × \$1.37$\frac{1}{2}$.

Perform the following operations:

21. $6\frac{7}{8} \times 6\frac{3}{4} \times 10\frac{2}{3}$.

22. $15\frac{2}{3} \times 33\frac{1}{3} \times 16\frac{7}{8}$.

23. $3\frac{1}{2} + 16\frac{2}{3} + 48\frac{3}{4}$.

24. $92\frac{1}{8} - 37\frac{3}{4} - 2\frac{1}{2}$.

25. Reduce the following to sixteenths:

$\frac{3}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ $\frac{12}{64}$ $\frac{24}{32}$ $\frac{1}{2}$ $\frac{20}{64}$ $\frac{16}{32}$

WRITTEN EXERCISE

1. The title-page of an old book has the date MDCXCIV. In what year was the book printed?

2. Make out a cash account as follows and balance it: Jan. 1, cash on hand, \$3.40. Receipts: Jan. 15, shoveling snow, 50¢; Jan. 20, delivering papers for the week, 85¢. Expenses: Jan. 3, repairing skates, 10¢; Jan. 3 to 29, street-car tickets, 45¢. Balanced on Feb. 1.

3. Write the tables of length and of square measure.

4. In a certain city there are eight blocks to a mile. If a boy lives ten blocks from the schoolhouse, how many rods does he go in walking from his home to the school?

5. Without any written computation, write the answer to this question: What will 64 gramophones cost at \$25 each?

6. Mt. Whitney lacks 1339 ft. of being 3 mi. high. What is its height in feet? in yards?

7. If each book is $1\frac{1}{4}$ in. thick, how many books in a pile 30 in. high? in a pile 25 in. high?

8. Find the cost of 112 yd. of silk at \$$1.12\frac{1}{2}$ a yard.

9. If a train travels a mile in $1\frac{1}{4}$ min., how far will it travel in $6\frac{1}{4}$ min.? in 25 min.? in 1 hr. $2\frac{1}{2}$ min.?

10. Measure the cover of your geography and draw a plan of it on the scale of $\frac{1}{4}$.

11. Which is cheaper, to buy a gross of articles for \$250 or to buy 12 doz. at \$22 a dozen? How much cheaper?

12. How many feet of fence will be needed to inclose a field 64 rd. 8 ft. long, and 42 rd. wide?

13. When a man is rowing at the rate of $1\frac{3}{4}$ mi. in 28 min., what is his rate per minute?

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

State the units used in measuring the following:

1. The length of dress goods; the width.

2. The quantity of a liquid.

3. The quantity of a dry substance, like grain.

4. Sugar, in buying it; in cooking.

5. Rice, in buying it; in cooking.

6. Tea, in buying it; in making it for the table.

7. Cocoa, in buying it; in making it for the table.

8. Salt, in buying it; in cooking.

9. Kerosene, in large quantities; in small quantities.

10. Water, in large quantities; in cooking.

11. The length of carpeting; the width.

12. The dimensions of a book cover; of a floor; of a field.

13. The area of a sheet of paper; of a floor; of a field.

14. The capacity of an aquarium.

15. Milk, when sold to a creamery; when sold to a dealer; when sold to a family.

16. Coffee, when sold in large quantities; when sold to families; when used for cooking.

17. Potatoes, when sold in large quantities; when sold in small quantities at a city grocery; when used in cooking.

18. Butter, when sold in large quantities; when sold in small quantities; when used in cooking.

19. Flour, when sold in large quantities; when sold in small quantities; when used in cooking.

III. INDUSTRIAL PROBLEMS

THE BOYS' CORN CLUB

1. A boy rented from his father, at $5 an acre, a field 40 rd. long and 11 rd. wide. How much did he charge to expenses as rent?

2. He estimated his labor at 10¢ an hour and the work of each horse at 25¢ an hour. In one month he put in 42 hr., during 4 hr. of which he had a horse. Find the amount.

3. He needed for seed 16 ears of corn to the acre. How many ears did he need in all? He paid for the seed at the rate of $2\frac{1}{2}$¢ an ear. How much did he pay?

4. He planted the hills 3 ft. 4 in. apart in rows 3 ft. 6 in. apart. How many hills could he put in, making an allowance of 1 ft. 6 in. at each end of the field and 2 ft. 6 in. at each side?

5. It cost him $12.48 per acre to raise the corn, including rental, seed, labor, and fertilizer. How much did it cost him in all? If the corn was worth 40¢ a bushel, and he raised $274\frac{3}{4}$ bu. in all, what was his total profit?

PAVING THE CITY STREETS

1. In preparing a street for paving, John saw that the workmen took out dirt to the average depth of 14 in. The street is 40 ft. wide and $\frac{1}{2}$ mi. long. How many cubic yards of dirt are to be removed, and how much will it cost at 50¢ per cubic yard?

2. John finds that they put in a foundation of 8 in. of concrete costing $1.05 per square yard. How much does this cost?

3. John finds that the concrete is to be covered with brick at $2.35 per square yard, or asphalt at $3.10 per square yard. What is the difference in cost for the street?

4. The curbing for the street costs 78¢ per running foot, and there is a curbing all along both sides, except where there are cross streets. John finds that 480 ft. is to be deducted on each side for intersecting streets. How much will the curbing cost?

5. John's father's lot has a frontage of 66 ft. on the street. If the city requires him to pay half the cost of the paving to the middle of the street, and half the cost of the curbing in front of his lot, how much must he pay if brick is used for the paving?

CHAPTER IV

DECIMAL FRACTIONS

124. Meaning of Decimals. You have learned that \$3.25 means 3 dollars + 25 cents, or $\$3 + \$\frac{25}{100}$.

Just as \$3.25 means $\$3 + \$\frac{25}{100}$,
so 3.25 ft. means 3 ft. + $\frac{25}{100}$ ft., or $3\frac{1}{4}$ ft.;
4.75 in. means 4 in. + $\frac{75}{100}$ in., or $4\frac{3}{4}$ in.;
2.5 means $2 + \frac{5}{10}$, or $2\frac{1}{2}$;
.5, or 0.5, means $\frac{5}{10}$, or $\frac{50}{100}$, or 0.50, or $\frac{1}{2}$;
.06, or 0.06, means $\frac{6}{100}$ or $\frac{3}{50}$;
and .125, or 0.125, means $\frac{125}{1000}$, or $\frac{1}{8}$.

125. Decimal Fraction. A fraction whose denominator is not written but is some power of 10 is called a *decimal fraction.*

Thus 0.5 (5 tenths) is a decimal fraction. If written in the form $\frac{5}{10}$ it becomes a common fraction.

The expression 0.5 means the same as .5. The 0 may be written or not, as we please. Those who have much computing to do usually find it better to write the 0 before .5, except where the numbers are written in columns, so there shall be no misunderstanding. It is perfectly proper to omit it, however, as is always done when we have columns that are ruled for dollars and cents, and as is often the case in printed numbers. You should accustom yourself to both forms.

Decimal fractions are usually called *decimals.* The period placed at the left of tenths is called the *decimal point.*

126. Reading Decimals. The names of the decimal places most frequently used are given in the following scheme:

Thousands	Hundreds	Tens	Units	(Decimal point)	Tenths	Hundredths	Thousandths	Ten-thousandths
8	3	9	5	.	2	7	6	5

This number is read "eight thousand, three hundred ninety-five and two thousand seven hundred sixty-five ten-thousandths." The orders beyond ten-thousandths are hundred-thousandths, millionths, ten-millionths, and so on. Practically, we do not have much need for decimals beyond thousandths.

Pupils should be given much exercise in reading such numbers from the blackboard, avoiding the use of "and" except between the units and tenths, and the vulgarism of "aught" for "naught."

127. Writing Decimals. Although we write a decimal fraction without its denominator, the denominator is shown by the decimal point and is indicated in reading.

Thus 0.9 has no printed denominator, but when we say "nine tenths" we indicate the fact that the denominator is ten, as well as state that the numerator is nine.

Therefore, *write only the numerator of the decimal fraction, inserting such zeros as may be necessary to indicate the denominator, and place the decimal point before the tenths.*

Thus, to write in figures two hundred one hundred-thousandths, we write 201 and prefix two zeros, preceded by a decimal point, so that the last figure is in hundred-thousandths' place. We then have 0.00201.

ORAL EXERCISE

Read the following numbers:

1. 1.25.
2. 3.78.
3. 2.95.
4. 6.33.
5. 0.125.
6. 48.675.
7. 0.375.
8. 67.005.
9. 32.375.
10. 100.025.
11. 91.075.
12. 300.075.

13. What is the name of the place to the right of tenths? to the right of that? What is the name of the fourth decimal place? the fifth? the sixth?

Since we rarely read the names of numbers beyond three decimal places, even when we write them, the chief drill should be on reading decimal fractions like those in Exs. 1–12.

Read the following numbers:

14. 16.004.
15. 75.137.
16. 82.3745.
17. 12.3147.
18. 91.82734.
19. 16.42398.
20. 68.998232.
21. 14.763458.

WRITTEN EXERCISE

Write the following numbers in figures:

1. Three tenths; five tenths; nine tenths.

2. Four and four tenths; seven and eight tenths.

3. Seventy-five hundredths; thirty-three hundredths.

4. One hundred forty-four thousandths; six hundred seventy-five thousandths; four ten-thousandths.

5. Five and seven thousandths; eight and forty-two thousandths; twenty-five thousandths; one millionth.

6. Twelve hundredths; fifty-five hundredths; nine hundredths; one hundredth; twenty hundredths.

7. One hundred twenty-five thousandths; forty-seven thousandths; ten thousandths; five thousandths.

128. Annexing Zeros to Decimals. Because we know that the fraction $\frac{5}{10} = \frac{50}{100}$, or $0.5 = 0.50$, we see that

Annexing a zero to a decimal fraction does not change its value.

Decimals that have the same number of decimal places are called *similar decimals.*

Therefore, *to change dissimilar decimals to similar decimals, give them the same number of decimal places by annexing or cutting off zeros.*

For example, change 0.25, 2.125, and 3.5 to similar decimals. We see that

$$0.25 = 0.250 \qquad 2.125 = 2.125, \qquad 3.5 = 3.500,$$

and 0.250, 2.125, and 3.500 are similar decimals.

WRITTEN EXERCISE

Change to similar decimals:

1. 0.2, 0.17.
2. 0.8, 0.08.
3. 0.7, 0.002.
4. 0.9, 0.130.
5. 0.6, 0.600.
6. 0.60, 0.600.
7. 0.6, 0.60.
8. 0.6, 0.123.
9. 0.5, 0.700.
10. 0.4, 0.050.
11. 0.1, 0.357.
12. 0.5, 0.50.
13. 0.5, 0.500.
14. 0.50, 0.500.
15. 0.12, 0.103.
16. 0.72, 0.008.
17. 0.69, 0.070.
18. 0.30, 0.700.
19. 0.7, 0.70.
20. 0.7, 0.700.
21. 0.70, 0.700.

22. Express as hundredths: 0.6; 0.7; 0.9; 0.750.

23. Express as tenths: 0.70; 0.90; 0.500; 0.900; 0.800.

24. Express as thousandths: 0.1; 0.3; 0.10; 0.30; 0.7; 0.70.

25. Compare 0.2 and 0.20; 0.2 and 0.200; 0.20 and 0.200.

26. Compare 2.5 and 2.50; $2\frac{1}{2}$ and 2.50; $2\frac{1}{2}$ and 2.500.

129. Changing Decimals to Common Fractions. Because the fraction $0.25 = \frac{25}{100}$, which equals $\frac{1}{4}$, therefore

To change a decimal to a common fraction, write the numerator and the denominator of the decimal and then reduce the common fraction to lowest terms.

In a case like that of $0.33\frac{1}{3}$ we proceed in the same way, thus:

$$0.33\frac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{1}{3}.$$

ORAL EXERCISE

Read these decimals as common fractions in lowest terms:

1. 0.1.
2. 0.3.
3. 0.7.
4. 0.9.
5. 0.11.
6. 0.17.
7. 0.2.
8. 0.4.
9. 0.6.
10. 0.25.
11. 0.5.
12. 0.50.
13. 0.40.
14. 0.80.
15. 0.75.

WRITTEN EXERCISE

Change to common fractions in lowest terms or to mixed numbers:

1. 0.8.
2. 0.60.
3. 0.20.
4. 0.40.
5. 0.35.
6. 0.65.
7. 0.45.
8. 0.55.
9. 0.125.
10. 0.375.
11. 0.625.
12. 0.875.
13. 1.25.
14. 2.75.
15. 5.125.
16. 10.375.
17. Express as hundredths: 0.3, 0.5, 0.1, 0.8, 4, 63.
18. Express as thousandths: 0.5, 0.25, 0.37, 0.09, 31.
19. Express as hundredths: 0.470, 0.680, 0.100, 0.040, 7.8.
20. Express 0.125 as eighths. Find 0.125 of $1600.
21. Express $0.33\frac{1}{3}$ as thirds. Find $0.33\frac{1}{3}$ of 621.
22. Express $0.37\frac{1}{2}$ as eighths. Find $0.37\frac{1}{2}$ of 1768.

130. Changing Common Fractions to Decimals. Since a common fraction indicates division (pages 47, 55), we may divide the numerator by the denominator and thus change the fraction to the decimal form. For example, consider the fraction $\frac{3}{8}$.

We have found that we may annex zeros after the decimal point without changing the value; that is, $3 = 3.000$. We may now divide exactly as in United States money, and the result is 0.375.

$$\begin{array}{r} 8\overline{)3.000} \\ 0.375 \end{array}$$

That this is allowable may be seen from the following: 3.0 means 30 tenths, and 30 tenths $\div$ 8 = 3 tenths with remainder 6 tenths. Again 6 tenths = 60 hundredths ($\frac{6}{10} = \frac{60}{100}$), and 60 hundredths $\div$ 8 = 7 hundredths with remainder 4 hundredths. But 4 hundredths = 40 thousandths, and 40 thousandths $\div$ 8 = 5 thousandths. Therefore $\frac{3}{8} = 0.375$.

Therefore, *to reduce a common fraction to a decimal, place a decimal point after the numerator and divide by the denominator, as in United States money.*

A common fraction cannot always be reduced to an exact decimal; thus $\frac{2}{3} = 0.6\frac{2}{3} = 0.66\frac{2}{3} = 0.666\frac{2}{3}$. It is customary to write a common fraction or the plus sign to indicate that the work is not complete; thus $\frac{2}{3} = 0.666\frac{2}{3}$, $0.666\cdots$, or $0.666+$. This is often written simply 0.666, using as many places as necessary.

WRITTEN EXERCISE

Change to decimals, carrying the reduction only to thousandths in the case of inexact decimals:

1. $\frac{1}{2}$.
2. $\frac{1}{4}$.
3. $\frac{1}{8}$.
4. $\frac{3}{4}$.
5. $\frac{3}{8}$.
6. $\frac{5}{8}$.
7. $\frac{7}{8}$.
8. $\frac{1}{5}$.
9. $\frac{2}{5}$.
10. $\frac{3}{5}$.
11. $\frac{4}{5}$.
12. $\frac{1}{16}$.
13. $\frac{5}{16}$.
14. $\frac{7}{16}$.
15. $\frac{9}{16}$.
16. $\frac{13}{16}$.
17. $\frac{1}{32}$.
18. $\frac{5}{32}$.
19. $\frac{1}{3}$.
20. $\frac{2}{3}$.
21. $\frac{1}{6}$.
22. $\frac{5}{6}$.
23. $\frac{7}{25}$.
24. $\frac{9}{25}$.
25. $2\frac{11}{25}$.
26. $4\frac{9}{75}$.
27. $0.1\frac{5}{8}$.
28. $8.2\frac{5}{8}$.
29. $4.3\frac{3}{5}$.
30. $8.1\frac{3}{4}$.

131. Adding and Subtracting Decimals. We have learned how to add and subtract such numbers as $1.25 and $2.10. We proceed in the same way with decimal fractions.

To add or subtract decimal fractions, write like orders under one another and add or subtract as with whole numbers.

1. Add 3.06, 9.475, and 0.1.

If the decimal points are in a column, units will come under units, tenths under tenths, and so on. Adding as with numbers representing dollars and cents or with whole numbers the sum is 12.635.

$$\begin{array}{r@{.}l} 3&06 \\ 9&475 \\ 0&1 \\ \hline 12&635 \end{array}$$

2. Subtract 0.26 from 4.1.

Arranging the numbers with the decimal points in a column and writing 4.1 as 4.10, the subtraction is performed in the same way as with United States money. The remainder is easily found to be 3.84.

$$\begin{array}{r@{.}l} 4&10 \\ 0&26 \\ \hline 3&84 \end{array}$$

WRITTEN EXERCISE

Add the following:

1.	2.	3.	4.	5.
$2.47	$15.25	28.92	68.23	32.301
3.62	7.42	6.87	.09	41.025
4.81	19.00	9.09	7.40	7.875

Add, and also subtract, the following:

6.	7.	8.	9.	10.
80.25	67.25	49.63	35.70	67.02
40.07	31.50	22.78	12.95	9.80

11.	12.	13.	14.	15.
42.57	48.05	540.00	128.42	142.00
6.64	6.97	.67	37.06	64.92

ORAL EXERCISE

1. A small geography cost $0.60 and a music book $0.35. How much did the two cost together?

2. An arithmetic cost $0.65 and a paint box $0.35. How much did the two cost together?

3. A history cost $0.60. I gave the dealer a 50-cent piece and a quarter of a dollar. How much change did I receive?

4. I bought a large geography for $1.25 and gave the dealer a dollar and a half. How much change did I receive?

5. How much more did the large geography cost than the small geography? How much more than the arithmetic?

WRITTEN EXERCISE

1. The three sides of a triangle are 2.25 in., 3.8 in., and 2.95 in. What is the perimeter?

2. Three pieces of land have areas as follows: 14.85 A., 26.35 A., and 49.8 A. What is the total area?

3. A man has a field of 38.75 A. He takes 1.2 A. for a garden and 9.8 A. for a pasture. How much is left?

Add the following:

4.	**5.**	**6.**	**7.**	**8.**
72.86	22.37	3.725	412.5	7.283
32.96	43.96	4.683	328.7	.796
41.83	77.63	5.928	587.5	.584
82.09	56.04	7.635	671.3	.963
73.57	29.37	4.009	428.9	4.020
61.23	92.08	5.700	571.1	5.000
4.72	70.63	6.302	266.6	.778
16.8	7.92	5.777	733.4	.610

Add, and also subtract:

9.	**10.**	**11.**	**12.**	**13.**
487.3	52.37	4.003	2.803	1.093
209.6	36.99	2.876	.196	.027

14.	**15.**	**16.**	**17.**	**18.**
22.05	39.26	43.01	$25.27\frac{1}{2}$	$34.33\frac{1}{3}$
13.75	13.54	34.17	$16.42\frac{1}{4}$	$21.66\frac{2}{3}$

19.	**20.**	**21.**	**22.**	**23.**
248.7	32.4	5.071	4.638	2.708
178.8	18.05	2.683	.989	1.819

24.	**25.**	**26.**	**27.**	**28.**
801.3	51.08	9.011	5.117	5.620
628.4	28.72	6.232	2.308	.875

29.	**30.**	**31.**	**32.**	**33.**
912.13	22.38	8.112	6.213	8.112
426.48	19.79	4.348	.487	6.023

34. What number must be added to 7.96 to make 19.8?

35. What number must be taken from 17.5 to leave 6.75?

36. What is the result of adding 4.93 to 17.89?

37. What is the result of subtracting 5.98 from 11.13?

38. What is the result of adding 2.69 to 3.968?

39. The distance from Chicago to Janesville is 91.1 mi., and from Chicago to Fond du Lac 176.7 mi. How far is it, by this route, from Janesville to Fond du Lac?

40. From a barrel of oil containing 31.5 gal. there is drawn off 17.75 gal. How much oil is left?

132. Multiplying Money. We have already studied, in the fourth grade, the multiplication of money by a whole number. For example, if we have to multiply $12.36 by 41 we multiply as if we had 41 × 1236, and then we place the decimal point in the product between the dollars and the dimes.

$$\begin{array}{r} \$12.36 \\ 41 \\ \hline 1236 \\ 4944 \\ \hline \$506.76 \end{array}$$

As an introduction to the multiplication of decimal fractions it is well to review the multiplication of money by a whole number. This is, of course, essentially the multiplication of a decimal fraction by an integer.

WRITTEN EXERCISE

1. If one pair of club skates costs $1.85, how much will 3 pairs cost? How much will 4 pairs cost?

2. If one tennis racket costs $2.50, how much will 5 rackets cost? How much will 3 rackets cost?

3. If one dog collar costs $1.35, how much will 2 dog collars cost? How much will 3 dog collars cost?

4. If 1 doz. tennis balls cost $4.85, how much will 3 doz. tennis balls cost?

5. If one basket ball costs $4.75, how much will 2 basket balls cost?

6. If one football costs $2.25, how much will 3 footballs cost?

7. If one baseman's mitt costs $1.75, how much will 5 such mitts cost?

8. If one baseball mask costs $2.45, how much will 3 such masks cost?

9. At $16.85 each, how much will 17 girls' bicycles cost?

10. At $2.85 each, how much will 33 croquet sets cost?

Multiply the following:

11. 2 × $4.26.
12. 3 × $5.08.
13. 7 × $6.30.
14. 9 × $4.77.
15. 6 × $5.32.
16. 8 × $6.41.
17. 5 × $4.43.
18. 9 × $9.96.
19. 5 × $3.40.
20. 2 × $6.66.
21. 8 × $7.43.
22. 4 × $8.84.
23. 7 × $5.07.
24. 3 × $2.99.
25. 6 × $9.89.
26. 12 × $3.26.
27. 56 × $8.22.
28. 94 × $3.28.
29. 81 × $4.40.
30. 23 × $4.46.
31. 67 × $3.41.
32. 85 × $8.23.
33. 92 × $3.32.
34. 34 × $5.81.
35. 78 × $8.22.
36. 66 × $4.92.
37. 73 × $7.20.
38. 45 × $6.96.
39. 89 × $9.90.
40. 57 × $8.09.
41. 62 × $12.48.
42. 28 × $20.09.
43. 94 × $66.82.
44. 73 × $81.62.
45. 55 × $70.00.
46. 39 × $32.01.
47. 61 × $83.40.
48. 25 × $31.12.
49. 84 × $42.38.
50. 30 × $80.09.
51. 4½ × $36.62.
52. 2½ × $41.14.
53. 6½ × $51.10.
54. 2¼ × $88.60.
55. 4¾ × $70.08.

56. Rob buys 3 fountain pens for presents. He pays $1.35 for each. How much does he pay for all the pens?

57. A jeweler sells 4 wrist watches at $6.75 each. How much does he receive for all the watches?

58. A book dealer sells 6 sets of Scott's novels at $2.43 a set. How much does he receive for all the sets?

59. A dealer buys 9 rifles at $14.65 each. How much does he pay for the lot?

60. Fred buys 2 minnow nets at $1.55 each. How much does he pay for both nets?

61. A dealer buys 3 pianos at $285.50 each. How much does he pay for all the pianos?

133. Multiplication of a Decimal by a Whole Number. There is a measure called the meter, 39.37 in. long. How many inches are there in 24 meters?

Here we have to multiply 39.37 by 24.

Since 7 hundredths multiplied by 4 is 28 hundredths, we know that this 8 will go in the hundredths' place.

Proceeding with the multiplication in the ordinary way, we finally place the decimal point in the product between the 4 and the 8 so as to have the right-hand figure in the hundredths' place.

$$\begin{array}{r} 39.37 \\ 24 \\ \hline 157\,48 \\ 787\,4 \\ \hline 944.88 \end{array}$$

In multiplying a decimal fraction by an integer, there are as many decimal places in the product as there are in the multiplicand.

This is evident from the above explanation. If we multiply tenths by a whole number the product is tenths; if we multiply hundredths the product is hundredths, and so on.

Multiplication by a common fraction or a mixed number is too rarely met to demand special treatment. It is easily understood from the above case.

WRITTEN EXERCISE

Multiply the following:

1. 27.25 by 25.
2. 61.82 by 37.
3. 42.12 by 45.
4. 62.09 by 35.
5. 65.232 by 32.
6. 34.375 by 81.
7. 68.325 by 50.
8. 42.175 by 62.
9. 313.45 by 72.
10. 46.925 by 93.
11. 20.005 by 75.
12. 29.407 by 87.

13. If a twelve-story city building averages 10.62 ft. to a story, how high is the building?

14. If one side of a square is 21.33 in., how long is the perimeter of the square?

15. The measure called the meter, studied later, is 39.37 in. long. How many inches in 62 meters?

134. Multiplying by Tenths. If we multiply 7 by $\frac{1}{2}$ we have $\frac{7}{2}$ as the product, and this we reduce to $3\frac{1}{2}$. In the same way,

$$\tfrac{1}{10} \text{ of } 7 = \tfrac{7}{10},$$

or, in decimal form, $0.1 \times 7 = 0.7.$

In the same way, $\frac{3}{10}$ of $250 = 3 \times \frac{250}{10} = 3 \times 25 = 75,$

or $0.3 \times 250 = 75.0 = 75.$

To multiply a whole number by tenths, multiply as by a whole number, and place a decimal point before the right-hand figure.

ORAL EXERCISE

Multiply the following by 0.1, or .1:

1. 25.
2. 37.
3. 426.
4. 384.
5. 270.
6. 360.
7. 400.
8. 408.
9. 2175.
10. 3286.

Multiply the following by 0.2, or .2:

11. 11.
12. 12.
13. 20.
14. 70.
15. 25.
16. 50.
17. 33.
18. 24.
19. 111.
20. 120.

Multiply the following by 0.3 and 0.5:

21. 10.
22. 20.
23. 30.
24. 40.
25. 11.
26. 12.
27. 100.
28. 200.
29. 111.
30. 101.

WRITTEN EXERCISE

1. How much is 0.7 of 5280 ft.? 0.8 of 1760 yd.?

2. How many feet in 0.6 mi.? in 0.9 mi.?

3. How many yards in 0.7 mi.? in 0.4 mi.? in 0.5 mi.?

4. The distance from Madison to Lancaster is 85.7 mi. This is 85 mi. and how many feet?

ORAL EXERCISE

Multiply the following:

1.	2 in.	$2	$\frac{2}{10}$	$\frac{2}{5}$	$\frac{2}{100}$	0.2	0.02
	3	3	3	3	3	3	3
2.	4 ft.	$4	$\frac{4}{10}$	$\frac{4}{5}$	$\frac{4}{100}$	0.4	0.04
	2	2	2	2	2	2	2

State rapidly the following products:

3.	$\frac{1}{10} \times \frac{3}{10}$	$\frac{2}{10} \times \frac{3}{10}$	$\frac{2}{10} \times \frac{5}{10}$	$\frac{5}{10} \times \frac{9}{10}$
4.	0.1 × 0.3	0.2 × 0.3	0.2 × 0.5	0.5 × 0.9

135. Multiplying by a Decimal Fraction. This is best understood from the following examples:

1. Multiply 6.2 by 3.5.

$$\begin{array}{r} 6.2 \\ \underline{3.5} \\ 3\,10 \\ \underline{18\,6} \\ 21.70 \\ 21.7 \end{array}$$

If we were to multiply 62 by 35 the result would be 2170. Since tenths multiplied by tenths is equal to hundredths, the result is 21.70; that is, $\frac{2170}{100}$ or $21\frac{70}{100}$ or 21.7.

So we see that we multiply as with whole numbers, and, beginning at the right, point off as many decimal places in the product as there are decimal places in both factors together.

2. Multiply 0.42 by 0.35.

$$\begin{array}{r} 0.42 \\ \underline{0.35} \\ 210 \\ \underline{126} \\ 0.1470 \end{array}$$

If we were to multiply 42 by 35 the result would be 1470. Since hundredths multiplied by hundredths is equal to ten-thousandths, the result is 0.1470; that is, 0.42 × 0.35 $= \frac{42}{100} \times \frac{35}{100} = \frac{1470}{10000}$ = 0.1470 = 0.147.

136. Decimal Places in the Product. In multiplying decimals,

Multiply as with whole numbers, and then, beginning at the right, point off as many decimal places in the product as there are decimal places in all the factors together.

ORAL EXERCISE

Multiply the following:

1. 0.7 × 3 0.9 × 4 0.8 × 5 0.3 × 7

2. 0.2 × 4 0.4 × 8 0.9 × 9 0.8 × 7

3. 0.1 × 0.1 0.1 × 0.2 0.2 × 0.3 0.2 × 0.4

4. 0.3 × 0.4 0.3 × 0.5 0.4 × 0.4 0.4 × 0.5

5. 0.6 × 0.7 0.6 × 0.8 0.7 × 0.8 0.8 × 0.9

6. Why will there be three decimal places in the product of 0.75 multiplied by 0.3? of 2.325 multiplied by 7?

7. How do you know the number of decimal places in the product of 0.62 and 2.79? of 2.756 and 3.123?

Without multiplying, tell the orders of the decimal parts of:

8. 2.8 × 7.9 2.04 × 6.09 1.28 × 2.67

9. 1.7 × 1.44 1.23 × 3.456 3.421 × 0.713

WRITTEN EXERCISE

Multiply the following:

1. 2 × 0.79.
2. 0.82 × 8.
3. 3 × 1.75.
4. 3 × 0.96.
5. 0.67 × 9.
6. 5 × 2.42.
7. 4 × 0.78.
8. 0.83 × 4.
9. 6 × 7.23.
10. 7 × 0.93.
11. 0.27 × 7.
12. 8 × 9.23.
13. 0.9 × 84.
14. 0.3 × 9.9.
15. 4.7 × 35.
16. 8.1 × 8.1.
17. 48 × 0.8.
18. 6.7 × 1.8.
19. 0.7 × 96.
20. 0.8 × 9.8.
21. 1.5 × 67.
22. 2.7 × 3.4.
23. 0.8 × 74.
24. 0.7 × 4.9.
25. 1.4 × 75.
26. 1.9 × 2.6.
27. 0.6 × 48.
28. 0.9 × 4.7.
29. 3.8 × 46.
30. 4.7 × 3.6.

Multiply the following:

31. 7.3 × 4.8.
32. 96 × 9.6.
33. 5.4 × 3.5.
34. 43.2 × 98.6.
35. 0.823 × 2946.
36. 3.49 × 80.23.
37. 2.99 × 124.63.
38. 89.1 × 49.3.
39. 72.9 × 87.09.
40. 36.4 × 49.87.
41. 0.600 × 0.500.
42. 6.50 × $28.43.
43. 0.3 × $333.33⅓.
44. 8.001 × 234.56.
45. 0.25 × $648.36.

46. At $17.50 per front foot (that is, per foot fronting the street), what will a piece of land with a frontage of 28.3 ft. cost?

47. A certain tank when full holds 42.8 gal. How much does it contain when 0.37 full?

48. A certain rectangle is 0.78 as wide as it is long. It is 4.9 in. long. What is the perimeter?

49. A certain rectangle is 3.42 times as long as it is wide. It is 4.8 ft. wide. What is the perimeter?

50. A city lot is 45.8 ft. wide and 76.3 ft. deep. What is the area of the lot?

51. The circumference of a circle is 3.1416 times the diameter. If the diameter is 42.27 ft., what is the circumference?

52. How many square rods of land are there in a piece of land 80 rd. wide and 114 rd. long? How many acres? How much is it worth at $87.50 an acre?

53. There are three towns, A, B, C, forming the three corners of a triangle. From A to B is 27.32 mi., from B to C is 1.4 times as far, and the perimeter of the triangle is 92.8 mi. How far is it from C to A?

In all applied problems of this kind, as well as in all abstract work, teachers should urge their pupils to estimate the results in advance, for the reason given on page 26. Every business man does this so as to avoid any absurd result due to an error in computation. In Ex. 53 a fair estimate would be 20 mi.

ORAL EXERCISE

State the values of:

1. $12 \times \frac{1}{12}$ $3 \times \frac{1}{3}$ $10 \times \frac{1}{10}$ 10×0.1
2. 10×0.2 10×0.6 $10 \times \frac{1}{100}$ 10×0.01
3. $100 \times \frac{1}{100}$ 100×0.01 100×0.02 100×0.09
4. $100 \times \frac{25}{100}$ $100 \times \frac{75}{100}$ 100×0.25 100×0.75
5. 100×0.38 100×0.75 $1000 \times \frac{25}{1000}$ 1000×0.025

6. We have found that to multiply a decimal fraction by 10, we need only move the decimal point one place in which direction? How is it when we multiply by 100? by 1000?

137. Multiplying by Multiples of 10. On page 22 we found how to multiply by 10, 100, 1000, and so on for other powers of 10. We also found how to multiply by such numbers as 40, 700, and so on. To multiply decimal fractions by such numbers is substantially the same as to multiply dollars and cents. Therefore, to multiply 487.7 by 600 is the same as to multiply by 6, moving the decimal point two places to the right.

To multiply 89.07 by 740 is the same as to multiply by 74, moving the decimal point one place to the right.

WRITTEN EXERCISE

Multiply the following:

1. 40 × 6.73.
2. 30 × 4.87.
3. 60 × 9.81.
4. 90 × 86.4.
5. 80 × 36.5.
6. 70 × 625.3.
7. 80 × 65.42.
8. 900 × 37.08.
9. 400 × 61.8.
10. 1000 × 1.25.
11. 2000 × 6.725.
12. 6970 × 43.2.
13. 9250 × 67.25.
14. 8220 × 350.4.
15. 6700 × 495.4.
16. 2500 × 374.65.
17. 3500 × 408.08.
18. 35,000 × 1.25.
19. 75 × 12.48.
20. 750 × 1.248.
21. 7500 × 0.1248.

138. Bills. If goods are not paid for by cash, which usually means that they are not paid for when they are purchased but at some later date, the dealer sends a *bill* showing the purchases, the prices, and the amount due. We studied such bills in the fourth grade. The following is a model bill, receipted by the dealers to show that it has been paid:

Columbus, Ohio, April 6, 1918

Mrs. M. A. Olds, 179 Cairo Street

Bought of DOE & ROE, Grocers,
8296 MAIN ST.

Apr.	3	2 tins sliced smoked beef	.25		50
	4	3 cans corned beef	.13		39
	5	6 cans tomato soup	.22	1	32
				2	21
		Received payment Doe & Roe			

WRITTEN EXERCISE

Make out bills for the following items, inserting the names of the purchaser and the dealer, and receipting the bills:

1. 1 baseball cap, 40¢; 1 catcher's mitt, $1.50; 1 baseball mask, $1.40; 1 catcher's body protector, $1.20.

2. 1 league ball, 85¢; 3 bats @ 60¢; 2 fielder's mitts @ 75¢; 3 canvas bases @ $1.10; 1 pitcher's toe plate, 13¢.

3. 4 hunting caps @ 30¢; 1 hunting vest, 69¢; 2 pr. corduroy hunting pants @ $1.45; 2 hunting coats @ $1.50.

Make out bills for the following, as on page 158:

4. 3 qt. Lima beans @ 15¢, 4 cans tomatoes @ 14¢, 6 cans French peas @ 22¢, 5 cans California asparagus @ 42¢, 2 cans imported asparagus @ 85¢, 4 cans sardines @ 18¢.

5. 5 boxes Smyrna figs @ 60¢, 4 boxes stuffed dates @ 40¢, 6 lb. California prunes @ 20¢, 3 lb. Malaga raisins @ 22¢, 4 tins crystallized ginger @ 40¢, 6 jars marmalade @ 28¢.

6. 6 heads lettuce @ 6¢, 9½ lb. butter @ 32¢, 7½ gal. oil @ 18¢, 12 lb. raisins @ 12½¢, 6 lb. coffee @ 32¢, 4½ doz. eggs @ 36¢, ½ doz. oranges @ 60¢, 6 lb. powdered sugar @ 7½¢.

7. 14 yd. flannel @ 45¢, 32 yd. sheeting @ 30¢, 4½ yd. velvet @ $1.60, 2½ yd. ribbon @ 40¢, 2 pr. gloves @ $1.75.

8. 5 gal. molasses @ 45¢, 10 lb. coffee @ 18¢, 15 lb. sugar @ 7¢, 6 boxes crackers @ 8¢, 6½ doz. eggs @ 34¢, 9 lb. butter @ 33¢, 6½ lb. cheese @ 12¢.

9. 36 yd. carpet @ $1.12½, 15 yd. border @ $1.25, 15½ yd. cotton @ 38¢, 22 yd. cloth @ 37½¢, 7½ yd. silk @ $1.40.

In making out a bill take the next higher cent when there is a fraction of a cent in a product.

10. 75 gro. bone buttons @ 22¢, 675 yd. cashmere @ 85¢, 840 yd. cambric @ 17½¢, 560 yd. chambray @ 22½¢, 620 yd. flannel @ 37½¢, 950 yd. sheeting @ 24½¢.

11. 7 gro. toothbrushes @ $9.65, 8 doz. brushes @ $17.75, 2½ gro. nailbrushes @ $26.80, ½ doz. clothes brushes @ $2.92, 6 doz. boxes tooth powder @ $2.04, 4½ doz. atomizers @ $19.60.

12. 3 rifles @ $12.50, 4 shot guns @ $17.50, 400 loaded paper shells @ $2.69 per 100, 3 hunting knives @ $1.85, 4 redwood cleaning rods @ 75¢, 2 trap targets @ $6.50.

13. 3 gun cases @ $4.25, 4 hunting coats @ $3.50, 3 hunting caps @ 33¢, 5 fishing rods @ $2.50, 7 silk casting lines @ 80¢.

139. Price List. The following is a price list of articles interesting to boys and girls. It is used in the problems on pages 160 and 161.

Boys' storm-proof boots, $1.50
Girls' school shoes, $1.45
Girls' skating cap, 60¢
Boys' sweater, $1.50
Boys' silver watch, $3.50
Girls' watch, $3.75
Rubber-stamp outfit, 40¢
Cartridge belt, $1.35
Baseball bat, 60¢
Football pants, $1.50
Bicycle, $16.75
Scout camera, $1.75
Hammock, $1.75
Small typewriter, $5
School drawing set, 30¢
School-book satchel, 50¢
Postcard projecting lantern, $2.50
Postcard album, 65¢
Single-barreled shotgun, $6.25
Rifle, $13.50
Catcher's mitt, $1.50
Catcher's mask, $2
Basket ball, $3.25
Banjo, $9.50
Guitar, $4.75

WRITTEN EXERCISE

Find the cost of the following:

1. 2 sweaters, 3 skating caps, a boys' watch, and 2 hammocks.

2. A bicycle, a guitar, and 2 catcher's mitts.

3. A pair of boys' storm-proof boots, 2 pairs of girls' shoes, and a school drawing set.

4. A rubber-stamp outfit, a shotgun, and a cartridge belt.

5. 4 baseball bats, a catcher's mitt, a basket ball, 2 catcher's masks, and a rifle.

6. 4 pairs of football pants, 2 basket balls, and a catcher's mask.

7. Make out three different lists of things you would like to buy, six items in each, and find the cost of each list.

Make out bills for the following:

8. 2 pairs of girls' shoes, 2 pairs of boys' boots, 1 school drawing set, 1 rubber-stamp outfit, 2 postcard albums.

9. 3 girls' skating caps, 2 boys' sweaters, 2 pairs of football pants, 1 catcher's mitt, 1 catcher's mask.

10. 1 boys' watch, 1 girls' watch, 1 basket ball, 3 school drawing sets, 3 postcard albums.

11. 2 cartridge belts, 1 shotgun, 1 rifle, 2 scout cameras, 3 baseball bats, 2 hammocks.

12. 2 bicycles, 3 typewriters, 2 projecting lanterns, 3 basket balls, 2 guitars, 3 banjos.

13. 6 hammocks, 2 typewriters, 4 girls' watches, 5 boys' watches, $\frac{1}{2}$ doz. girls' skating caps.

14. $\frac{1}{2}$ doz. boys' sweaters, 8 pairs of boys' boots, 4 cartridge belts, $\frac{1}{2}$ doz. baseball bats, 4 pairs of football pants.

15. $\frac{1}{2}$ doz. scout cameras, $\frac{1}{2}$ doz. catcher's masks, 8 catcher's mitts, 4 rifles, 2 shotguns.

16. 8 projecting lanterns, 9 postcard albums, 25 school drawing sets, 7 typewriters, 3 school satchels.

17. 5 basket balls, 4 banjos, 4 guitars, 8 hammocks, 9 baseball bats, 7 pairs of football pants, 8 rubber-stamp outfits.

18. 15 rifles, 16 single-barreled shotguns, 9 cartridge belts, 9 baseball bats, 6 pairs of football pants, 2 bicycles, 8 basket balls, 4 scout cameras, 6 hammocks, 2 postcard projecting lanterns.

Teachers should have the pupils obtain price lists, either at the local stores or from catalogues, and make bills of goods in which they are interested. The pupils should see that some of the bills in this exercise are such as might represent goods purchased by them or by their parents, while some of the bills include such numbers of articles as would be purchased only by dealers. An effort should always be made to relate a topic of this kind to the personal and home interests of the pupil.

140. Marketing. The following is a price list of household supplies. It may be used in the examples on pages 162 and 163, or the current prices in the various places may be used.

Oranges, per dozen, 40¢
Butter, per pound, 38¢
Eggs, per dozen, 44¢
Sugar, 25-pound bag, $1.30
Oysters, per dozen, 25¢
Coffee, per pound, 30¢
Celery, per bunch, 10¢
Apples, per peck, 40¢
Cheese, per pound, 14¢
Cranberries, per quart, 14¢
Turkey, per pound, 26¢
Halibut, per pound, 15¢

WRITTEN EXERCISE

Find the cost of the following:

1. 2½ doz. oranges and 3½ lb. of butter.
2. 3½ doz. eggs and a 25-pound bag of sugar.
3. 2 doz. oysters and 5 lb. of coffee.
4. 3 bunches of celery and ½ bu. of apples.
5. 2½ lb. of cheese and 4 lb. of coffee.
6. An 8-pound turkey and a quart of cranberries.
7. 4 lb. of halibut and 2½ doz. eggs.
8. ½ doz. oranges, 2½ lb. of butter, 2 doz. eggs, and a 25-pound bag of sugar.
9. 3 doz. oysters, a 9-pound turkey, 2 qt. of cranberries, and a peck of apples.
10. 6 lb. of coffee, 4 doz. eggs, 6½ lb. of butter, 4 bunches of celery, and 6 lb. of halibut.
11. 3 doz. oranges, 1½ lb. of butter, 2½ doz. eggs, 25 lb. of sugar, 1 pk. of apples, 3 lb. of cheese.
12. 2 doz. oysters, 7 lb. of coffee, 4 doz. eggs, 2½ pk. of apples, 3 qt. of cranberries, and a 12-pound turkey.

Make out bills for the following, inserting the names of purchaser and dealer and receipting the bills:

13. 4 bunches of celery, 2 pk. of apples, 2 lb. of butter, 2 doz. oysters, 4 lb. of halibut.

14. 4 qt. of cranberries, a 14-pound turkey, 2 lb. of cheese, 25 lb. of sugar, 3 doz. oranges.

15. A 15-pound turkey, 3 qt. of cranberries, 1 pk. of apples, 4 bunches of celery, 3 doz. eggs, 1 doz. oranges.

16. 2 lb. of butter @ 38¢, 2 lb. of domino sugar @ $8\frac{1}{2}$¢, 2 packages of puffed rice @ 14¢, 3 doz. eggs @ 34¢, 2 lb. of powdered sugar @ 6¢, 2 lb. of coffee @ 30¢.

17. 2 doz. oranges @ 60¢, 3 doz. eggs @ 33¢, 2 lb. of butter @ 44¢, 7 lb. of granulated sugar @ 5¢, $\frac{1}{2}$ lb. of tea @ 90¢, 2 grapefruits @ 15¢, 4 tins of sugar wafers @ 25¢.

18. 2 bottles of malt vinegar @ 22¢, 3 packages of corn flakes @ 10¢, $1\frac{1}{2}$ doz. oranges @ 60¢, 3 doz. eggs @ 44¢, $2\frac{1}{2}$ lb. of butter @ 42¢, 4 cans of pea soup @ 29¢.

19. 1 pt. of olive oil, 50¢, 2 boxes of domino sugar @ 17¢, $1\frac{1}{2}$ lb. of tea @ 90¢, $\frac{1}{2}$ doz. cakes of soap @ $1.08 per doz., 3 doz. oranges @ 40¢, $\frac{1}{2}$ lb. of milk chocolate @ 50¢.

20. 2 baskets of figs @ 30¢, 3 doz. cakes of soap @ $1.06 a dozen, 2 bottles of vinegar @ 22¢, 2 lb. of chocolate mints @ 80¢, 2 boxes of layer figs @ 55¢.

21. 4 bottles of ammonia @ 24¢, 3 boxes of salt @ 10¢, $\frac{1}{2}$ gal. of olive oil @ $3 per gallon, 3 cans of mustard @ 25¢, 2 bags of flour @ 85¢, 4 lb. of confectioners' sugar @ $6\frac{1}{2}$¢.

22. $1\frac{1}{2}$ lb. of pork tenderloin @ 38¢, $1\frac{3}{4}$ lb. of French lamb chops @ 28¢, $2\frac{1}{2}$ lb. of steak @ 26¢, 8 lb. of roast beef @ 34¢, $1\frac{1}{2}$ lb. of fish @ 22¢, 2 packages of sausage @ 25¢.

141. Simple Accounts. We have already learned (pages 15–17) how to keep simple accounts. We shall now review the subject briefly. The following is a sample page from a boy's account book:

1916		RECEIPTS			1916		PAYMENTS		
March	1	On hand . .	1	18	March	2	Papers . .		25
“	6	From father		25	“	3	Car fare .		10
“	14	Errand . . .		10	“	4	Pencil . .		05
“	20	Paper route .		75	“	31	On hand .	2	03
“	29	Errand . . .		15					
			2	43				2	43

In order to train pupils to keep simple accounts, an accomplishment that every one should possess, teachers are requested to have at least one exercise a week in which the children make a record of their income and expenses. This exercise need not occupy an entire period. Except when class instruction is needed, the records may be made before school or at odd times.

The accounts should be of actual moneys belonging to the pupils, if that is possible. It is better to have a briefer account and have it a real one than to fill it with imaginary transactions. The accounts should indicate briefly but adequately the sources and dispositions of income: not "Rec. from Aunt Mary," but "Gift from Aunt Mary"; not "Paid out," but "Paid for Candy"; not "Earned," but "Shoveling Snow." The headings at the top of the pages should be uniform through each book, as "Received," "Paid"; or "Income," "Outgo"; or "Receipts," "Payments." Balances should be computed at the close of each month. These balances should be properly brought down by the use of red lines, if possible, and carried forward to the next month's account.

Ask the pupils to bring to school bills that have been used in business transactions. This is review work, and pupils should become proficient in making and receipting bills of various sorts that are used in the common business of life. Make the work as practical as possible.

The examples in any textbook must necessarily be imaginative, and those given on page 165 may be used in case it is not possible to carry out the work in the spirit above set forth.

The dollar sign and the decimal point are commonly omitted when the ruling of the account book shows clearly, as in the above example, which column is dollars and which is cents.

WRITTEN EXERCISE

Rule a sheet of paper, or use an account book properly ruled, and write the following accounts, balancing each:

1. April 2, on hand, \$1.25. Receipts: April 9, gift from Uncle John, 30¢; April 20, errand, 15¢; April 24, errand, 10¢. Payments: April 13, ball, 15¢; April 14, bat, 60¢.

2. June 1, on hand, \$1.35. Receipts: June 8, from Rob for money borrowed, 10¢; June 18, paper route, 60¢; June 23, care of horse, 50¢. Payments: June 13, catcher's mask, \$1.15; June 24, 2 bats @ 60¢.

Write the following accounts, inserting dates and items, and balancing each:

3. On hand, \$1.60. Receipts: \$2.20, 20¢, 10¢, 15¢. Payments: \$1.50, 25¢, 75¢, 10¢.

4. On hand, \$1.25. Receipts: 20¢, 40¢, 75¢, 30¢. Payments: 20¢, 30¢, 5¢, 60¢.

5. On hand, \$3.60. Receipts: \$1, 45¢, 15¢, 25¢, 5¢. Payments: \$1.35, 25¢, 60¢, 10¢, 10¢.

6. On hand, \$4.30. Receipts: 60¢, \$1.30, 40¢, 20¢, 10¢, 5¢. Payments: \$1.75, 30¢, 40¢, 60¢, 10¢, 2¢, 1¢.

7. On hand, \$2.30. Receipts: \$5, 20¢, 30¢, 2¢, 10¢. Payments: \$1.25, \$1, 40¢, 65¢, 75¢, 2¢, 10¢.

8. On hand: \$3.35. Receipts: 5¢, 25¢, 60¢, \$1, \$2.25. Payments: \$2, \$1.20, 60¢, 2¢, 35¢, 4¢, 5¢.

9. On hand: \$2.75. Receipts: \$1, 75¢, 60¢, \$2, 40¢, 5¢. Payments: 40¢, 10¢, 3¢, 20¢, 25¢, 10¢, 15¢.

10. On hand: \$4.40. Receipts: 30¢, 70¢, \$1, 10¢, 15¢, 20¢. Payments: 25¢, \$2.50, 30¢, 45¢, 5¢, 15¢, 2¢.

142. Dividing by a Whole Number. The following examples will illustrate the division of a decimal fraction by a whole number.

1. In 13 da. a man traveled 925.6 mi. in his automobile. What was the average distance per day?

We have to divide 925.6 mi. by 13.

We see that 92 tens ÷ 13 = 7 tens, and there remains 15.6 to be divided.

We see that 15 ÷ 13 = 1, and there remains 2.6 to be divided.

We see that 2.6 ÷ 13 = 0.2, so that the quotient is 71.2, and the average rate was 71.2 mi. per day.

```
     71.2
13)925.6
   91
    15
    13
     26
     26
```

2. If the man in Ex. 1 had traveled 814 mi. in 12 da., what would have been the average distance per day?

We have to divide 814 mi. by 12.

Since 814 = 814.0 = 814.00, we may place a decimal point after the 814, followed by as many zeros as we please. Dividing 814.0 by 12 as in Ex. 1, we have for the quotient 67.8, and 0.4 remainder.

We may write the quotient 67.8⅓, or we may annex more zeros and continue to divide as far as we please. The quotients 67.8⅓, 67.83⅓, 67.833⅓, and so on, all have the same value. For practical purposes such a result would be written simply 67.8.

```
     67.8
12)814.0
   72
    94
    84
    100
     96
      4
```

ORAL EXERCISE

Divide, in turn, by 3, by 4, and by 5:

1. 360	$\frac{360}{100}$	0.60	0.6	1.2	0.12
2. 240	$\frac{240}{100}$	0.24	2.4	24	0.024

Divide, in turn, by 6 and by 7:

3. 42	4.2	0.42	420	0.042	$\frac{42}{10000}$
4. 84	8.4	0.84	840	8400	0.084

WRITTEN EXERCISE

1. The perimeter of a triangle whose sides are all equal is 1237.8 in. What is the length of each side?

2. The perimeter of a six-sided figure, with equal sides, is 7.62 in. What is the length of each side?

3. Four boys are camping on the shore of a lake and pay $17.64 for the outfit. How much is each boy's share?

4. The distance from Chicago to Cedar Rapids is 219.4 mi. How many miles in $\frac{1}{4}$ of the distance?

Divide, carrying each quotient to one decimal place:

5. 2087.2 ÷ 5.
6. 6039.3 ÷ 7.
7. 4276.8 ÷ 6.
8. 3748.1 ÷ 9.
9. 5263.7 ÷ 8.
10. 843.2 ÷ 72.
11. 342.8 ÷ 66.
12. 802.7 ÷ 69.
13. 534.8 ÷ 92.
14. 417.6 ÷ 87.
15. 52.3 ÷ 48.
16. 62.9 ÷ 69.
17. 34.1 ÷ 128.
18. 46.7 ÷ 293.
19. 99.8 ÷ 487.

Divide, carrying each quotient to two decimal places:

20. 274.1 ÷ 36.
21. 428.3 ÷ 41.
22. 720.6 ÷ 84.
23. 293.4 ÷ 73.
24. 820.9 ÷ 29.
25. 873.4 ÷ 37.
26. 607.4 ÷ 49.
27. 700.6 ÷ 58.
28. 801.1 ÷ 63.
29. 803.7 ÷ 96.
30. 42.38 ÷ 46.
31. 73.42 ÷ 68.
32. 82.37 ÷ 49.
33. 64.86 ÷ 38.
34. 42.73 ÷ 19.
35. 68.42 ÷ 37.
36. 72.26 ÷ 35.
37. 90.71 ÷ 54.
38. 82.32 ÷ 94.
39. 72.08 ÷ 33.
40. 4287 ÷ 693.
41. 2909 ÷ 426.
42. 4702 ÷ 328.
43. 5309 ÷ 772.
44. 6027 ÷ 428.
45. 5009 ÷ 762.
46. 8121 ÷ 406.
47. 9303 ÷ 500.
48. 7000 ÷ 228.
49. 6280 ÷ 600.

ORAL EXERCISE

1. Divide 10 by 2; 100 by 20. Compare the results.

2. Divide \$2.50 by 5; \$25 by 50. Compare the results.

3. If multiplying both dividend and divisor by the same number does not change the quotient, how does the quotient of 360 ÷ 4 compare with that of 36 ÷ 0.4?

143. Division by a Decimal. Since both divisor and dividend may be multiplied by the same number without changing the quotient, therefore,

In the division of decimal fractions, multiply both divisor and dividend by such a power of 10 as shall make the divisor an integer, and divide as in United States money.

Divide 6.25 by 2.5.

Multiplying 2.5 and 6.25 by 10 we have 25 and 62.5. The result is the same if we divide 62.5 by 25.

Dividing as in United States money, 62.5 ÷ 25 = 2 with 12.5 still to be divided. Then 12.5 ÷ 25 = 0.5. Therefore the quotient is 2.5.

We might, if we desired, multiply both 2.5 and 6.25 by 100, giving 625 to be divided by 250. The result would be the same.

$$\begin{array}{rr} & 2.5 \\ 25) & \overline{62.5} \\ & 50 \\ & \overline{12\,5} \\ & \underline{12\,5} \end{array}$$

Instead of moving both decimal points to the right, we may draw a line to show the position of the decimal point in the quotient, as in the annexed solution. That is,

In the division of decimal fractions, draw a vertical line as many places to the right of the decimal point in the dividend as there are decimal places in the divisor. Then divide as with whole numbers, placing the decimal point in the quotient above this vertical line.

$$\begin{array}{rr} & 2.5 \\ 2.5) & \overline{6.2|5} \\ & 5\,0 \\ & \overline{1\,2\,5} \\ & \underline{1\,2\,5} \end{array}$$

Both of these plans are in common use. The school should adopt either plan and adhere to it in teaching.

ORAL EXERCISE

1. How long will it take a railway train traveling 0.7 mi. a minute to go 14.7 mi.? to go 28 mi.?

2. How long will it take a railway train traveling 0.8 mi. a minute to go 32.8 mi.? to go 40 mi.?

3. A drawing 9.6 in. long and 0.8 in. wide is how many times as long as wide?

4. How many times is 0.1 in. contained in 3.8 in.?

Divide the following:

5. \$6 ÷ \$3.
6. $\frac{6}{10} \div \frac{3}{10}$.
7. 0.6 ÷ 0.3.
8. 0.06 ÷ 0.03.
9. 1.2 ÷ 0.4.
10. 2.5 ÷ 0.5.
11. 3.6 ÷ 0.9.
12. 3.5 ÷ 0.7.
13. 2.8 ÷ 0.7.
14. 8.1 ÷ 0.9.
15. 7.2 ÷ 0.8.
16. 5.6 ÷ 0.7.

WRITTEN EXERCISE

1. How long will it take an aeroplane traveling at the rate of 1.2 mi. a minute to go 16.8 mi.?

2. The speedometer of an automobile shows that the car has gone 23.9 mi. on a trip of 143.4 mi. What part of the total distance has the car gone?

Divide the following:

3. 125 ÷ 25.
4. 12.5 ÷ 2.5.
5. \$1.25 ÷ \$0.25.
6. \$125 ÷ \$0.25.
7. \$1250 ÷ \$2.50.
8. 6.232 ÷ 0.82.
9. 462 ÷ 22.
10. 46.2 ÷ 2.2.
11. 4.62 ÷ 2.2.
12. 0.462 ÷ 0.21.
13. 11.56 ÷ 3.4.
14. 17.466 ÷ 7.1.
15. 625 ÷ 0.25.
16. 600 ÷ 0.06.
17. 629 ÷ 0.37.
18. 7.14 ÷ 4.2.
19. 30.5 ÷ 0.61.
20. 62.32 ÷ 8.2.

ORAL EXERCISE

State the following results:

1.	10×0.4	$4 \div 10$	10×1.4	$14 \div 10$
2.	$10 \times \$0.04$	$\$0.40 \div 10$	10×0.04	$0.4 \div 10$
3.	$10 \times \$0.25$	$\$2.50 \div 10$	10×0.25	$2.5 \div 10$
4.	10×12.5	$125 \div 10$	100×0.04	$4 \div 100$

144. Dividing by Powers of 10. From the above exercise and from the work on page 30, we find the following:

To divide a number by 10, move the decimal point one place to the left.

Any whole number may be thought of as having a decimal point after units. That is, 6 is the same as 6. or 6.0 or 6.00.

To divide a number by 100, move the decimal point two places to the left.

Thus $7 \div 100 = 0.07$ or .07.

To divide a number by 1000, move the decimal point three places to the left.

$$\boxed{\begin{array}{r} 6\cancel{0}\cancel{0})\underline{2.16} \\ 0.36 \end{array}}$$

To divide 216 by 600 is the same as to divide 2.16 by 6.

WRITTEN EXERCISE

Divide the following:

1. $133 \div 70$.

2. $10.8 \div 60$.

3. $32.9 \div 70$.

4. $3745 \div 500$.

5. $2291 \div 290$.

6. $296.1 \div 470$.

7. $273.6 \div 120$.

8. $2984 \div 4000$.

9. $7344 \div 9000$.

10. $625 \div 25{,}000$.

11. $961 \div 31{,}000$.

12. $882 \div 21{,}000$.

13. $1728 \div 1200$.

14. $144 \div 60{,}000$.

15. $872 \div 22{,}000$.

16. $7260 \div 2500$.

17. $154 \div 12{,}000$.

18. $145.7 \div 4700$.

19. $57.19 \div 190$.

20. $571.9 \div 1900$.

21. $162.5 \div 250$.

145. Illustrative Problems. 1. Divide 16.35 by 500.

Here it is better to divide both divisor and dividend by 100, canceling the zeros and moving the decimal point two places to the left.

$$\begin{array}{r} 5\cancel{0}\cancel{0}\underline{)\,0.1635} \\ 0.0327 \end{array}$$

2. Divide 3.82 by 2.5, carrying the quotient to one decimal place.

Multiplying both numbers by 10, we have $38.2 \div 25 = 1.5$ with .7 remainder. If we do not write the fraction $\frac{7}{25}$ in the quotient, the quotient may be written 1.5 +, meaning 1.5 and a fraction less than $\frac{1}{2}$.

3. Divide 0.049 by 0.17, carrying the quotient to two decimal places.

Multiplying both numbers by 100, we have $4.9 \div 17 = 0.28$ with .14 remainder. If we do not write the fraction $\frac{14}{17}$ in the quotient, the quotient may be written 0.29 –, meaning 0.28 and a fraction $\frac{1}{2}$ or greater than $\frac{1}{2}$.

WRITTEN EXERCISE

Divide the following:

1. $17.25 \div 500$.
2. $3.248 \div 200$.
3. $817.6 \div 700$.
4. $82.48 \div 8000$.
5. $0.7364 \div 7000$.
6. $74.34 \div 9000$.
7. $15.72 \div 12{,}000$.
8. $133.1 \div 11{,}000$.
9. $404.3 \div 13{,}000$.
10. \$6844.50 ÷ 900.
11. \$7337.75 ÷ 500.
12. \$1247.01 ÷ 300.

Divide, carrying the quotients to three decimal places:

13. $17.78 \div 1.5$.
14. $62.23 \div 0.23$.
15. $19.26 \div 3.1$.
16. $21.43 \div 3.7$.
17. $33.43 \div 6.8$.
18. $423.6 \div 0.27$.
19. $167.8 \div 3.7$.
20. $8.348 \div 2.68$.
21. $41.73 \div 2.18$.
22. $44.87 \div 23.7$.
23. $97.634 \div 0.125$.
24. $483.62 \div 42.7$.
25. $826.3 \div 12.5$.
26. $42.78 \div 3.162$.
27. $75.41 \div 2.807$.

146. Estimating Results. In the following exercise pupils should state the approximate answers orally, and these should be written down. The problems should then be solved on paper and the real result compared with the approximate one. This is one of the best checks against absurd results, and should frequently be made a subject of class drill with other written exercises.

Requiring pupils to estimate results carries out the policy already referred to on page 26. Teachers should not expect to receive the same approximations from all pupils. A natural approximation for Ex. 1 is $3\frac{1}{2} \times 22¢$.

WRITTEN EXERCISE

1. At 22¢ a pound, how much will $3\frac{5}{8}$ lb. of steak cost?

2. At 18¢ a gallon, how much will 6.25 gal. of oil cost?

3. At 26¢ a pound, how much will $6\frac{7}{8}$ lb. of butter cost?

4. At $12\frac{1}{2}$¢ a yard, how much will $8\frac{3}{4}$ yd. of ribbon cost?

5. At $2 a yard, how much will 2.8 yd. of velvet cost?

6. A dealer buys 200 lb. of raisins for $19. How much do they cost a pound?

7. A dealer pays $18.83 for 7 rocking-chairs. How much does he pay for each?

8. A dealer sells 17 pails of mackerel for $40.80. What is the price per pail?

9. A man pays $1560 a year for the rent of a city house. How much does he pay a month?

10. Traveling at the rate of 47 mi. an hour, how far will a train go between 8.45 A.M. and 11.30 A.M.?

11. A grocer buys a 50-gallon barrel of Porto Rico molasses for $11.50. How much does it cost per gallon?

12. A grocer sells a half chest of tea, 140 lb. to the chest, for $19.60. What is the price per pound?

147. Cancellation with Decimals. A dealer has two sizes of boxes in which he sells candy. The larger box is 4.2 in. by 8.6 in. by 2.1 in., and the smaller one is 4.2 in. by 4.3 in. by 1.4 in. The larger box holds how many times as much as the smaller one?

$$\frac{\cancel{4.2} \times \overset{\cancel{2}}{\cancel{8.6}} \times \overset{3}{\cancel{2.1}}}{\cancel{4.2} \times \cancel{4.3} \times \underset{\cancel{2}}{\cancel{1.4}}}$$

We see that we must divide 4.2 × 8.6 × 2.1 by 4.2 × 4.3 × 1.4.

Writing the work like a fraction, as here shown, we cancel common factors.

Evidently 4.2 may be canceled at once.

We also see that 8.6 = 2 × 4.3, so we may cancel the factor 4.3, leaving 2 over the 8.6 in the dividend.

We now have left 2 × 2.1 in the dividend and 1.4 in the divisor. We may now cancel the factor 0.7 from 1.4 and 2.1, and then cancel the 2.

Therefore the dividend is 3 times the divisor, and so the larger box is 3 times the smaller one.

If we wish to do so, in this example, we may multiply both dividend and divisor by 1000, thus having 42 × 86 × 21 to be divided by 42 × 43 × 14, and avoiding decimals entirely. This is often the better plan.

WRITTEN EXERCISE

Find the values of the following:

1. $\frac{3.9 \times 0.84}{15.6}$.
2. $\frac{1.4 \times 0.6}{0.63}$.
3. $\frac{2.2 \times 2.4}{4 \times 7.7}$.
4. $\frac{5.5 \times 0.42}{0.77}$.
5. $\frac{4.5 \times 6.3}{8.1}$.
6. $\frac{5.6 \times 0.3}{0.63 \times 0.8}$.
7. $\frac{2.1 \times 0.4}{0.015}$.
8. $\frac{0.21 \times 0.6}{0.126}$.
9. $\frac{0.8 \times 1.5 \times 0.6}{3.6 \times 0.2 \times 10}$.
10. $\frac{1.2 \times 0.39 \times 1.4}{2.6 \times 0.42 \times 2}$.
11. $\frac{0.2 \times 3.6 \times 1.43}{9.9 \times 0.4 \times 2.6}$.
12. $\frac{6.3 \times 4.2 \times 0.17}{2.7 \times 3.4 \times 0.49}$.

13. A block of marble 3.5 ft. by $4\frac{1}{4}$ ft. by 2.2 ft. weighs how many times as much as one that is $2\frac{1}{2}$ ft. by 7.7 ft. by 0.85 ft.?

14. If 4 tennis rackets cost $8, how much will 2 rackets cost? Solve by one division.

15. If 12 fishing reels cost $7.80, how much will 3 fishing reels cost? How much will 4 fishing reels cost? In each case solve by one division.

16. If 3 shotguns cost $29.25, how much will 15 shotguns cost? Solve by one multiplication.

17. If 3 cameras cost $14.25, how much will 4 cameras cost? Solve by one division and one addition, or else by one multiplication by a mixed number.

18. If 4 spoon hooks cost $1.76, find how much 2 spoon hooks will cost, and then find how much 6 spoon hooks will cost.

19. If 6 trout baskets cost $5.40, find how much 3 trout baskets will cost, and then find how much 9 trout baskets will cost.

20. If 2 folding camp cots cost $4.50, find how much one will cost, and then find how much 3 cots will cost.

21. If 6 telegraph outfits for learners cost $17.10, how much will 2 outfits cost? How much will 4 outfits cost?

22. If 3 music bags cost $2.94, how much will 9 music bags cost? How much will 11 music bags cost?

23. If 2 German accordeons cost $7.30, how much will 3 German accordeons cost?

24. If 2 violins cost $9.50, how much will 5 violins cost?

25. If 3 banjos cost $6.75, how much will 7 banjos cost?

26. If 4 mandolins cost $11.40, how much will 6 mandolins cost? How much will 9 mandolins cost?

27. If 6 boxes of writing paper cost $1.80, how much will 9 boxes cost? How much will 12 boxes cost? How much will 15 boxes cost? How much will 24 boxes cost?

WRITTEN REVIEW

1. A man has two farms, one containing 175.4 A., and the other 163.8 A. How many acres of land has he in both farms?

2. At $75 an acre, how much will 62.4 A. of land cost?

3. An automobile travels 28.6 mi. the first hour and 22.9 mi. the second hour. How far does it travel in the two hours?

4. In going from Denver to Pueblo, a distance of 119.4 mi., the train passes through Colorado Springs, 74.9 mi. from Denver. How far is it from Colorado Springs to Pueblo?

5. A train leaving Denver at 7.45 P.M. reaches Santa Fe the next day at 4.25 P.M., a distance of 405.9 mi. Find the average rate per hour. Remember that 4.25 P.M. means 4 hr. 25 min. P.M.

6. In traveling from Chicago to Winona, a distance of 296.1 mi., we pass through Madison, 129.7 mi. from Chicago. The distance from Chicago to Madison lacks how much of being half the distance from Chicago to Winona?

7. One train leaves Chicago at 10.30 A.M. and arrives at Omaha at 11.59 P.M. Another train leaves Chicago at 7 P.M. and arrives at Omaha at 7.20 the next morning. The distance being 491.3 mi., find the average rate of each train per hour.

8. A man, on an automobile trip, stops for luncheon when the indicator on his car shows that he has traveled 37.8 mi. At the end of the trip he finds he has traveled 113.4 mi. What part of the trip had he made when he stopped for luncheon?

9. Two cousins, Frank and Tom, visit their uncle in Denver. Frank lives in Colorado Springs, 74.9 mi. from Denver, and Tom lives in Carbondale, 373 mi. from Denver. Tom tells his uncle that Carbondale is five times as far from Denver as Colorado Springs is, but Frank says it is not. Which is right?

Find the cost of the following:

10. 27.9 A. of land at $62.50 per acre.

11. 72.8 A. of land at $87.50 per acre.

12. A city lot having a frontage of 62.8 ft., at $375 per foot.

13. 165.48 A. of land at $125 per acre; at $140 per acre.

14. 27.5 T. of hay at $16.40 per ton; at $17.50 per ton.

15. 48.6 T. of hay at $14.75 per ton; at $15.25 per ton.

16. 23.48 mi. of road at $1875 per mile; at $2350 per mile.

17. 148.72 mi. of railway at $21,748.50 per mile.

18. 61.82 mi. of canal excavating at $164,872.25 per mile.

19. Kate's brother Fred is in college. Last year his entire expenses were $625.75. At the same rate, how many boys could be kept in college for a year for the amount paid for a single battleship that cost $12,515,000?

20. A passenger train leaves Clinton at 5.10 P.M. and reaches Anamosa, a distance of 71.4 mi., at 8.05 P.M. A freight train leaves Clinton at 4.15 A.M. and reaches Anamosa at 11 A.M. By how many miles an hour does the average rate of the passenger train exceed that of the freight train?

21. Marquette is 71.7 mi. from Hancock. A train leaves Marquette at 4.45 and reaches Hancock at 8.10, while another leaves at 7.45 and reaches Hancock at 10.30. Which train makes the greater average distance per hour, and how much greater?

22. The distance from Milwaukee to St. Paul is 325.6 mi. The distance from Milwaukee to Wyeville, 153.9 mi., lacks how much of being half the distance from Milwaukee to St. Paul?

23. A farmer buys three adjoining pieces of land, the first containing 142.7 A., the second 115.6 A., and the third 168.7 A. He then sells half of the land. How many acres does he sell?

24. A man has three adjacent lots with a frontage of 148.5 ft. One of the lots has a frontage of 52.75 ft. and another of 49.5 ft. What is the frontage of the third lot?

25. A man who had a farm of 215.8 acres bought an adjacent farm of $192\frac{3}{4}$ acres. He sold a field of $72\frac{1}{3}$ acres and another of 68.25 acres. How much land had he left?

26. The perimeter of a triangle is 27.25 ft., and two of the sides are $8.33\frac{1}{3}$ ft. and $9.75\frac{2}{3}$ ft. What is the third side?

27. The distance from New Orleans to Baton Rouge is 89.4 mi., and from New Orleans to Natchez by way of Baton Rouge, 214.1 mi. How far is it from Baton Rouge to Natchez?

28. The distance from Chicago to Lincoln is 561.4 mi., and from Chicago to Colorado Springs by way of Lincoln, 1072.2 mi. How far is it from Lincoln to Colorado Springs?

29. If the four sides of a field are 22.75 rd., 17.65 rd., 28.6 rd., and 18.35 rd., what is the perimeter of the field?

30. From the sum of 218.7 lb. and 306.4 lb. subtract the sum of 175.4 lb. and 163.7 lb.

31. How many bushels of potatoes will 24 acres produce if an average acre produces 125 bu.?

32. If a dealer buys three barrels of sugar containing respectively 313.6 lb., 314.8 lb., and 315.5 lb., how many pounds does he buy in all?

33. A man bought three barrels of sugar containing respectively 312.75 lb., 314.75 lb., and 316.5 lb. How many pounds did he buy in all?

34. A man has to make a railway journey of 254.7 miles. After he has gone $\frac{1}{3}$ of the distance he receives a telegram calling him halfway back to his starting point. After he gets there, how many miles is it to the end of his journey?

LITTLE EXAMINATIONS

I. **1.** Change to common fractions or to mixed numbers:

0.6 $\quad 0.75$ $\quad 0.33\frac{1}{3}$ $\quad 0.87\frac{1}{2}$ $\quad 3.25$

2. Change to decimals, carrying the results to thousandths:

$\frac{2}{3}$ $\quad \frac{3}{8}$ $\quad \frac{3}{5}$ $\quad 2\frac{3}{16}$ $\quad 32\frac{5}{8}$ $\quad 33\frac{1}{3}$

3. $72.49 + 36.84 + 90.73 + 86.87 + 230.4 + 2.75.$

4. $52 + 78.26 + 140.3 + 9.625 + 41.7 - 129.967.$

5. $25 \times 37.42.$ **7.** $5.36 \times 21.7.$ **9.** $16.074 \div 4.8.$
6. $8.42 \times 264.$ **8.** $84.76 \div 46.$ **10.** $12.446 \div 0.23.$

II. **1.** Change to common fractions or to mixed numbers:

$0.37\frac{1}{2}$ $\quad 0.66\frac{2}{3}$ $\quad 0.16\frac{2}{3}$ $\quad 0.06\frac{1}{4}$ $\quad 4.75$

2. Change to decimals, carrying the results to thousandths:

$\frac{5}{8}$ $\quad 3\frac{3}{8}$ $\quad 7\frac{7}{8}$ $\quad 3\frac{5}{16}$ $\quad 2\frac{1}{32}$ $\quad 5\frac{3}{64}$

3. $26.98 + 4.764 + 314.9 + 34.69 + 2.786.$

4. $27.99 + 6.783 + 427.6 + 96.49 - 75.496.$

5. $75 \times 68.64.$ **7.** $6.31 \times 41.82.$ **9.** $9.648 \div 0.75.$
6. $5\frac{3}{4} \times 29.48.$ **8.** $78.75 \div 2.5.$ **10.** $234.8 \div 1.25.$

III. **1.** $2\frac{3}{8} \times 74.64.$ **4.** $7.3\frac{1}{3} \times 45.9.$ **7.** $3.75 \div \frac{3}{8}.$
2. $0.12\frac{1}{2} \times 85.6.$ **5.** $8 \div 1.25.$ **8.** $52.83 \div 0.9.$
3. $3\frac{2}{3} \times 42.27.$ **6.** $0.014 \div \frac{7}{8}.$ **9.** $5.001 \div 0.03.$

10. Multiply 2.008 by $5\frac{3}{4}$ and divide the product by $2\frac{7}{8}$.

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. A corner stone in a building is dated MDCCCLXXXIX. In what year was the building erected?

2. Write in figures seven million, ninety-six and five tenths.

Add the following:

3.	**4.**	**5.**	**6.**	**7.**
$27.36	$52.37	$61.72	$82.73	$48.73
4.83	26.42	3.89	61.54	60.07
23.49	3.58	55.70	20.09	2.96
17.06	47.63	6.32	17.27	40.37
5.27	73.58	12.47	38.46	6.84

8. Multiply 17 ft. 3 in. by 4; by 7; by $2\frac{1}{2}$.

9. Divide 11 ft. 8 in. by 4; by 5; by 2.5.

10. Find the sum of 17 rd. $9\frac{1}{2}$ ft. and 26 rd. $8\frac{1}{3}$ ft.

11. From 3 A. 40 sq. rd. subtract $2\frac{1}{2}$ A.

12. From the sum of $296.40 and $37.65 subtract $198.79.

Write as rapidly as you can the results of the following:

13. $\frac{1}{2}+\frac{1}{4}$.

14. $\frac{1}{2}\times\frac{1}{4}$.

15. $\frac{1}{2}\div\frac{1}{4}$.

16. $\frac{3}{4}-\frac{1}{2}$.

17. $\frac{3}{4}\times\frac{1}{2}$.

18. $\frac{3}{4}\div\frac{1}{2}$.

19. $\frac{1}{2}+\frac{1}{8}$.

20. $\frac{1}{2}\times\frac{1}{8}$.

21. $\frac{1}{2}\div\frac{1}{8}$.

22. $\frac{7}{8}-\frac{1}{2}$.

23. $\frac{5}{8}\times\frac{3}{4}$.

24. $\frac{7}{8}\div\frac{3}{4}$.

Add the following:

25.	**26.**	**27.**	**28.**	**29.**
240.4	64.28	33.21	169.28	147.26
802.2	85.	150.5	3.7	38.09
79.8	6.32	7.32	45.94	140.6
388.7	108.51	46.48	2.33	3.98

30.	**31.**	**32.**	**33.**	**34.**
4.028	0.387	1.043	0.2909	0.7864
.396	.0426	.427	.0628	.3429
.285	.0539	6.091	.0703	.0008
4.72	.2064	.5	.5076	.703
3.942	.0708	.709	.0289	.0682
6.587	.0684	.792	.0587	.5209

Add, and also subtract, the following:

35.	**36.**	**37.**	**38.**	**39.**
90.32	209.8	6.273	81.04	50.93
4.86	49.2	.992	6.98	27.98

40.	**41.**	**42.**	**43.**	**44.**
1.035	0.2809	82.481	64.035	70.068
.876	.1589	2.909	37.029	39.579

Multiply or divide as indicated:

45. 1.4 × 6.64.
46. 3.6 × 72.08.
47. 888.16 ÷ 2.8.
48. 103.6 ÷ 0.56.
49. 1.5 × 553.35.
50. 5.8 × 439.002.
51. 825.468 ÷ 4.2.
52. 1131.264 ÷ 4.8.
53. 1.6 × 140.64.
54. 6.2 × 53.134.
55. 20.72 ÷ 0.28.
56. 38.8692 ÷ 5.4.
57. 388.692 ÷ 5.4.
58. 3.88692 ÷ 5.4.
59. 3886.92 ÷ 0.54.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. If you are multiplying dollars and cents by a fraction, where do you place the decimal point in the product?

2. If you are dividing dollars and cents by a whole number, how do you know where to place the decimal point in the quotient?

3. If you are multiplying one decimal fraction by another, how many decimal places must you have in the product? Why is this the case? Illustrate by an example.

4. If you are dividing a decimal fraction by a whole number, how can you tell where to put the decimal point in the quotient? Illustrate by an example.

5. If you are dividing a decimal fraction, and the divisor is not a whole number, how may you make it such without altering the quotient? Give your reason.

6. If you know the length of each side of a triangle, how do you find the perimeter, or distance around the triangle?

7. If you know the length of only one side of a square, how do you find the perimeter? Illustrate by an example.

8. If you know the length of only one side of a square, how do you find the area of the square? Illustrate by an example.

9. If you know the cost of several baseball bats, how do you find the cost of one?

10. If you know the cost of baseballs by the dozen, how do you find the cost of each ball? Could you probably buy one at this price? If not, why?

11. If you know the cost of a certain number of tennis balls, how do you find the cost of a certain other number?

III. INDUSTRIAL PROBLEMS

DOES IT PAY TO HAVE GOOD ROADS?

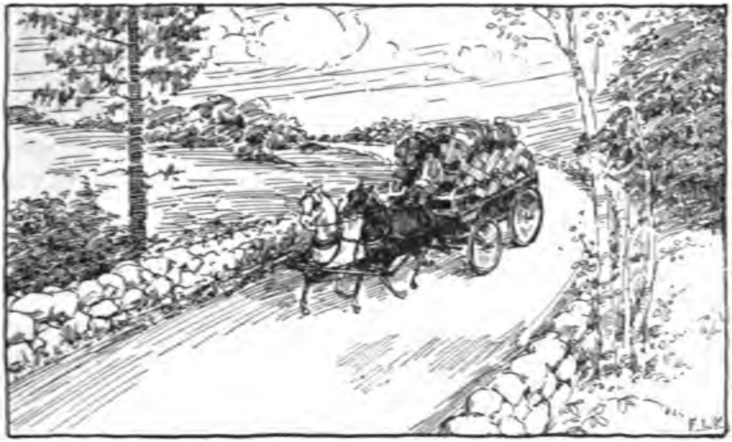
1. Do good roads pay? Here is a picture of a bad road in the country. A teamster had to haul $7\frac{1}{2}$ tons of barbed wire from the railway, a distance of 23 mi. He found that he could haul only 1500 lb. to a load and that it took him $1\frac{1}{2}$ da. to do it. How long did it take him to haul the $7\frac{1}{2}$ tons of wire, and how much did it cost at $4 per day for man and team?

2. In Ex. 1, what was the cost of hauling 1 ton 1 mi., or the cost of 1 ton-mile, as it is ordinarily called?

3. The next picture shows the same road after improvement. The teamster finds that with the same team he can now easily haul $2\frac{1}{2}$ tons to the load and do the work in 1 da. What does it now cost to haul a ton of wire? What is the cost per ton-mile?

4. Comparing the results in Exs. 2 and 3, what common fraction less is the cost per ton-mile in Ex. 3, owing to good roads?

5. If a teamster has to haul 30 tons of merchandise 23 mi. over this road, what is the cost if the road is in the bad condition referred to in Ex. 1? What is the cost if the road is in the good condition referred to in Ex. 3 and shown in the following picture?



6. A farmer lives 10 mi. from the railway. The road was formerly so bad that with a two-horse team he could haul only 40 bu. of wheat, and it took a day to make the round trip. If the farmer reckoned $4 a day for man and team, how much did it cost per bushel to haul the wheat?

7. The roads were recently improved. The farmer can now haul 75 bu. with his team. Find the cost per bushel of hauling the wheat now.

8. Comparing Exs. 6 and 7, how much more money does the farmer really get per bushel now that he has good roads?

9. Taking 60 lb. as the weight of a bushel of wheat and comparing Exs. 6 and 7, what has been the reduction in cost of cartage per ton-mile?

10. The average cost of hauling cotton to market in the Southern states is 80¢ per bale. If these states market 16,250,000 bales a year, what is the cost of hauling it?

11. A state often borrows money to build good roads. New York State has borrowed $100,000,000 for this purpose, and pays each year as interest 0.04 of this sum. How much interest does the state pay per year? New York City, because of its wealth, pays $\frac{7}{10}$ of this. What is left for the rest of the state to pay?

12. Sometimes a state neglects its roads after they are improved. In this way a certain road went to pieces in a single year, and it cost $1400 a mile to restore it. If it had been kept in repair, the total cost would have been only $608 a mile. On a road 78 mi. long, how much did the state waste by its carelessness?

13. Automobiles help to pay for the roads they use. In a recent year there were 1,900,000 motor vehicles of all kinds in this country, and they paid on an average $10 a year for registration and licenses. How much did they pay to the states?

14. A county produces an average of 30 bu. of wheat to the acre, and this wheat has to be hauled an average of 5 mi. If good roads reduce the hauling cost 5¢ per ton-mile, how much do they increase the value of the wheat per acre? At 60 lb. to the bushel, what part of a ton is produced to the acre?

15. Having solved Ex. 14, let us find the saving on this part of a ton for 5 mi. at 5¢ per ton-mile.

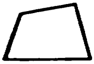
16. Following the work in Ex. 15, let us now consider how much this means per square mile. There being 640 acres to a square mile, how much is the saving per square mile?

17. If the county mentioned in Exs. 14–16 contains 300 sq. mi., and 0.6 of the land is used for growing wheat, how much does the county gain in value of wheat per year by having good roads?

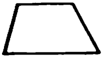
CHAPTER V

PRACTICAL MEASUREMENTS

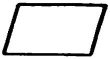
148. Figures commonly Measured. We are already familiar with most of the figures commonly measured. We shall now review some of these and learn how to measure others.



Quadrilateral

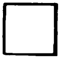


Trapezoid



Parallelogram

Rectangle



Square

149. Quadrilateral. A figure which is bounded by four straight lines is called a *quadrilateral.*

150. Trapezoid. A quadrilateral which has one pair of opposite sides parallel is called a *trapezoid.*

151. Parallelogram. A quadrilateral which has its two pairs of opposite sides parallel is called a *parallelogram.*

152. Rectangle. A parallelogram which has all its angles right angles is called a *rectangle.*

153. Square. A rectangle which has its sides all equal is called a *square.*

154. Polygon. A figure which is bounded by straight lines is called a *polygon.* The figures pictured above are all special kinds of polygons.

As a preliminary to this chapter the tables on pages 108–119 should be reviewed; and in order that this review should be thorough, much oral work is provided on pages 186–187.

ORAL EXERCISE

1. Express 640 rd. as miles.
2. Express 64 oz. as pounds.
3. Express 2 A. as square rods.
4. Express 33 ft. as rods; as yards.
5. Express 54 cu. ft. as cubic yards.
6. Express 9 ft. as inches; as yards.
7. Express 11 da. as hours; as weeks.
8. Express 17 qt. as pints; as gallons.
9. Express 9 pk. as quarts; as bushels.
10. How many cubic inches in 10 cu. ft.?
11. How many rods in 3 mi.? in 10 mi.?
12. How many yards in 8 rd.? in 10 rd.?
13. How many cubic feet in 2 cd. of wood?
14. Express 300 min. as hours; as seconds.
15. How many quarts in a barrel of $31\frac{1}{2}$ gal.?
16. How many pints in a 60-gallon hogshead?
17. Express 80 sq. rd. as a fraction of an acre.
18. How many right angles in 270°? in 180°?
19. How many days from May 23 to June 25?
20. How many days from June 11 to July 17?
21. What part of a 16-carat ring is pure gold?
22. Express 11 yd. as rods; as feet; as inches.
23. Express 30 pt. as quarts; as gills; as gallons.
24. How many feet in 2 rd. 8 ft.? in 2 rd. 10 ft.?
25. How many feet in 10 rd.? in 5 rd.? in 2 rd.?
26. How many degrees in $540'$? in $420'$? in $660'$? in $720'$?

27. How many mills in $2.75?

28. How many days from August 15 to October 15?

29. How many acres in a tract of land 10 mi. square?

30. How many days from January 17 to February 17?

31. How many square feet in 10 sq. yd.? in 5 sq. yd.?

32. Using 6 qt. a day, how long will 3 bu. of oats last a horse?

33. What is the perimeter of a square that is $4\frac{1}{2}$ ft. on a side?

34. What is the side of a square whose perimeter is 14 ft. 4 in.? of a square whose perimeter is $4\frac{1}{2}$ ft.?

35. Since a gallon contains 231 cu. in., how many gallons in 693 cu. in.?

36. How many square feet in a surface having an area of 720 sq. in.?

37. Express a long ton (2240 lb.) as a short ton and hundredths of a short ton.

38. What is the perimeter of an equilateral triangle that is 3 ft. 8 in. on a side?

39. If school closes June 21, how many days from that day to the Fourth of July?

40. How many quarts will a 10-bushel bin hold? a 5-bushel bin? How many pecks in each?

41. A man buys a building lot that has a frontage of 2 rd. How many square feet in his sidewalk if it is 5 ft. wide?

42. A cellar 15 ft. by 21 ft. is to be excavated to a depth of 6 ft. How many loads (cubic yards) of earth must be removed?

43. If a man buys a load of coal weighing 1 T. 500 lb., how many pounds of coal does he buy? Express this weight in tons and tenths of a ton.

WRITTEN EXERCISE

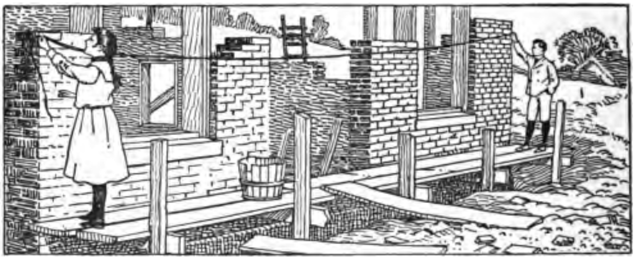
1. A baseball "diamond," or "infield," is a square 90 ft. on a side. Draw this to the scale of 1 in. to 20 ft.

As we found on page 110, this means that we represent 20 ft. by 1 in. Therefore 90 ft., which is $4\frac{1}{2} \times 20$ ft., will be represented by $4\frac{1}{2}$ in. Such work is very practical.

2. On the infield the pitcher's plate is 60.5 ft. from the point where two sides of the diamond meet at the home base. Draw another plan of the infield, using the scale of 1 in. to 30 ft., and mark the position of the pitcher's plate.

3. The first, second, and third bases are placed in squares 15 in. on a side. Draw one of these squares to the scale $\frac{1}{5}$.

This means that 5 in. is to be represented by 1 in.



4. Fred and Rose measure a house that is being built near the school. They find it to be 20 ft. by 30 ft. Draw a ground plan of the house to the scale of 1 in. to 5 ft.

5. They find that the front door is 4 ft. by 7 ft. 6 in. Draw a plan of the door to the scale $\frac{1}{12}$.

6. A brick is 8 in. long and 4 in. wide. Draw a plan of the top to the scale $\frac{1}{2}$.

Teachers should give much work in measuring and in drawing to scale.

7. Rob and Mary are measuring a cellar that is being dug for a house. They use a tape that is 100 ft. long. It is divided into feet, inches, half inches, and quarter inches. How many inches in the tape? How many half inches? How many quarter inches?

8. They find that the workmen have dug only part of the cellar, this part being 6 ft. wide. Express the width in yards. Express the width in inches.

9. They find that the excavation is 22 ft. 6 in. long. Express this in feet and a fraction; in yards and a fraction.

10. The distance from where Rob is to the house is 412 ft. 6 in. A rod being $16\frac{1}{2}$ ft., express the distance in rods.

11. If a field is 17 rd. long, what is the length in feet?

12. If a field is 19 rd. long and 12 rd. wide, what length of fence is needed to inclose it?

13. If Jack steps 27 in. each time, and takes 450 steps in coming from his home to the school, how many feet does he come?

In each half year the teacher should give plenty of work in practical measuring. The pupils should measure the lengths of their ordinary steps, and should learn to pace such distances as the length of the room and the width of the playground. They should become perfectly familiar with the divisions of the ruler, and should have practice in estimating the height of the room, of trees, and of the schoolhouse; the width of the street and of pieces of land; and the distances they travel in coming to school. In order to be real, this work must apply to local conditions and must therefore be given by the teacher.

DRAWING TO SCALE

1. Draw a plan of a floor 16 ft. long and 12 ft. wide, representing 1 ft. by $\frac{1}{8}$ in.

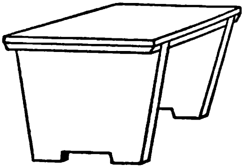
2. What part of 1 ft. is $\frac{1}{8}$ in.? Then the drawing in Ex. 1 is what part as long as the floor that it represents?

3. Draw a plan of a box cover 6 in. by 10 in., to the scale $\frac{1}{4}$.

4. Draw a plan of a school garden 56 ft. long by 48 ft. wide, to the scale of $\frac{1}{8}$ in. to 1 ft.

5. Draw a plan of a rug that the class is weaving. The rug is to be 10 in. by 16 in. Use the scale $\frac{1}{4}$.

6. This is a drawing of a doll's table made by the class. The top is to be 10 in. by 12 in. The end supports are to be 10 in. at the top, 7 in. at the bottom, and 8 in. high. Draw plans for the top and ends, using the scale $\frac{1}{4}$. The end supports are indented 1 in. at the distance of 2 in. from each bottom corner.



7. Draw a plan of a book cover that the boys are making. The cover is to be 5 in. by 8 in. Use the scale $\frac{1}{2}$.

8. A corridor in a school building is 12 ft. by 128 ft. Draw a floor plan of this corridor, using the scale of $\frac{1}{16}$ in. to 1 ft.

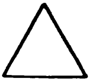
9. A playground near our school is 65 ft. by 125 ft. Draw a plan of the playground, using the scale of $\frac{1}{16}$ in. to 1 ft.

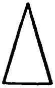
10. A schoolroom is 40 ft. long and 20 ft. wide. Draw a plan of the floor, using the scale $\frac{1}{80}$.

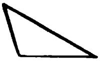
11. A sitting room is 14 ft. by 18 ft. Draw a plan of the floor, using any convenient scale.

12. A bedroom is 12 ft. 6 in. by 16 ft. Draw a plan of the floor, using any convenient scale.

155. Equilateral Triangle. A triangle in which the three sides are equal is called an *equilateral* triangle.

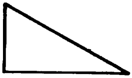

Equilateral

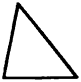

Isosceles

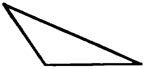

Scalene

156. Isosceles Triangle. A triangle in which two of the sides are equal is called an *isosceles* triangle.

157. Scalene Triangle. A triangle in which no two sides are equal is called a *scalene* triangle.


Right


Acute


Obtuse

158. Right Triangle. A triangle in which one of the angles is a right angle is called a *right* triangle. The side opposite the right angle is called the *hypotenuse.*

159. Acute Triangle. A triangle in which all the angles are acute angles is called an *acute* triangle.

160. Obtuse Triangle. A triangle in which one of the angles is an obtuse angle is called an *obtuse* triangle.

ORAL EXERCISE

1. How long is the side of an equilateral triangle if the perimeter is 45 in.? 9 ft. 6 in.? $9\frac{1}{2}$ yd.? 63 ft. 9 in.?

2. How long is the side of a square if the perimeter is 84 in.? 8 ft. 4 in.? 16 yd. 8 in.? $60\frac{1}{2}$ rd.? 120 in.?

3. How long is the fourth side of a quadrilateral if the perimeter is 85 in. and three of the sides are 10 in., 20 in., 30 in.?

WRITTEN EXERCISE

1. How long is the perimeter of an equilateral triangle $275\frac{2}{3}$ ft. on a side?

Draw figures of the correct shape in all examples on this page, not to scale but only approximately.

2. A triangle has a perimeter of 342 ft. 8 in. One side is 90 ft. and another is half as much longer; how long is the third?

3. The base of an isosceles triangle is 35 ft., and each of the equal sides is $\frac{1}{5}$ longer. How long is the perimeter?

4. Cut off the three angles of a triangle and place them together. The sum is equal to how many right angles?

Find the sides of equilateral triangles of which the perimeters are as follows:

5. 113 ft. 3 in.
6. 127 yd.
7. 88 ft. 3 in.

Find the sides of squares of which the perimeters are as follows:

8. 110 ft. 4 in.
9. 42 ft. $3\frac{1}{2}$ in.
10. 61 yd. 2 ft.
11. 2 ft. 4 in.
12. 118 ft. 8 in.
13. 227 mi. 40 rd.

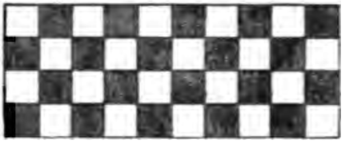
14. The perimeter of an isosceles triangle is 100 ft., and the unequal side is 33 ft. Find the length of one of the equal sides.

15. The perimeter of an isosceles triangle is 88 ft., and the unequal side is $\frac{1}{5}$ of the perimeter. Required the sides.

16. The perimeter of a triangle is 24 ft. One side is $\frac{1}{3}$ of the perimeter and another is $\frac{3}{8}$ of the perimeter. How many feet are there in each of the three sides?

17. The perimeter of a triangle is 120 ft. One side is $\frac{1}{4}$ of the perimeter and another is $\frac{1}{3}$ of the perimeter. How many feet are there in each of the three sides?

161. Area of a Rectangle. A school building has an entrance hall 10 ft. long and 4 ft. wide. The floor is made of marble squares 1 ft. on a side. What is the easiest way of finding the number of squares?



There are 10 squares in each row, and there are 4 rows. Hence there are 4 × 10 squares.

In the same way we see that the area of the floor is

$$4 \times 10 \text{ sq. ft.} = 40 \text{ sq. ft.}$$

162. Base and Height. The line on which a figure appears to stand is called the *base*. The distance from the highest point above the base to the base is called the *height* or *altitude* of a figure.

163. Finding the Area of a Rectangle. From the above example and definitions we see that

The area of a rectangle equals the product of the base and height.

It must always be understood that such a rule refers to abstract numbers. We do not mean that we multiply 10 ft. by 4 ft. in § 161, but that

4 × 10 = 40, the *number* of square feet.

WRITTEN EXERCISE

Find the areas of the following rectangles, using short methods:

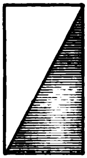
1. 37½ ft. by 48 ft.
2. 33⅓ yd. by 42 yd.
3. 75 ft. by 96 ft.
4. 66⅔ yd. by 48 yd.
5. 125 ft. by 160 ft.
6. 250 ft. by 324 ft.
7. 625 ft. by 720 ft.
8. 87½ in. by 96 in.
9. Find the area of the floor of a room 24 ft. by 28 ft. 6 in.

The teacher should give plenty of practical work in finding the areas of floors, platforms, desks, the school grounds, the sidewalk, and the like.

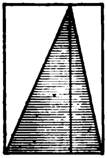
ORAL EXERCISE

1. How is the area of a rectangle found?

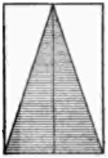
2. How do the areas of these triangles compare with the areas of the rectangles? Then a triangle is what part of a rectangle of the same base and the same height?



A



B



C

3. If the rectangle *A* is 2 in. wide and 4 in. high, what is its area? What is the area of the shaded triangle in *A*?

4. If the rectangle *B* is 3 ft. wide and 5 ft. high, what is its area? What is the area of the shaded triangle in *B*?

5. If the rectangle *C* is 4 yd. wide and 6 yd. high, what is its area? What is the area of the shaded triangle in *C*?

6. What is the area of a rectangle with base 6 ft. and height 9 ft.? of a triangle with base 6 ft. and height 9 ft.?

7. How do you find the area of a triangle?

8. Do you know of some triangular piece of ground the area of which you might wish to find? What measurements would you take to find the area of such a piece?

9. What is the area of a triangular flower bed with base 20 ft. and height 10 ft.?

Find the areas of triangles with bases and heights as follows:

10. 4 in., 3 in.
11. 8 in., 6 in.
12. 5 in., 4 in.
13. 7 in., 6 in.
14. 8 in., 10 in.
15. 9 ft., 4 ft.

164. Area of a Triangle. As shown on the preceding page,

The area of a triangle equals half the product of the base and altitude.

For example, what is the area of a triangle of base 18 in. and altitude 7 in.?

$\frac{1}{2}$ of 18×7 sq. in. = 63 sq. in.

ORAL EXERCISE

Using aliquot parts, state the areas of triangles with bases and altitudes as follows:

1. 80 in., 75 in.
2. 16 in., 125 in.
3. 60 in., $33\frac{1}{3}$ in.
4. 80 in., 25 in.
5. 44 in., 50 in.
6. 120 in., $16\frac{2}{3}$ in.

WRITTEN EXERCISE

Find the areas of triangles with bases and altitudes as follows:

1. 43 ft., 32 ft.
2. 7.9 in., 8.7 in.
3. 8.7 in., 6.3 in.
4. 15.2 in., 9.4 in.
5. $37\frac{1}{4}$ ft., $42\frac{1}{2}$ ft.
6. 2 yd. 32 in., 27 in.
7. 3 ft. 4 in., 2 ft. 8 in.
8. 2 ft. $2\frac{1}{2}$ in., 1 ft. $8\frac{1}{4}$ in.

9. What is the area of an equilateral triangle with perimeter 37 ft. 6 in. and altitude 10.8 ft.?

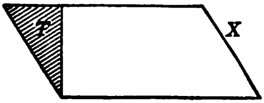
10. What is the area of a triangle with altitude 6.4 rd. and base 17 rd.? altitude 7.2 in. and base 17 in.?

11. The two sides forming the right angle of a right triangle are 30 rd. and 40 rd. What is the area in square rods? in acres?

12. What is the number of square feet of slating required for a steeple, the surface of which is made up of 6 triangles, each with base 20 ft. and altitude 65 ft.?

ORAL EXERCISE

1. If we cut triangle *T* from this parallelogram and place it where *X* is, what kind of a figure do we have? What does this tell us about the area of a parallelogram?



This work should actually be done, a parallelogram being cut from paper and then transformed into a rectangle as suggested.

State the areas of parallelograms with bases and heights as follows:

2. 11 ft., 9 ft.
3. 2 yd., $1\frac{1}{2}$ ft.
4. 6 ft., $9\frac{1}{2}$ ft.
5. 4 in., $6\frac{3}{4}$ in.
6. 8 ft., $10\frac{1}{4}$ ft.
7. 10 ft., $5\frac{1}{2}$ ft.

State the areas of triangles with bases and altitudes as follows:

8. 7 ft., 6 ft.
9. 8 ft., $4\frac{1}{2}$ ft.
10. 3 ft., 6 in.

165. Area of a Parallelogram. *The area of a parallelogram equals the product of the base and altitude.*

WRITTEN EXERCISE

Find the areas of parallelograms with bases and heights as follows:

1. $19\frac{3}{4}$ in., 16.8 in.
2. 171.4 ft., 27.2 ft.
3. 6 ft. 4 in., 4 ft. 6 in.
4. 14.7 ft., 16.5 ft.
5. 3.75 in., $2.33\frac{1}{3}$ in.
6. 16 ft. 6 in., 8.25 ft.

Find the areas of triangles with bases and altitudes as follows:

7. 31.2 in., 27.3 in.
8. 32 ft. 6 in., 12 ft.
9. 16.75 in., 14.2 in.
10. 17.3 ft., 1 rd.

11. A man has a piece of land in the form of a parallelogram. The base of the parallelogram is 36 rd. and the height is 24 rd. How many acres are there in the piece of land?

166. Altitude of a Parallelogram. What is the altitude of a parallelogram with base 10 ft. and area 60 sq. ft.?

If the base is 10 ft. and the altitude is 1 ft., the area is 10 sq. ft.

Therefore the number of feet in the height is 60 sq. ft. ÷ 10 sq. ft., or 6. In practical computation we do not label the numbers; we simply divide 60 by 10.

Teachers should review with care the statement on page 18 with respect to abstract and concrete numbers. They should also encourage the pupils to find the areas of pieces of land in the forms of triangles and parallelograms if such can be found in the vicinity of the school. Only part of the examples on pages 196 and 197 need be taken by any one class.

WRITTEN EXERCISE

Find the areas of parallelograms with bases and heights as follows:

1. 6.7 in., 3.9 in.
2. 42.5 ft., 27.8 ft.
3. 16.9 ft., 7.2 ft.
4. 3 ft., 2 ft. 7 in.
5. 5 rd. 6 ft., $2\frac{1}{2}$ rd.
6. 17 ft. 8 in., 27 in.
7. 9 ft. 6 in., 6 ft. 9 in.
8. 345 ft., 127 ft.
9. 2 yd. 17 in., 2 ft. 3 in.
10. 17 yd. 23 in., 5 yd. 2 in.

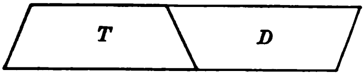
Find the heights of parallelograms with bases and areas as follows:

11. $15\frac{1}{2}$ in., 108.5 sq. in.
12. 25 ft., 275 sq. ft.
13. 22 ft. 4 in., 268 sq. ft.
14. 24 ft. 6 in., 588 sq. ft.
15. 129 ft., 10,320 sq. ft.
16. 31 ft. 2 in., 187 sq. ft.
17. 16 ft. 8 in., 100 sq. ft.
18. 17 ft. 6 in., 105 sq. ft.

Find the heights of triangles with bases and areas as follows:

19. 4 in., 6 sq. in.
20. 9 in., 36 sq. in.
21. 10 in., 45 sq. in.
22. 12 in., 60 sq. in.
23. 14 ft., 70 sq. ft.
24. 12 yd., 72 sq. yd.

167. Area of a Trapezoid. If a trapezoid T has its double cut from paper and turned over and fitted to it, like D, the two together make a parallelogram. This is easily seen by cutting figures out of paper. Therefore



The area of a trapezoid equals half the area of a rectangle with the same altitude and with a base equal to the sum of the two parallel sides of the trapezoid.

For example, what is the area of a trapezoid of altitude 4 in. and parallel sides 8 in. and 10 in.?

The area is $\frac{1}{2}$ of $4 \times (8 + 10)$ sq. in., or $\frac{1}{2}$ of 4×18 sq. in., or 36 sq. in.

ORAL EXERCISE

State the areas of trapezoids with altitudes first given below, followed by the two parallel sides:

1. 6 in., 7 in. and 13 in.
2. 10 in., 12 in. and 13 in.
3. 14 in., 14 in. and 16 in.
4. 15 in., 8 in. and 12 in.
5. 17 in., 9 in. and 11 in.
6. 11 in., 11 in. and 19 in.

WRITTEN EXERCISE

Find the areas of trapezoids with altitudes first given below, followed by the two parallel sides:

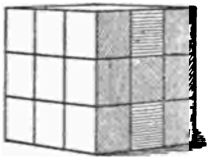
1. $13\frac{1}{2}$ in., $6\frac{1}{4}$ in. and 14 in.
2. 22 in., $11\frac{1}{2}$ in. and $9\frac{3}{4}$ in.
3. 1.5 in., 1.5 in. and 3.4 in.
4. 2.5 in., 2.1 in. and 4.2 in.
5. $3\frac{1}{2}$ ft., 4 ft. and $5\frac{1}{2}$ ft.
6. 17 rd., 16 rd. and 23 rd.

7. If the area of a trapezoid is $8\frac{1}{4}$ sq. in. and the parallel sides are $3\frac{1}{2}$ in. and $4\frac{3}{4}$ in., what is the altitude?

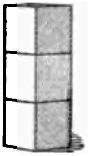
8. If the altitude of a field in the form of a trapezoid is 10 rd., and the parallel sides are $31\frac{1}{4}$ rd. and $16\frac{3}{4}$ rd., what is the area?

ORAL EXERCISE

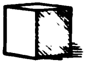
1. If $C = 1$ cu. in., what is the volume of B? the volume of A?



A



B



C

2. What is the volume of a work box 3 in. by 4 in. by 2 in.?

State the volumes of solids of the following dimensions:

3. 4 in., 5 in., 6 in.

4. 3 in., 3 in., 7 in.

5. 2 in., 3 in., 10 in.

6. 6 in., 8 in., 10 in.

168. Rectangular Solid. A solid having six sides, each side being a rectangle, is called a *rectangular solid.* If the sides are all squares, the solid is called a *cube.*

169. Volume of a Rectangular Solid. *The volume of a rectangular solid equals the product of the three dimensions.*

In finding volumes we do not write 3 in. × 4 in. × 2 in., but 3 × 4 × 2 cu. in. It is customary, however, in giving dimensions, to write 3 in. × 4 in. × 2 in., or $3'' \times 4'' \times 2''$, reading either of these expressions "3 inches by 4 inches by 2 inches."

ORAL EXERCISE

Find the volumes of solids of the following dimensions:

1. $3'' \times 4'' \times 7''$.

2. $6' \times 8' \times 10'$.

3. $3' \times 8' \times 11'$.

4. $2'' \times 8'' \times 9''$.

5. $2' \times 3' \times 6'$.

6. $5' \times 6' \times 10'$.

7. $4' \times 7' \times 10'$.

8. $6' \times 9' \times 10'$.

9. $20' \times 30' \times 5'$.

10. $11'' \times 11'' \times 10''$.

11. 6 yd. × 8 yd. × 2 yd.

12. 6 ft. × 8 ft. × 11 ft.

WRITTEN EXERCISE

1. How many cubic inches in an aquarium 16 in. long, 8½ in. wide, and 9½ in. deep?

2. How many cubic feet of masonry in a wall 40 ft. long, 30½ ft. high, and 1½ ft. thick?

3. A cellar 24 ft. by 39 ft. by 6 ft. is to be excavated. How much will it cost at 40¢ a load (cubic yard)?

4. How many cords in a pile of 4-foot wood 5 ft. high and 56 ft. long? 6 ft. high and 128 ft. long?

5. How many loads of earth must be taken out in excavating a cellar 24 ft. by 18 ft., and 7 ft. deep?

6. How much will it cost to excavate a ditch 150 ft. by 2 ft. by 6 ft., at 50¢ per cubic yard? at 62½¢ per cubic yard?

7. A schoolroom is 32 ft. 8 in. long, 18⅓ ft. wide, and 12½ ft. high. What is the number of cubic feet in the room?

8. A tunnel 720 ft. long has a cross-section area of 180 sq. ft. How much earth and rock were excavated? (Imagine the cross section 10 ft. by 18 ft.)

9. The dimensions of a room are 14 ft., 12 ft., and 9 ft. How many square feet of floor? of walls? of ceiling?

10. In preparing a flower bed in a park it was necessary to fill in a space 60 ft. by 100 ft. to an average depth of 1½ ft. The earth cost 33⅓¢ a load. What was the total cost of the earth used?

11. The interior of a freight car is 8 ft. wide, 34 ft. long, and 7 ft. high. How many cubic feet does it contain? If it is filled with grain to a height of 5 ft., what is the weight of the grain at 60 lb. to the bushel, allowing 1¼ cu. ft. to the bushel?

Teachers should frequently require the pupils to estimate results in advance.

170. Practical Equivalents. In the problems given below, refer to these measurements without memorizing them.

31½ gal. = 1 bbl.
231 cu. in. = 1 gal.
24¾ cu. ft. = 1 perch
7½ gal. = 1 cu. ft.
4⅕ cu. ft. = 1 bbl.
1¼ cu. ft. = 1 bu.

The three values in the right-hand column are the ones usually taken, but they are not exact. For example, a bushel is 2150.42 cu. in.

In estimating brickwork, take 22 bricks with the mortar as 1 cu. ft.

A ton of hay varies from about 450 cu. ft. to 550 cu. ft., depending on the quality and on how much the hay has settled, but 500 cu. ft. may be taken in solving the problems. The volume of a ton of coal depends upon the size and kind of the coal, but 35 cu. ft. may be taken for our purposes.

In examples like the following, all measures refer to inside dimensions.

WRITTEN EXERCISE

1. How many bushels in a bin 6 ft. by 7 ft. by 4 ft.?

2. How many bushels in a bin 5 ft. by 9 ft. by 3 ft.?

3. How many bushels in a box 18 in. wide, 3 ft. long, and 2 ft. deep?

4. How many gallons in a tank 5 ft. by 6 ft. by 4 ft.?

5. How many gallons in an aquarium 14 in. long, 9 in. wide, and 11 in. deep?

6. How many gallons in a tank 6¼ ft. long, 3½ ft. wide, and 2⅓ ft. deep?

7. A cistern is 6 ft. square at the bottom and 5 ft. deep. How many barrels will it hold?

8. An iron tank is 8 ft. long, 6½ ft. wide, and 4¾ ft. deep. How many barrels will it contain?

9. A watering trough is 6 ft. long, 18 in. wide, and 14 in. deep. How many barrels will it contain?

10. A coal bin is 21 ft. long, 13 ft. wide, and 8 ft. deep. About how many tons of coal will it hold?

11. A coal bin is 14 ft. long, $8\frac{1}{2}$ ft. wide, and $6\frac{1}{4}$ ft. deep. About how many tons of coal will it hold?

12. A haymow is 22 ft. long, 14 ft. wide, and 9 ft. high. About how many tons of hay will it hold?

13. A barn has two haymows, each 24 ft. long, 15 ft. 6 in. wide, and 10 ft. high. About how many tons of hay will it hold?

14. How many cords of wood in a pile 4 ft. wide, 7 ft. high, and $32\frac{1}{2}$ ft. long?

15. How many cords of wood in a pile 4 ft. wide, $6\frac{1}{2}$ ft. high, and 44 ft. long?

16. If a schoolroom is 30 ft. long, 15 ft. wide, and 12 ft. high, and 180 cu. ft. of air should be allowed to each person, how many will the room accommodate?

17. A tank holds 82,500 gal. About how many cubic feet does it contain?

18. An aquarium contains $3\frac{4}{5}$ cu. ft. About how many gallons does it hold?

19. An aquarium is 18 in. long, $9\frac{1}{4}$ in. wide, and $10\frac{1}{8}$ in. deep. How many gallons does it hold?

20. A can containing 1732.5 cu. in. is full of milk. What is the milk worth at 6¢ a quart?

21. A wall is $32\frac{1}{2}$ ft. long, 2 ft. thick, and $32\frac{1}{2}$ ft. high. How many perches of masonry does it contain?

22. A brick wall is to be built 42 ft. long, $1\frac{1}{2}$ ft. thick, and $6\frac{1}{4}$ ft. high. About how many bricks will be needed?

23. How much will a mason charge for laying a wall 20 ft. long, $1\frac{1}{2}$ ft. thick, and $5\frac{1}{2}$ ft. high, at $\$1.87\frac{1}{2}$ a perch?

24. The standard Florida orange box for shipping is 12 in. by 12 in. by 26⅝ in. How many cubic inches does it contain?

Use cancellation in all such cases.

25. In shipping Georgia peaches a crate 8 in. by 12½ in. by 22 in. is used. This crate contains how many cubic inches more than 1 bu.?

26. In New Jersey a bushel cranberry box is 12 in. by 8¾ in. by 22 in. How does this differ from the standard bushel?

27. California apples and pears for local use are shipped in a 50-pound box 22 in. by 10 in. by 12 in. How does this differ in capacity from the standard bushel?

28. A California cantaloupe crate is 38 in. long, 16 in. wide, and 15 in. deep. How many cubic inches does it contain?

29. The California orange box is sometimes 26½ in. by 11¼ in. by 11¼ in., and sometimes 22 in. by 7¾ in. by 17½ in. Which holds the more, and how many cubic inches more?

30. Dried fruits are shipped in 50-pound or 25-pound boxes. The 50-pound box is 15¼ in. by 9 in. by 9 in. How many cubic inches in the box?

31. The 25-pound box mentioned in Ex. 30 is sometimes 13¾ in. by 9¾ in. by 5¾ in., and sometimes 16 in. by 9 in. by 5 in. Which box holds the more, and how much more?

32. For shipping fruit out of the state, the sizes of some of the boxes used in California are as follows:

Plums, 18½ in. by 11¾ in. by 4 in.
Cherries, 16½ in. by 10⅜ in. by 2½ in.
Pears, 18½ in. by 11¾ in. by 8½ in.
Peaches, 18½ in. by 11¾ in. by 4⅞ in.

Find the number of cubic inches in each of these boxes.

171. Other Problems Relating to Volumes. Instead of having the three dimensions of a rectangular solid given to find the volume, we may have the volume and certain dimensions given to find other dimensions.

1. How deep is a box 5 ft. long and 4 ft. wide, containing 60 cu. ft.?

$$\boxed{\frac{60}{5 \times 4} = 3}$$

Since it is 5 ft. long and 4 ft. wide, the area of the base is 20 sq. ft. If the box were 1 ft. deep, the volume would be 20 cu. ft.

Therefore the box is as many times 1 ft. deep as 60 cu. ft. ÷ 20 cu. ft., or 3. Therefore the box is 3 ft. deep.

The number of cubic units of volume, divided by the product of the numbers of units of two dimensions, equals the number of units of the third dimension.

This is sometimes expressed thus: *The volume divided by two dimensions equals the third dimension.*

2. What is the area of the base of a block $2\frac{1}{2}$ in. high, containing 70 cu. in.?

If the block were only 1 in. high, the volume would be 70 cu. in. ÷ $2\frac{1}{2}$, or 28 cu. in. But the area of the base of such a block is 28 sq. in. Therefore the area of the base is 28 sq. in.

$$\boxed{70 \div \frac{5}{2} = \frac{2 \times 70}{5} = 28}$$

This may be briefly stated as follows: *The volume divided by the height equals the area of the base.*

3. What is the thickness of a block containing 100 cu. in., the base containing 30 sq. in.?

If the block were 1 in. thick, it would contain 30 cu. in. Therefore it is as many times 1 in. thick as 100 cu. in. ÷ 30 cu. in., or $3\frac{1}{3}$. Hence it is $3\frac{1}{3}$ in. thick.

$$\boxed{\frac{100}{30} = 3\frac{1}{3}}$$

This may be briefly stated as follows:

The volume divided by the area of the base equals the height.

WRITTEN EXERCISE

1. How many cubic feet in a hall 60 ft. by 80 ft. by 23 ft.?

The pupils should measure the schoolroom and then apply Ex. 16, page 202.

2. A tank with base 9 ft. by $16\frac{2}{3}$ ft. contains 900 cu. ft. How deep is it?

3. A box with base 13 in. by 20 in. contains 3900 cu. in. How deep is it?

4. A tank 4 ft. deep contains 450 cu. ft. What is the area of the base?

5. A hall 22 ft. high and 30 ft. wide contains 28,600 cu. ft. Find the length.

6. A block containing 700 cu. ft. has a base area of 56 sq. ft. How high is it?

7. A hall contains 41,341.3 cu. ft. It is 41.3 ft. long and 36.4 ft. wide. How high is it?

8. A box contains 42,159 cu. in. The area of the bottom is 1081 sq. in. How deep is the box?

9. A block of granite is 14 in. long and 8.3 in. wide. It contains 836.64 cu. in. How thick is it?

10. The floor of a hall contains 1386 sq. ft. The hall contains 15,246 cu. ft. Required the height.

11. A storeroom is 14.3 ft. long, 13 ft. 6 in. wide, and has a capacity of 2316.6 cu. ft. How high is the room?

12. The floor of a bedroom is square and contains 144 sq. ft. If the room contains 1368 cu. ft., find the three dimensions.

First find the two equal factors of 144.

13. A room containing 2110.68 cu. ft. is 16.4 ft. long, and has a floor area of 234.52 sq. ft. What are the three dimensions of the room?

14. How many cubic feet in a bin that is 6 ft. 4 in. long, 3 ft. wide, and 3 ft. 6 in. deep?

15. Express in bushels the volume of a bin 4 ft. by 6 ft. by 3 ft. $6\frac{1}{2}$ in., allowing 2150 cu. in. to the bushel.

16. How many bars of soap, each 4 in. long, $2\frac{3}{4}$ in. wide, and $1\frac{1}{2}$ in. thick, can be packed in a box 16 in. long, $13\frac{3}{4}$ in. wide, and 1 ft. deep? Will the bars fit in the box?

17. At $9 a thousand, what is the value of the bricks in a kiln 60 ft. long, 25 ft. wide, and 9 ft. high, each brick being 4 in. by 8 in. by 2 in.?

18. The length of a tank is twice as great as the width, and the width is twice as great as the depth. If the width is 2 yd., how many gallons does the tank hold?

19. A cord of stone having the same volume as a cord of wood, what is a pile of cut stones 27 ft. long, 5 ft. wide, and 6 ft. high worth at $4.20 a cord?

20. If it takes 550 cu. ft. of clover hay to make a ton, how many tons of clover hay will fill a mow 32 ft. long, the width being double the depth and half the length?

21. How many gallons of water are there in a watering trough that is 6 ft. long and 16 in. wide, the depth of the water being three fourths of the width?

22. A bin 21 ft. 6 in. long and 10 ft. wide is filled with wheat to the depth of 5 ft. Allowing 2150 cu. in. to the bushel, what is the wheat worth at 80¢ a bushel?

23. A cellar is 32 ft. long, 17 ft. wide, and 8 ft. 6 in. deep. How much will it cost to build the wall, 1 ft. thick, at 35¢ per cubic foot, no allowance being made for openings, and the length of the wall being determined by the outside measure, thus doubling the corners?

172. Capacity and Weight. The laws of the various states usually specify the weight of a bushel, requiring it to weigh at least the amount specified below. This table should not be committed to memory, but used for reference.

Corn (in the ear),	70 lb.	Oats,	32 lb.
Barley,	48 lb.	Dried Apples,	26 lb.
Corn (shelled), Rye,	56 lb.	Timothy Seed,	45 lb.
Wheat, Clover Seed, Peas,	60 lb.	Dried Peaches,	33 lb.
Beans, White Potatoes,	60 lb.	Sweet Potatoes,	55 lb.
Buckwheat,	48 lb.	Onions,	48 lb.

WRITTEN EXERCISE

1. How many tons to 1000 bu. of wheat? to 1000 bu. of oats?

2. How many bushels of white potatoes to the ton?

3. How many bushels to a ton of corn in the ear? How many bushels to a ton of shelled corn?

4. How many tons to 1000 bu. of corn in the ear? How many tons to 1000 bu. of shelled corn?

Find the number of bushels to a ton of the following:

5. Oats.

6. Buckwheat.

7. Clover seed.

8. Onions.

9. Sweet potatoes.

10. Timothy seed.

11. A bushel of rye weighs how many times a bushel of oats?

12. A bushel of oats is what part as heavy as a bushel of shelled corn?

13. A bushel of timothy seed is what part as heavy as a bushel of buckwheat? It is what part as heavy as a bushel of peas?

LITTLE EXAMINATIONS

I. 1. Find the area of a rectangle 47 ft. 6 in. by 32 ft. 6 in.

2. Find the area of a triangle with base 16 ft. and height 6 ft.

3. Find the area of a parallelogram with base 7 ft. 6 in. and height 4 ft. 2 in.

4. Find the area of a trapezoid with parallel sides 4 ft. 6 in. and 3 ft. 4 in., and height 2 ft. 6 in.

5. Find the volume of a rectangular solid 2 ft. 3 in. long, 1 ft. 8 in. wide, and 13 in. deep.

II. 1. Find the area of a rectangle 1 yd. 9 in. by 2 yd. 16 in.

2. Find the area of a triangle with base 14 rd. 7 ft. and height 9 rd. 3 ft.

3. Find the area of a rectangle $8\frac{1}{4}$ ft. by $4\frac{1}{2}$ rd.

4. Find the area of a parallelogram with base 127 rd. $8\frac{1}{4}$ ft. and height 64 rd.

5. Find the volume of a rectangular solid 6 ft. 8 in. by 5 ft. 4 in. by 2 ft. 6 in.

III. 1. Find the area of a rectangle 84 rd. $4\frac{1}{8}$ ft. by 126 rd.

2. Find the area of a triangle with base $12\frac{1}{2}$ ft. and height 17 in.

3. Find the area of a rectangle $6\frac{1}{2}$ rd. by 146 ft.

4. Find the area of a triangle with base 37 rd. $4\frac{1}{8}$ ft. and height 36 ft.

5. Find the volume of a rectangular solid 3 ft. $9\frac{1}{2}$ in. long, 2 ft. $7\frac{1}{4}$ in. wide, and 16 in. deep.

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. Write in our common numerals the number sixty million, two hundred seventy-seven thousand, eight hundred nine.

2. Write in our common numerals the number seventy-two and sixty-four thousandths.

Add the following:

3.	**4.**	**5.**	**6.**	**7.**
\$2.96	\$8.61	\$5.42	\$8.87	\$6.43
4.83	7.43	6.84	2.93	4.21
5.77	9.36	.76	8.40	8.36
6.09	2.08	1.20	1.20	5.60
.73	1.39	9.08	.67	3.57
4.86	2.57	6.77	.42	5.79

8. From the sum of \$72.86 and \$47.63 subtract \$107.59.

Perform the following operations:

9. $\frac{2}{3} + \frac{1}{2}$.
10. $\frac{3}{4} - \frac{3}{8}$.
11. $\frac{7}{8} + \frac{3}{4}$.
12. $\frac{7}{8} - \frac{1}{4}$.
13. $\frac{1}{2} - \frac{1}{3}$.
14. $\frac{3}{16} + \frac{1}{2}$.
15. $\frac{9}{16} - \frac{1}{2}$.
16. $\frac{1}{2}$ of $\frac{9}{16}$.
17. $\frac{7}{16} \div \frac{1}{2}$.
18. $\frac{9}{16} \div \frac{3}{4}$.
19. $1\frac{1}{2} + 2\frac{7}{8}$.
20. $3\frac{1}{3} + 2\frac{1}{2}$.
21. $6\frac{1}{4} - 3\frac{7}{8}$.
22. $7\frac{2}{3} - 2\frac{1}{2}$.
23. $2\frac{1}{2} \times 2\frac{1}{2}$.
24. 0.7×1.2.
25. 1.8×2.3.
26. 1.6×0.9.
27. 0.6×0.91.
28. $1.44 \div 1.2$.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. I have measured the length, width, and height of this room. How do I find the number of square feet in the walls and ceiling, including all openings?

2. I have measured the length and width of a room. How do I find the number of square yards of linoleum needed to cover a floor?

3. I know the dimensions of this room and the dimensions of all openings. How do I find the number of square yards of plastering needed for the walls and ceiling, allowing for the doors and windows?

4. How do you find the number of square feet of flooring needed to cover the floor of your schoolroom?

5. How do you find the number of square yards of plaster needed for this room?

6. If you know the cost of shingles by the bunch and the number of bunches necessary to cover a given area, how do you find the cost of shingles for a roof of given dimensions?

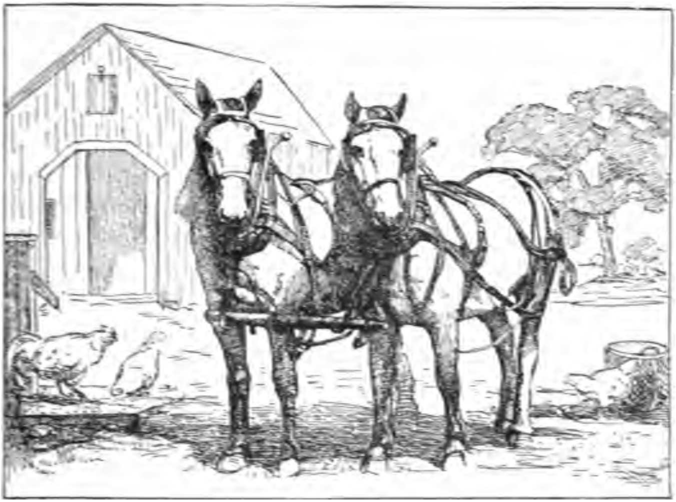
7. How do you find the cost of a sidewalk to be laid in front of the schoolhouse?

8. Given the length, breadth, and thickness of a rectangular solid, how do you find the volume?

9. Given the volume of a rectangular solid and two dimensions, how do you find the third dimension?

10. If you know the number of bricks needed for a cubic foot of wall, how do you estimate the number of thousand bricks needed to build a wall?

III. INDUSTRIAL PROBLEMS



GETTING HORSES READY FOR MARKET

1. To one lot of horses a dealer fed corn. In this case the average weight increased from 1436 lb. to 1674 lb. in 112 da. What was the average increase per day?

2. To another lot he fed three parts corn to one of oats, and the average weight increased from 1468 lb. to 1734 lb. in 112 da. What was the average increase per day?

3. To a third lot he fed half corn and half oats, the increase being from 1463 lb. to 1736 lb. in 112 da. What was the average increase per day in this case?

Examples like this, representing scientific experiments, not only give the pupils exercise in computation but also give them some idea of the way in which the government is trying to solve the problem of how to improve stock. In rural schools similar problems may be brought to class by the pupils.

THE PARCEL POST

1. The captain of our school baseball team finds that the catalogue from which he orders supplies tells the weight of the articles. The parcel post rates (1915) for shipping in four zones are:

Pounds	*Local Rate*	*First Zone*	*Second Zone*	*Third Zone*	*Fourth Zone*
1	$0.05	$0.05	$0.05	$0.06	$0.07
2	.06	.06	.06	.08	.11
3	.06	.07	.07	.10	.15
4	.07	.08	.08	.12	.19
5	.07	.09	.09	.14	.23
6	.08	.10	.10	.16	.27
7	.08	.11	.11	.18	.31
8	.09	.12	.12	.20	.35
9	.09	.13	.13	.22	.39
10	.10	.14	.14	.24	.43

The first zone is from 1 mi. to 50 mi., the second is from 50 mi. to 150 mi., the third is from 150 mi. to 300 mi., and the fourth is from 300 mi. to 600 mi. Our school is 75 mi. from the city where the captain buys the goods. He buys three bats at 45¢, the shipping weight of each being 50 oz. How much money does he have to send for the bats and postage?

Any weight between 1 lb. and 2 lb. is counted as 2 lb., and so on. The limit of weight in the first and second zones is 50 lb., in other zones 20 lb.

2. The captain wishes two catcher's mitts. They cost $2 each, and the shipping weight of each is 25 oz. How much money does he have to send for the mitts and postage?

3. Another team is 170 mi. from the city where it buys supplies. The captain of that team sends for two baseball masks. They cost $1.40 each, and the weight of each is 24 oz. How much money must he send for the masks and postage?

CHAPTER VI

HOW TO SOLVE PROBLEMS

173. Necessity for Using Short Methods. We have now learned how to perform all the common operations with numbers, but before we complete this part of arithmetic we should review the short methods of computing. If Ralph sells 8 doz. eggs at 25¢ a dozen, it would be absurd for him to take a pencil and some paper to find the amount received. He should calculate this "in his head" by using a short method.

ORAL EXERCISE

1. Multiply by 10: 27, 2.7, and 270.
2. State the short method of multiplying by 10; by 100.
3. Multiply by $\frac{10}{2}$: 20, 44, 88, and 440.
4. State the short method of multiplying by 5.
5. Multiply by $\frac{100}{4}$: 40, 88, and 160.
6. State the short method of multiplying by 25.
7. Multiply by $\frac{100}{3}$: 3, 9, 18, and 66.
8. State the short method of multiplying by $33\frac{1}{3}$.
9. Multiply by $\frac{1000}{8}$: 8, 16, 24, and 32.
10. State the short method of multiplying by 125.

Teachers should observe that Exs. 2, 4, 6, 8, and 10 depend respectively on Exs. 1, 3, 5, 7, and 9. It is desirable to give a few oral exercises applying this work at once to problems of local interest. The work is summarized formally on page 214.

174. Summary of Short Methods in Multiplication. There are several short methods of multiplication that are valuable, particularly because certain multipliers are so often used.

To multiply by 10, move the decimal point one place to the right, annexing a zero if necessary.

To multiply by 100, move the decimal point two places to the right, annexing zeros if necessary.

To multiply by 5, multiply by 10 and divide by 2.

To multiply by 25, multiply by 100 and divide by 4.

To multiply by 125, multiply by 1000 and divide by 8.

To multiply by $33\frac{1}{3}$, *multiply by 100 and divide by 3.*

To multiply by 9, multiply by 10 and subtract the multiplicand.

To multiply by 11, multiply by 10 and add the multiplicand.

WRITTEN EXERCISE

State the short method of multiplying by each of the following numbers and give the explanation:

1. 1000. **2.** 10,000. **3.** $12\frac{1}{2}$. **4.** $66\frac{2}{3}$. **5.** $16\frac{2}{3}$.

Multiply, in turn, by 10, by 100, by 5, by 25, and by 125:

6. 4832 7648 9176 7048 6112

7. 52.56 7.176 348.8 9.008 0.736

8. 27.28 8.416 245.6 7.024 0.824

9. 17.68 6.808 765.6 2.008 0.744

Multiply, in turn, by $33\frac{1}{3}$, *by 9, and by 11:*

10. 7074 5415 2211 4701 9036

11. 42.51 37.71 610.2 5.505 0.711

12. 81.36 61.26 504.3 7.263 0.504

13. 10.11 72.36 281.1 5.604 0.123

175. Short Methods in Division. There are certain divisors which are used so frequently in business that it is desirable to know the shortest methods of dividing by them. These include the numbers 5, 10, 25, 100, 125 (with 12.5 or $12\frac{1}{2}$), and $33\frac{1}{3}$.

ORAL EXERCISE

1. Divide by 10: $420, $5700, 330, $425, 375 lb., 6 mi.

2. State the short method of dividing by 10.

3. Since $\frac{1}{5}=\frac{2}{10}$, we may take $\frac{1}{5}$ of a dollar by taking how many tenths of it? State the short method of dividing by 5, or of taking $\frac{1}{5}$ of a number.

4. Divide by 5: $1110, $3240, 5280 ft., 4230 mi., 3221.

5. Divide by 100: $600, 7000, 4800, $560, $275, $48.

6. State the short method of dividing by 100.

7. State the short method of finding $\frac{1}{25}$ (or $\frac{4}{100}$) of a dollar; of dividing a number by 25.

8. Divide by 25: $2200, $1200, $3100, 4100, $32, $11.

9. State the short method of dividing by 125. ($\frac{1}{125}=\frac{8}{1000}$.)

10. Divide by 125: $11,000, 2000, $4000, $70, $600.

11. State the short method of dividing by $33\frac{1}{3}$. (Take $\frac{3}{100}$.)

12. Divide by $33\frac{1}{3}$: $200, 500, $7, $12, $30,000, $11,000.

13. In a certain year Delaware had 346.2 mi. of railroad, and the District of Columbia had $\frac{1}{10}$ as many miles. How many miles had the latter?

14. In the same year France had 26,600 mi. of railroad, and Korea had 0.001 as many. How many had Korea?

15. In the same year the United States had 193,300 mi. of railroad, and Ecuador had 0.001 as many. How many had Ecuador?

176. Summary of Short Methods in Division. *To divide by 10, move the decimal point one place to the left.*

To divide by 100, move the decimal point two places to the left, and so on for other powers of 10.

To divide by 5, multiply by 2 and divide by 10.

To divide by 25, multiply by 4 and divide by 100.

To divide by 125, multiply by 8 and divide by 1000.

To divide by $33\frac{1}{3}$, *multiply by 3 and divide by 100.*

WRITTEN EXERCISE

State the short method of dividing by each of the following numbers, and give the explanation:

1. 1000. **2.** 10,000. **3.** $12\frac{1}{2}$. **4.** $66\frac{2}{3}$. **5.** $16\frac{2}{3}$.

Divide by short methods:

6. $6484 \div 66\frac{2}{3}$.
7. $2937 \div 16\frac{2}{3}$.
8. $9237 \div 12\frac{1}{2}$.
9. $6889 \div 33\frac{1}{3}$.
10. $4936 \div 125$.
11. $7280 \div 66\frac{2}{3}$.
12. $4831 \div 16\frac{2}{3}$.
13. $7396 \div 12\frac{1}{2}$.
14. $4959 \div 33\frac{1}{3}$.
15. $9276 \div 66\frac{2}{3}$.
16. $67,400 \div 125$.
17. $9134 \div 16\frac{2}{3}$.
18. $8187 \div 12\frac{1}{2}$.
19. $8892 \div 33\frac{1}{3}$.
20. $39,888 \div 125$.

21. Divide by 25: 13,428, \$37,642, \$135,621, \$427,693.

22. Divide by 125: 17,200, \$68,300, \$276,400, \$317,500.

23. Divide by $33\frac{1}{3}$: 69,720, \$32,960, \$48,750, \$276,320.

24. At 25¢ a yard, how many yards of cloth can be bought for \$37.25? for \$42.25? for \$45.75?

25. At $33\frac{1}{3}$¢ a yard, how many yards of cloth can be bought for \$98? for \$66? for \$105?

26. Divide 3,742,697 by 25, using long division, and see how many seconds it takes. Do the same by the short method.

ORAL EXERCISE

1. To multiply by $\frac{1}{2}$ is the same as to divide by what number? To divide by $\frac{1}{2}$ is the same as to multiply by what number? Illustrate each answer on the blackboard.

2. Multiply by $\frac{1}{2}$: 48, 250, 380, 460, 666, 1110, 1234.

3. Divide by $\frac{1}{2}$: 70, 35, 60, 300, 450, 700, 1450, 8112.

4. To multiply by $\frac{1}{3}$ is the same as to divide by what number? To divide by $\frac{1}{3}$ is the same as to multiply by what number? Illustrate each answer on the blackboard.

5. Multiply by $\frac{1}{3}$: 36, 90, 63, 45, 75, 300, 123, 321.

6. Divide by $\frac{1}{3}$: 30, 70, 50, 25, 80, 400, 2000, 4211.

7. Multiply by $\frac{2}{3}$: 30, 60, 90, 33, 63, 300, 600, 1230.

8. Divide by $\frac{2}{3}$: 20, 12, 16, 10, 60, 100, 200, 500, 1000.

9. Multiply by $\frac{1}{4}$: 8, 16, 32, 48, 64, 84, 100, 200, 1000.

10. Divide by $\frac{1}{4}$: 9, 11, 21, 25, 60, 70, 91, 125, 250, 800.

11. Multiply by $\frac{3}{4}$: 8, 16, 20, 32, 40, 44, 48, 60, 80, 444.

12. Divide by $\frac{3}{4}$: 9, 12, 15, 21, 30, 60, 66, 90, 120, 300.

13. Since $\frac{1}{5} = \frac{2}{10}$, what short method do you see of multiplying by $\frac{1}{5}$? of dividing by $\frac{1}{5}$?

14. Multiply by $\frac{1}{5}$: 21, 31, 34, 123, 112, 321, 132, 125.

15. Divide by $\frac{1}{5}$: 22, 300, 34, 160, 444, 664, 486, 1010.

16. Since $\frac{2}{5} = 0.4$, state the short methods of multiplying and dividing by $\frac{2}{5}$. Do the same for $\frac{3}{5}$ and $\frac{4}{5}$.

17. Multiply by $\frac{5}{6}$: 30, 60, 66, 120, 600, 720, 840, 1200.

18. Because $0.12\frac{1}{2} = \frac{1}{8}$, what is the short method of multiplying by $0.12\frac{1}{2}$?

19. At $12\frac{1}{2}$¢ a yard, what will 16 yd. of lawn cost? What will 72 yd. cost at the same rate?

177. Problem Solving. We have now studied the fundamental processes with integral numbers, common fractions, decimal fractions, and denominate numbers, the short methods of working with numbers, and the measurement of simple figures. The remaining part of arithmetic relates chiefly to business problems, and a few directions on the solution of problems are desirable before we proceed. In considering a problem there are three matters which are especially important, namely:

1. *Work rapidly and accurately.* The business man demands this first of all. Rapidity not only means money because of the time saved but it usually means accuracy, for when a man works rapidly he keeps his mind on his work. To insure accuracy, *always check your work.* One of the best checks is that of estimating the answer in advance. If the answer is not about what was expected, there is probably a mistake.

2. *Work as the business man does.* The business man does not write his work in steps, with signs like ÷ and +; he works in columns, because it is quicker and he can more easily check his work. He also makes his work look neat, because he finds that slovenly work leads to inaccuracy.

3. *Analyze the problems.* This means that we are to state the reasons. The business man does not write out his analyses, but he thinks them. We are asked to give analyses both orally and in writing, so that we may learn to think as accurately as the business man thinks.

It should be remembered that the greatest defect in arithmetic is seen in the inability of the pupil to add, subtract, multiply, and divide, these being the processes most frequently employed in business. The defect can be overcome only by constant drill and by attention to rapidity and accuracy. The solution of every type of problem that may be met in after life, and particularly the solution of the technical problems of the shop, the bank, and the broker's office, is not to be expected in the school, but accuracy in the fundamental operations is always possible and should be attained.

178. Oral Analysis. Many problems are so easy that they may be solved without pencil and paper. Merchants solve their problems in this way, and we should solve our own problems in this way whenever it is possible. In analyzing a problem, speak briefly and accurately. Use your own language; do not feel that you must use the exact language of the book or of the teacher.

1. If a dealer pays $10 for 8 books of the same kind, what is the cost of each book?

No one would think of solving such a problem on paper. If asked to analyze (explain) it, we should say, "Since 8 books cost $10, 1 book costs $\frac{1}{8}$ of $10, or $1.25."

2. If a stationer buys 100 sheets of blotting paper for $3.50 and retails it at 5¢ a sheet, how much does he gain?

Since he sells 1 sheet for 5¢, he sells 100 sheets for 100 times 5¢, or $5. (Do not stop to say "500¢"; reduce this to $5 mentally.) He therefore gains the difference between $3.50 and $5, or $1.50.

3. A dealer pays $24 for 120 doz. pencils. How much does one pencil cost?

Since 120 doz. pencils cost $24, 1 doz. costs $\frac{1}{120}$ of $24, or 20¢. Therefore 1 pencil costs $\frac{1}{12}$ of 20¢, or $1\frac{2}{3}$¢.

4. If a dealer pays $3.60 for a dozen knives and sells them at 50¢ each, how much does he gain? His gain is what part of the cost?

Since he sells 12 knives at 50¢ each, he receives 12 times 50¢, or $6. He therefore gains $6 – $3.60 = $2.40. Then $2.40 is $\frac{2.40}{3.60} = \frac{24}{36} = \frac{2}{3}$ of the cost.

5. If you can buy baseballs at $5.50 a dozen and are allowed the dozen rate for $\frac{1}{2}$ doz., how much do you save in buying 6 baseballs by the half dozen instead of buying at 55¢ each?

Since 6 baseballs cost 6 × 55¢, or $3.30, if bought singly, and $\frac{1}{2}$ of $5.50, or $2.75, if bought by the half dozen, the saving is $3.30 – $2.75, or 55¢.

ORAL EXERCISE

Using short processes when possible, find the cost of the following:

1. 25 yd. of felt @ \$1.60; @ \$1.44; @ \$1.20.
2. 12½ yd. of denim @ 16¢; @ 20¢; @ 24¢.
3. 7½ yd. of dimity @ 16¢; @ 20¢; @ 24¢.
4. 12½ lb. of feathers @ 48¢; @ 56¢; @ 64¢.
5. 32 yd. of cambric @ 10¢; @ 12½¢; @ 11¢.
6. 8 yd. of cashmere @ 75¢; @ 80¢; @ 90¢.
7. 16 yd. of gingham @ 10¢; @ 12½¢; @ 11¢; @ 15¢.
8. 22 yd. of buckram @ 15¢; @ 11¢; @ 20¢; @ 22¢.
9. 20 yd. of crinoline @ 11¢; @ 10¢; @ 12½¢; @ 15¢.
10. 11 yd. of percaline @ 20¢; @ 22¢; @ 25¢; @ 21¢.
11. 10 pieces of beading @ 23¢; @ 24¢; @ 26¢; @ 30¢.
12. 25 yd. of eider down @ 60¢; @ 64¢; @ 68¢.
13. 12 yd. of chambray @ 25¢; @ 20¢; @ 30¢.
14. 22 yd. of nainsook @ 20¢; @ 25¢; @ 30¢; @ 21¢.
15. 12 yd. of linen lawn @ 75¢; @ 70¢; @ 80¢.
16. 20 whalebones @ 20¢; @ 22¢; @ 24¢; @ 30¢.
17. 7 gross of bone buttons @ 22¢; @ 21¢; @ 25¢.
18. 10 yd. of jute burlap @ 35¢; @ 37¢; @ 37½¢.
19. 3 doz. spools cotton @ 40¢; @ 41¢; @ 42¢; @ 43¢.
20. 7 yd. of torchon lace @ 10¢; @ 11¢; @ 12¢.
21. 4 doz. spools of basting cotton @ 22¢; @ 23¢; @ 25¢.
22. 11 yd. of cotton scrim @ 22¢; @ 23¢; @ 24¢; @ 25¢.
23. 30 yd. of cheesecloth @ 5¢; @ 6¢; @ 7¢; @ 8¢.
24. 40 yd. of chiffon @ 75¢; @ 80¢; @ 95¢; @ \$1.25.
25. 12 yd. of velvet @ \$2.50; @ \$3; @ \$4; @ \$5.

179. Comparison of Numbers. A comparison of the numbers in a problem will often lead to a simpler solution. For example:

1. If 6 tons of hay cost $72, what will 18 tons cost?

Since 18 is 3 times 6, the cost of 18 tons will be 3 × $72, or $216.

2. At $12 a ton, what will 17,500 lb. of hay cost?

Since 2000 lb. cost $12, the cost of 1 lb. is $\frac{1}{2000}$ of $12. Hence we have

$$\frac{17500 \times \$12}{2000} = \$105.$$

From Ex. 2 we see that, given the price per ton, to find the cost of a given number of pounds we may proceed as follows:

Point off three figures from the number of pounds, divide by 2, and multiply the price per ton by the quotient.

WRITTEN EXERCISE

1. If 2000 ft. of lumber cost $84, what will 8000 ft. cost?

2. If 3 T. of hay cost $36, what will 15 T. cost?

3. If 15 school desks cost $90, what will 90 desks cost?

Find the cost of the following at the given prices per ton:

4. 9500 lb. of hay @ $9.

5. 12,500 lb. of hay @ $12.

6. 7420 lb. of coal @ $4.

7. 8600 lb. of straw @ $3.

8. 18,115 lb. of coal @ $7.50.

9. 81,320 lb. of hay @ $12.50.

10. 92,400 lb. of hay @ $12.

11. 17,280 lb. of slack @ $1.25.

In some parts of the country such examples are common.

12. If a pound of solder is used to seal 60 cans of tomatoes, how many pounds of solder are needed for 1800 cans?

13. If $\frac{5}{8}$ A. produces 25 bu. of corn, how many bushels will $\frac{5}{16}$ A. produce at the same rate?

180. Written Analysis. In the written work we wish to find out two things: whether we can solve problems quickly and accurately, as they do in business; whether we clearly understand the solution. For this reason give both the analysis and the computation, as in the following example, unless the teacher directs omitting one. In the simpler problems there is only one step in the analysis, as in the following example:

A cubic foot of water weighs $62\frac{1}{2}$ lb. What is the weight of the water that will fill a cubic can 6 in. on an edge?

Analysis

6 in. = $\frac{1}{2}$ ft.

Since the can is $\frac{1}{2}$ ft. on an edge, the volume is $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of 1 cu. ft. = $\frac{1}{8}$ cu. ft.

Since the volume is $\frac{1}{8}$ cu. ft., the weight is $\frac{1}{8}$ of $62\frac{1}{2}$ lb. = $7\frac{13}{16}$ lb. = 7 lb. 13 oz.

Work

$$\boxed{\begin{array}{r} 8\overline{)62\frac{1}{2}} \\ 7\frac{6\frac{1}{4}}{8} \\ 7\frac{13}{16} \end{array}}$$

WRITTEN EXERCISE

1. At $37\frac{1}{2}$¢ a yard, how much will $15\frac{1}{2}$ yd. of cloth cost?

2. At 680 bu. to a car, how many freight cars will it take to carry 11,560 bu. of wheat?

3. At 40,000 shingles to a car, how many freight cars will it take to carry 760,000 shingles?

4. If the President of the United States receives a salary of $75,000 a year, how much does he receive a month?

5. If a bushel of wheat weighs 60 lb., how many bushels of wheat are there in a carload that weighs 21,240 lb.?

6. If a bushel of oats weighs 32 lb., how many bushels of oats are there in a carload that weighs 20,000 lb.?

Teachers should frequently advise the pupils to estimate a result in advance, and use such an estimate as a check on the work.

7. How many chests of tea, weighing 25 lb. each and costing 48¢ a pound, can be bought for $1500?

8. If 32 bricks will pave 1 sq. yd., how many bricks will be needed to pave a space 36 ft. by 63 ft.?

9. If a homing pigeon makes a flight of 1000 mi. in 2 da. 9¾ hr., what is the average speed per hour?

10. How many railway ties placed 2 ft. 6 in. from center to center will be required for a double line of road 13 mi. long?

11. One of the yachts in the international races made 30 mi. in 3 hr. 12 min. 30 sec. What was the average time per mile?

12. A cubic foot of pure cast gold weighs 1204 lb., and a cubic foot of granite weighs 172 lb. Gold is how many times as heavy as granite?

13. How many tons' pressure will a hurricane exert on the front of a city building 150 ft. high and 100 ft. wide, allowing 45 lb. pressure to the square foot?

14. How many minutes will it take a line of soldiers, the line being ½ mi. long, to pass a fixed point, if the soldiers take 75 steps a minute, and the steps average 2 ft. 3 in. in length?

15. In a certain arid region in the West 250 mi. of main irrigating ditches were put in at an average cost of $10,000 a mile, and 400 mi. of lateral ditches at an average cost of $1250 a mile. Required the total cost.

16. A man sold a load of hay for $17.76. The wagon and the hay together weighed 4160 lb. and the wagon was known to weigh 1200 lb. At what price per ton was the hay sold?

17. A fair breeze blows 880 ft. a minute, and a hurricane 7040 ft. a minute. How many feet does each blow in a second? How many miles does each blow in an hour? Answer to two decimal places only.

181. Problems in Profit and Loss. In business the merchant's chief problem relates to the profit on his goods.

For example, a merchant bought 800 yd. of linen lawn at 67½¢ a yard, and sold 726 yd. at 80¢ a yard, and the rest at a bargain sale at 65¢ a yard. Find his profit.

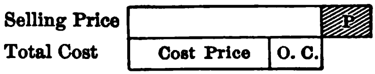
$0.67½	$726	$0.65	$580.80
800	.80	74	48.10
400	$580.80	2 60	$628.90
536		45 5	540.
$540		$48.10	$88.90

The written analysis is as follows:

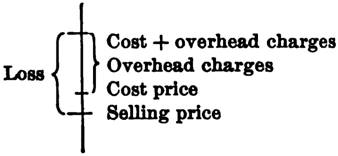
1. 800 × $0.67½ = $540, the cost.
2. 726 × $0.80 = $580.80, received at regular sale.
3. 800 yd. − 726 yd. = 74 yd., sold at bargain sale.
4. 74 × $0.65 = $48.10, received at bargain sale.
5. $580.80 + $48.10 = $628.90, total receipts.
6. $628.90 − $540 = $88.90, profit.

The merchant always has certain "overhead charges," such as rent, help, insurance, freight, and interest on the money invested. These must be counted in when we consider the total cost. It is not always necessary to mention these in the problems, but they are mentioned often enough to call attention to their importance.

Teachers sometimes find it advantageous to use diagrams to indicate the general conditions of a problem. For example, the accompanying diagram shows the cost price, overhead charges (O. C.), selling price, and profit (P.).



It is also possible to represent these numbers as we would represent temperatures on a thermometer. If we sell above the cost price plus overhead charges, we have a profit; if we sell below this point, we have a loss, as shown in the figure.



WRITTEN EXERCISE

1. A dealer buys 75 sewing machines at $38 each. He pays $56.25 freight on the machines and sells them at $45 apiece. His overhead charges are $1.20 per machine. Find his profit.

2. A merchant buys 840 yd. of muslin at 6¢ a yard and sells it at the rate of a dozen yards for a dollar. His overhead charges are taken as $\frac{3}{4}$¢ per yard. How much is his profit on the lot?

3. A merchant buys 12 gross of spools of silk at $5.76 a gross. He sells the spools at 60¢ a dozen. Not counting overhead charges, how much is his profit?

4. A dealer buys 275 lb. of twine at 26¢ a pound. He sells it at 35¢ a pound. His overhead charges are reckoned as $\frac{3}{8}$¢ a pound. How much is his profit per pound? on the whole lot?

5. If a dealer buys 960 yd. of scrim for $300 and sells three fourths of it at 35¢ a yard and the rest at 30¢ a yard, how much is his profit, not counting overhead charges?

6. A dealer buys 250 reams of paper at 1.22\frac{1}{2}$ a ream. He sells 7 reams, which were damaged, at 75¢ a ream, and the rest at $1.35 a ream. Find his profit, not counting overhead charges?

7. A dealer buys 640 yd. of damask linen at 87$\frac{1}{2}$¢ a yard. At what price per yard must he sell it to make a profit of $80 on the whole amount, not counting overhead charges?

8. If a dealer buys 1600 yd. of Victoria lawn for $300 and the overhead charges are $40, for how much per yard must he sell it to make a profit of 3$\frac{5}{8}$¢ a yard? Find his whole profit.

9. A dry goods dealer buys 720 yd. of percaline at 18¢ a yard, and the overhead charges are estimated as 1$\frac{5}{8}$¢ a yard. He sells 500 yd. at 25¢ a yard, 120 yd. at 23¢ a yard, and the rest at 15¢ a yard. How much is his profit?

LITTLE EXAMINATIONS

I. 1. $\frac{1}{8}$ of 488; 125 × 0.488.

2. $\frac{1}{3}$ of 1527; $33\frac{1}{3}$ × 1.527.

3. $\frac{1}{4}$ of 7264; 25 × 72.64 mi.

4. 6 × 5874; 5874 lb. ÷ $16\frac{2}{3}$.

5. 8 × 1295; 12.95 ft. ÷ $12\frac{1}{2}$.

6. 74.2 ft. ÷ $\frac{2}{3}$.

7. $\frac{2}{3}$ of $4\frac{3}{4}$ mi.

8. 27.3 ft. ÷ $\frac{1}{3}$.

9. $33\frac{1}{3}$ × 54.9 mi.

10. 125 × 62.48 ft.

II. 1. $\frac{1}{4}$ of 6012; 25 × 6.012 mi.

2. 9 × $3.78; 9 × $48.34.

3. 11 × $876; 11 × $72.06.

4. $\frac{1}{2}$ of 376; 50 × $3.76.

5. 3 × 429; 4.29 mi. ÷ $33\frac{1}{3}$.

6. 6.75 mi. ÷ 1.25.

7. $840 ÷ $1.33\frac{1}{3}$.

8. $2\frac{1}{2}$ × 448 yd.

9. 2 yd. 4 in. ÷ 10.

10. 7 ft. 6 in. ÷ 9.

III. *Estimate each result, write down the estimate, and then solve:*

1. If 32 bricks will pave 1 sq. yd., how many bricks will be needed to pave 175 sq. yd.?

2. If a cubic foot of granite weighs 172 lb., how much will a block 2 ft. 6 in. long, 2 ft. wide, and 13 in. thick weigh?

3. At $14.50 a ton, how much will $15\frac{1}{2}$ tons of hay cost?

4. A merchant buys some goods for $743.75, and sells them for $875. He pays $26.48 in freight and cartage, and reckons his overhead charges as 0.01 of the cost of goods, freight, and cartage. Find his profit.

5. A dealer buys 625 yd. of lawn at 23¢ a yard, and sells it at 30¢ a yard. He pays $6.35 freight, and reckons his overhead charges at $17.50. Find his profit.

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. The last chapter of a book is Chapter XLIX, and I have finished Chapter XVIII. How many more chapters are there?

2. Write the present year in Roman numerals.

3. Write in figures five hundred and three thousandths.

4. Write the tables of liquid measure and dry measure.

5. Write the lines in the tables of denominate numbers suggested by the numbers 3, 8, $16\frac{1}{2}$, 27, 5280, 1728.

6. If a square contains 144 sq. in., find the perimeter.

7. If a cube contains 1728 cu. in., find the area of each face.

8. In 1 gal. there are 231 cu. in. How many gallons of water will it take to fill a rectangular tank 7 ft. by 11 ft. by 3 ft.?

9. A pumping plant to furnish irrigation for 20 A. cost as follows: dam, $150; engine, $37.50; belt, $9; pump, $26; freight, $5; pipe, $36; sheet-iron house, $25. What was the average price per acre irrigated?

10. A rectangular field containing 16 A. is 64 rd. long. How many rods of fence will it take to inclose it?

Perform the following operations:

11. $\frac{1}{8} + \frac{3}{16}$.

12. $\frac{7}{8} - \frac{3}{16}$.

13. $1\frac{1}{2} + 2\frac{7}{8}$.

14. $3\frac{1}{8} - 1\frac{3}{4}$.

15. $\frac{1}{2} \times \frac{7}{8}$.

16. $\frac{3}{4} \times \frac{2}{3}$.

17. $\frac{3}{8} \div \frac{2}{3}$.

18. $\frac{2}{3} \div \frac{3}{8}$.

Find, in turn, the sum, difference, product, and quotient of the following pairs of numbers, in every case of division dividing the greater by the less:

19. 67,956, 9708.

20. 24,438, 4073.

21. 45,081, 5009.

22. 14.49, 20.7.

23. 273.6, 34.2.

24. 2.115, 0.423.

25. $\frac{7}{8}$, $\frac{1}{2}$.

26. $\frac{5}{8}$, $\frac{1}{4}$.

27. $\frac{3}{4}$, $\frac{3}{8}$.

28. To the difference between 4287.6 and 1939.8 add the sum of 428.6 and 9007.9.

Find the sums and also the differences:

29.		**30.**		**31.**		**32.**	
ft.	in.	ft.	in.	lb.	oz.	yd.	in.
29	2	30	1	16	4	38	14
9	7	18	9	8	7	19	29

Multiply, and also divide, the first number by the second:

33. 22 ft. 8 in.; 4.

34. 43 ft. 6 in.; 6.

35. 36 yd. 4 in.; 5.

36. 37 rd. $1\frac{1}{2}$ ft.; 4.

37. 12 lb. 8 oz.; 5.

38. 30 yd. 26 in.; 7.

Multiply, in turn, by 10, by 25, by 50, and by 75:

39. 44.

40. 56.

41. 72.

42. 96.

43. 144.

44. 240.

45. 4 ft. 8 in.

46. 16 yd. 24 in.

Multiply, in turn, by 9, and by 11:

47. 1728.

48. 5280.

49. 9280.

50. 3174.

51. 12,365.

52. 14,807.

53. 26,342.

54. 31,287.

Multiply, in turn, by 10, by 100, by 25, by 75, and by $33\frac{1}{3}$:

55. 120.

56. 360.

57. 252.

58. 480.

59. 1200.

60. 1224.

61. 4836.

62. 4848.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. If you know the cost of a certain number of things and wish to know the cost of one, how do you proceed?

2. If you know the cost of one thing and wish to find the cost of a certain number of things, how do you proceed?

3. If you know the cost of a fraction of anything and wish to know the cost of the whole, how do you proceed?

4. If you know the cost of a certain number of things and wish to know the cost of a certain other number of the same things, how do you proceed?

5. If you know the cost of a gallon of anything and wish to know the cost of a pint, how do you proceed?

6. If you know the price of cloth per yard, how do you find the cost of a certain number of inches?

7. If you know the price of a certain kind of cloth per yard, how do you find the cost of a certain number of yards plus a certain number of inches?

8. If you know the circumference of a wheel, how do you find how many times the wheel will turn in going a mile?

9. If you know how many times a wheel turns in going a mile, how do you find the circumference?

10. If you know the freight rate per hundred pounds, how do you find the freight charge on a certain number of tons plus a certain number of pounds?

11. If you have the product of several numbers to be divided by the product of several others, how may you perform the operations with least labor?

III. INDUSTRIAL PROBLEMS

INDUSTRY OF THE COOKING CLASS

1. If in putting up a certain kind of fruit we use 9 oz. of sugar to $2\frac{1}{4}$ lb. of fruit, the weight of the sugar is what fraction of the weight of the fruit?

2. If in putting up another kind of fruit we use 4 oz. of sugar to $1\frac{3}{4}$ lb. of fruit, the weight of the sugar is what fraction of the total weight of the fruit and sugar?

3. How much sugar must be used with 5 lb. of apples if the weight of the sugar is $\frac{1}{4}$ of the weight of the apples?

4. In Ex. 3 we add as many ounces of water as ounces of sugar. Allowing a pint to a pound, and 2 cups to a pint, how many cups of water do we add?

5. If we use 4 lb. of sugar to 8 lb. of raspberries, the weight of the sugar is what fraction of the total weight?

6. In putting up strawberries the class uses 12 qt. of berries to 6 qt. of sugar. The weight of sugar is what fraction of the weight of berries? of the total weight of berries and sugar?

7. A recipe calls for 4 lb. of fruit, 2 lb. of sugar, and 1 pt. of water. If we wish to use 15 lb. of fruit, how much sugar and how much water must we use, allowing 1 pt. of water to a pound?

8. What multiplier must you use in enlarging a recipe that calls for 3 lb. of fruit, when you wish to use 16 lb.? What multiplier must you use when you wish to use 25 lb.?

MAKING A LIVING

1. Mrs. Wiggs receives $1.25 a day for doing washing. How much does she make in a week of six working days if she spends 5¢ a day for car fare? How much does she make in 51 wk.?

2. Mr. Wiggs left her some life insurance which she invested so as to receive $45 a year. How much is this a month? If her month's rent is $9, what part of the rent will this pay?

3. John helps his mother by working before and after school. He receives $1.50 a week for delivering morning papers, and on Sunday mornings he earns on an average 85¢ more by selling extra papers. He averages 35¢ every evening by selling papers. If he also makes $1.10 a week by selling the *Saturday Evening Post*, what is his total weekly contribution to the family income?

4. A neighbor offers John's sister Jennie 10¢ an hour for wheeling the baby. If she works on the average 1½ hr. a day, missing only ten days, how much will she earn in a year?

5. Their brother Frank borrows $1.50 from John to pay for plowing and seed and $1.50 from Jennie to pay for a spade and hoe. If he raises $4.25 worth of cabbages, $3.25 worth of tomatoes, $6.15 worth of potatoes, and $7.75 worth of other vegetables, and then pays back the money he borrowed, how much is he able during the year to give his mother toward the family expenses?

6. They all think they can do better by keeping chickens. They buy six hens and a rooster for $5. Frank makes a coop out of a box that costs 15¢, and he spends 35¢ for nails, hinges, screws, and some nest eggs. What is the total investment?

7. Frank feeds each week ½ pk. of oats costing 40¢ a bushel, 7½ lb. of bran costing $1.30 per 100 lb., and the table scraps. If the hens lay on an average 15 eggs a week, and Frank sells the eggs at 36¢ a dozen, how much does he make in a week?

HOW TOM EARNED HIS SPENDING-MONEY

1. Tom's father offers him 35¢ a week for spending-money or a dozen hens with which to earn what he can. How much would 35¢ a week amount to in a year? If the hens lay 100 eggs apiece in a year and eggs average 28¢ a dozen, how much more would Tom receive by keeping hens?

2. Tom figured this out, but forgot to deduct the expense of feeding his hens. Suppose this to be $1 per year for each hen, and suppose that the hens averaged 125 eggs apiece, how much better would Tom's annual income be than 35¢ a week?

3. Tom chooses the hens. He finds that there are more table scraps than he had counted on, so that he needs each week only 1 pk. of corn costing 60¢ a bushel, $\frac{1}{2}$ pk. of oats costing 36¢ a bushel, and 5 lb. of bran costing $1.40 per 100 lb. What is the expense of the feed per year?

4. During the year the hens averaged $2\frac{1}{4}$ doz. eggs a week, and these he sold at an average price of 25¢ a dozen. What is his income per year? Deducting his expenses, does this leave him more than or less than 35¢ a week, and how much for the year?

5. Tom learns that by selling eggs for hatching purposes in the late winter he can get 75¢ for a setting of 15 eggs. If he sells 20 doz. at this price, how much does it add to his yearly income? What does this make his average income per week?

6. Dick keeps bees, getting 33 lb. of honey a year from each of four hives, and selling the honey for 18¢ a pound. Is he doing better or worse than Tom, and how much per week?

7. Tom decides to keep bees also. If he buys two hives of bees for $4.25 a hive, and needs in addition $1.80 worth of supplies, how many weeks will he have to save his income, as found in Ex. 5, in order to pay for the bees and supplies?

CHAPTER VII

PERCENTAGE AND ITS APPLICATIONS

182. The Need of knowing about Per Cents. Fred Roberts is captain of the baseball team of the sixth grade of our school. He finds that he can buy at the store a catcher's mitt for $2. The next day he sees that it has been marked down 10 per cent. He now begins to think that it is necessary that he should know what per cent means.

If any one in the class knows, he may tell the meaning, otherwise it may be taken up after § 183. This page is merely a kind of preparatory reading lesson.

Mrs. Roberts, Fred's mother, wishes to buy a suit of clothes for Fred. The price was $15, but the suit has been marked down 20 per cent. If she wishes to buy the suit, she must know what this means. Do you know what it means?

The teacher has to make a report of the number of pupils tardy or absent last week. She says in the report that 5 per cent of the pupils were tardy and 2 per cent were absent. Do you know what this means?

A man wishes to buy an automobile. He can buy a new one of the kind he likes for $1160, but one that is nearly new is offered for 25 per cent less than this price. Do you know how to find what the man would have to pay for this car?

It is not necessary that any pupil should answer these questions. What is of importance is that all the class should see that per cents are frequently met and that every one must know what they mean.

183. Per Cent. Another name for "hundredths" is *per cent.* For example, instead of saying "ten hundredths" we may say "ten per cent." The two expressions mean exactly the same.

If Fred Roberts finds that a catcher's mitt which is marked $2 can be bought for 10 per cent less than the marked price, this means that it can be bought for $\frac{10}{100}$, or $\frac{1}{10}$, less than $2; that is, it can be bought for $2 less $\frac{1}{10}$ of $2, or $2 – $0.20, or $1.80.

Such simple illustrations should be drawn from local cases, even before the subject of discount is taken up, as it will be a little later.

184. Symbol for Per Cent. There is a special symbol for per cent, %. Thus we write 20% for 20 per cent, and it means the same as $\frac{20}{100}$, or 0.20.

185. Hundredths written as Per Cents. Because hundredths and per cents are the same, any fraction with 100 for its denominator may be written in the form of per cent. Thus

$$\frac{3}{100} = 3\% \qquad \frac{1}{100} = 1\% \qquad \frac{21}{100} = 21\% \qquad \frac{75}{100} = 75\%$$

ORAL EXERCISE

Read the following as per cents:

1. $\frac{2}{100}$.
2. $\frac{4}{100}$.
3. $\frac{5}{100}$.
4. $\frac{7}{100}$.
5. $\frac{9}{100}$.
6. $\frac{11}{100}$.
7. $\frac{25}{100}$.
8. $\frac{32}{100}$.
9. $\frac{48}{100}$.
10. $\frac{62}{100}$.
11. $\frac{68}{100}$.
12. $\frac{72}{100}$.
13. $\frac{80}{100}$.
14. $\frac{90}{100}$.
15. $\frac{100}{100}$.

Read the following as hundredths:

16. 10%.
17. 25%.
18. 30%.
19. 42%.
20. 50%.
21. 72%.
22. 92%.
23. 99%.
24. 100%.
25. 200%.

26. Read $\frac{125}{100}$ as per cent, and 250% as hundredths.

27. Read as per cents: $\frac{12\frac{1}{2}}{100}$, $\frac{37\frac{1}{2}}{100}$, $\frac{\frac{1}{2}}{100}$, $\frac{33\frac{1}{3}}{100}$, $\frac{\frac{1}{3}}{100}$, and $\frac{66\frac{2}{3}}{100}$.

186. Halves and Fourths as Per Cents. Mr. Roberts says that he sold an old automobile for 50 % of what it cost him. What does this mean? He sold the car for how many hundredths of what it cost him? Express this fraction in lowest terms.

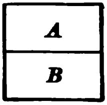
If a baseball team plays 32 games and loses only 25 % of them, how many games does it lose? The team loses how many hundredths of the games? Express this fraction in lowest terms.

From these two examples we see that

$$50\% = \tfrac{1}{2} \qquad 25\% = \tfrac{1}{4}$$

Other simple examples having local interest to the pupils should be given. The following oral exercise emphasizes the relations of $\frac{1}{2}$ and $\frac{1}{4}$ to per cents.

ORAL EXERCISE

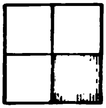


1. If this square represents a floor containing 100 sq. ft., how many square feet in B? Then B is how many hundredths of the square? What per cent of the square is B? Then A is what per cent of the square?

2. If asked to find 50 % of a number, you may find what fraction of the number instead?

Find 50 % of each of the following numbers:

3. 24. **4.** 32. **5.** 44. **6.** 50. **7.** 200.



8. If this square represents a box cover containing 100 sq. in., how many square inches are shaded? How many fourths are shaded? How many hundredths are shaded? What per cent is shaded?

9. If asked to find 25 % of a number, you may find what fraction of the number instead?

Find 25 % of each of the following numbers:

10. 24. **11.** 32. **12.** 88. **13.** 8. **14.** 160.

ORAL EXERCISE

1. Half of a square is what per cent of it?
2. A fourth of a square is what per cent of it?
3. An eighth of a square is what per cent of it?

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187. Important Per Cents. We have found that

$50\% = \frac{1}{2}$ $\qquad 25\% = \frac{1}{4}$ $\qquad 12\frac{1}{2}\% = \frac{1}{8}$

---

4. How much is 50% of 20? of 40? of 50? of 7?
5. How much is 25% of 32? of 60? of 120? of 5?
6. How much is $12\frac{1}{2}\%$ of 40? of 72? of 88? of 100?
7. How much is $12\frac{1}{2}\%$ of 32? of 16? of 160? of 320?
8. In your class, if 50% are boys, how many are boys?

WRITTEN EXERCISE

Find the values of the following:

1. 50% of 1632.
2. 25% of 2592.
3. 50% of 12,370.
4. 25% of $3416.80.
5. 50% of 978 bu.
6. 25% of 96 ft. 8 in.
7. $12\frac{1}{2}\%$ of $91.28.
8. $12\frac{1}{2}\%$ of $73.68.

9. A class at school contains 48 pupils, and 50% of them are girls. How many are girls? How many are boys?

10. A boy had 64 marbles and lost 25% of them. How many marbles did he lose and how many did he have left?

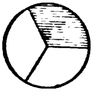
11. A hawk captured $12\frac{1}{2}\%$ of 152 chickens. How many were captured? How many were left?

12. Of 1408 sheep owned by a farmer $12\frac{1}{2}\%$ were sold. How many were sold? How many remained?

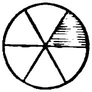
ORAL EXERCISE

1. If this circle is called 100, how much is the shaded part? how much is the unshaded part? If the circle is called 1, what per cent is the shaded part?

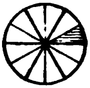
2. Read $0.33\frac{1}{3}$, using the words "per cent"; using the word "hundredths." How many thirds are there in 1? Then how many times is $33\frac{1}{3}\%$ contained in 1?



3. If this circle is called 100, the shaded part is what part of $33\frac{1}{3}$? How much is this? Then the shaded part is what per cent of the circle?



4. How much is $\frac{1}{2}$ of $16\frac{2}{3}\%$? The shaded part of this circle is what per cent of the whole circle? Then $\frac{1}{12}$ is what per cent?



188. Important Per Cents. We have found that

$$16\frac{2}{3}\% = \frac{1}{6} \qquad 33\frac{1}{3}\% = \frac{1}{3} \qquad 66\frac{2}{3}\% = \frac{2}{3}$$

5. How much is $33\frac{1}{3}\%$ of 90? of 120? of 45? of 60?

6. How much is $66\frac{2}{3}\%$ of 30? of 66? of 90? of 300?

7. How much is $16\frac{2}{3}\%$ of 36? of 24? of 66? of 72?

8. How much is $33\frac{1}{3}\%$ of \$6? of \$15? of \$30?

WRITTEN EXERCISE

Find the values of the following:

1. $33\frac{1}{3}\%$ of 891.
2. $66\frac{2}{3}\%$ of 711.
3. $16\frac{2}{3}\%$ of 504.
4. $16\frac{2}{3}\%$ of \$41.10.
5. $16\frac{2}{3}\%$ of \$159.72.
6. $66\frac{2}{3}\%$ of 747 rd.
7. $16\frac{2}{3}\%$ of 9774 bu.
8. $33\frac{1}{3}\%$ of 222 ft. 9 in.
9. $16\frac{2}{3}\%$ of 726 bu. 6 qt.
10. $66\frac{2}{3}\%$ of 141 lb. 15 oz.

189. Per Cents as Common Fractions. Since per cent means hundredths, we know that

$$62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}.$$

To express per cent as a common fraction, write the number indicating the per cent for the numerator and 100 for the denominator, and reduce this fraction to lowest terms.

For example, 50% means 0.50, or $\frac{50}{100}$, and this equals $\frac{1}{2}$.

It is often convenient to use one form, and often another. Thus, if we are multiplying by 13.5%, it is more convenient to think of the multiplier as 0.135; but if we are multiplying by $33\frac{1}{3}\%$, it is better to think of it as $\frac{1}{3}$ instead of $0.33\frac{1}{3}$.

We know that $\frac{2}{3}$ may be reduced to hundredths by multiplying each term by $33\frac{1}{3}$. We then have

$$\frac{2}{3} = \frac{66\frac{2}{3}}{100} = 66\frac{2}{3}\%.$$

To express a common fraction as per cent, reduce it to hundredths, and then write the numerator followed by the symbol for per cent.

We may also reduce $\frac{2}{3}$ to hundredths by dividing 2 by 3. We then have

$$\frac{2}{3} = 0.66\frac{2}{3} = 66\frac{2}{3}\%.$$

WRITTEN EXERCISE

Express as common fractions in lowest terms:

1. 32%.
2. 68%.
3. 24%.
4. $6\frac{1}{4}\%$.
5. 36%.
6. 125%.
7. 17%.
8. 375%.
9. $82\frac{1}{2}\%$.
10. $33.3\frac{1}{3}\%$.

Express the following as per cents:

11. $\frac{3}{8}$.
12. $\frac{5}{8}$.
13. $\frac{7}{8}$.
14. $\frac{1}{5}$.
15. $\frac{2}{5}$.
16. $\frac{3}{5}$.
17. $\frac{4}{5}$.
18. $\frac{1}{16}$.
19. $\frac{3}{16}$.
20. $\frac{15}{16}$.

190. Finding Per Cents. We have now found that certain important fractions like $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{2}{3}$ are easily expressed as per cents; that any fraction can be expressed as a per cent if we first reduce it to hundredths; and that per cents can easily be expressed as common fractions.

If we wish to find 50%, 25%, $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, $33\frac{1}{3}\%$, or $66\frac{2}{3}\%$ of a number, it is easier to use the equivalent common fractions, as in the following example:

If Ralph wishes to buy a camping suit that is marked $8, and finds that it is to be marked down $12\frac{1}{2}\%$, how much will the suit cost him after it is marked down?

Since $12\frac{1}{2}\% = \frac{1}{8}$, we simply take $\frac{1}{8}$ of $8, and this tells the amount the suit is to be marked down.

Then we have $\frac{1}{8}$ of \$8 = \$1;

and \$8 − \$1 = \$7, cost.

If we wish to find such a per cent as 15% of a number, it is better to use decimal fractions, as in the following example:

Kate wishes a new dress. The price of the dress she wants is $14, but she finds that to-morrow at a bargain sale this dress is to be marked down 15%. How much will it then cost her?

Because 15% = 0.15, we multiply $14 by 0.15. We then have

15% of \$14 = \$2.10;

and \$14 − \$2.10 = \$11.90, cost.

$14	$14
.15	2.10
70	$11.90, cost
14	
$2.10	

To find a per cent of a number, express the per cent as a decimal fraction and multiply the number by this decimal.

As stated above, in cases like 50% it is better to use a common fraction than a decimal. By so doing we can frequently solve a problem mentally that would otherwise require written work.

ORAL EXERCISE

1. If you have 50 words in a spelling test and are marked 90%, how many words did you spell correctly?

2. If you have 10 questions in arithmetic and are marked 80%, how many questions did you answer correctly?

3. If you have 12 questions in geography and are marked 75%, how many questions did you answer correctly?

4. If 60% of a class of 40 have not missed a day this year, how many have been present every day?

5. If a merchant pays 2¢ apiece for pencils and sells them at a profit of 50%, how much does he receive for each?

6. If a milk dealer pays 5¢ a quart for milk and sells it at a profit of 20%, how much does he receive per quart?

7. If a butcher buys meat at 18¢ a pound and sells it at a profit of $16\frac{2}{3}\%$, at what price does he sell it per pound?

WRITTEN EXERCISE

1. In the world's championship series in a certain year, the players' share was $135,000, of which the Giants received 40%. How much money did they receive?

2. In the same year the Athletics received 60% of the players' share, which was $135,000, as stated in Ex. 1. How much money did they receive?

3. The St. Louis club was at bat 4800 times in one season and the base hits were 25%. How many base hits did they make?

Find the values of the following:

4. 22% of 5475.
5. 6% of $2496.
6. 95% of $625.
7. 65% of $9200.
8. 87% of 4700 ft.
9. $66\frac{2}{3}\%$ of $4131.

10. In a school of 350 pupils 8% were in the sixth grade. How many pupils were there in that grade?

11. In a school of 250 pupils 12% are in the third grade and 14% in the first grade. How many pupils are there in each of these grades?

12. Of 320 pupils 5% have been tardy or absent this month. How many pupils have not been tardy or absent?

In such a case, use a common fraction for per cent.

13. If there should be an increase of $3\frac{1}{3}$% next year in a school of 330 pupils, how many pupils would there then be?

14. In a school of 246 pupils $16\frac{2}{3}$% were not in the school last year. How many new pupils are there this year?

In such a case, use a common fraction for per cent.

15. An ice wagon started out with a load of ice. One cake weighed 190 lb. at starting, but before it was delivered lost 4% by melting. How much did it then weigh?

16. The area of Illinois is 56,600 sq. mi. Of this 63% is underlain with coal beds. How many square miles of coal lands are there in the state?

17. A class in industrial arts bought 36 yd. of cloth, and when the cloth was washed the shrinkage was $\frac{1}{2}$%. How much did the cloth shrink? What was the length after shrinkage?

Since per cent means hundredths, $\frac{1}{2}$% means $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$. So we can find $\frac{1}{2}$% of a number by taking $\frac{1}{200}$ of it.

18. In the class in cooking it was found that some food costing $8.50 had deteriorated in value 5% by being kept for a week. What was the amount of the deterioration expressed in cents?

When an answer is a certain number of cents and a fraction, leave the fraction in the answer except in the case of bills, which are considered elsewhere.

19. One park has an area of 539 acres, and another has 69% as much. What is the area of the second park?

20. In a village of 975 inhabitants 20% are school children. How many school children are there?

21. A man bought a piece of land for $1500. He sold it so as to gain 8% on what it cost him. How much did he gain?

22. A farmer irrigated 240 acres of land last year and 15% more this year. How many acres did he irrigate this year?

23. In a school of 250 pupils, if there should be an increase of 4% next year, how many pupils would there then be?

24. In a school of 300 pupils $4\frac{2}{3}$% are absent to-day. How many pupils are absent?

25. If $14\frac{1}{2}$% of the pupils in a school of 400 pupils were not in the school last year, how many new pupils are there this year?

26. If 24% of the pupils in a school of 250 pupils are in the sixth and seventh grades, how many are in those grades?

27. If 25% of the pupils in a school of 372 pupils are in the first and second grades, how many are in those grades? in all the other grades?

28. If 55% of the pupils in a school of 280 pupils are girls, how many girls are there in the school? how many boys?

29. How much must be added to 250 ft. to increase it 17%? to increase it 25%? to increase it 100%?

30. How much must be added to 148 lb. to increase it 25%? to increase it 50%? to increase it 75%?

31. A column of mercury in a thermometer is 3.6 in. high. If it increases 10% in height, how high is it then?

32. A book is $\frac{4}{5}$ in. thick. Each of the two parts of the cover is $12\frac{1}{2}$% of this thickness. How thick is each part?

33. A rectangle $4\frac{1}{4}$ in. long and $3\frac{1}{2}$ in. wide is increased 40% in length and $33\frac{1}{3}$% in width. What are the length and width now?

34. A certain water tank holds 60,000 gal. A large pump at the waterworks pumps 5% of this amount in a single stroke. How many gallons does it pump in one stroke?

35. A certain building in San Francisco cost $1,240,000, and another cost 50% as much. How much did the second cost?

36. If the receipts at the St. Louis docks were 125 carloads on one day and 20% less the next day, what were the receipts on the second day?

37. The value of the property in Chicago at the time of the great fire of 1871 was estimated as $600,000,000, and 31% of it was destroyed. Find the value of the property destroyed.

38. If 500 tons of ore are shipped to Hoboken, and 15% of it is loaded on a steamer, how many tons are loaded on the steamer?

39. A large electric company has certain engines of 22,000 horse power, and certain others of 66% as much horse power. Find the horse power of the latter engines.

40. If you have 25 questions in arithmetic and are marked 80%, how many questions did you answer correctly?

41. If you have 30 words in a spelling test and are marked 90%, how many words did you spell correctly?

42. A man bought a bicycle for $40 and sold it at a loss of 35%. How much did he lose?

43. A dealer bought 83 rocking-chairs at $6 apiece and sold them at a profit of 15%. How much did he gain?

44. A class set out a tree on the school grounds. The tree was 12 ft. 6 in. high when it was set out, and the next year it had increased 25% in height. How high was it then?

191. Debtor and Creditor. A man who owes money to another is called a *debtor* (Dr.). The man to whom he owes money is called the *creditor* (Cr.). So if Rob has bought a baseball from Fred and owes him 50¢, Rob is the debtor and Fred is the creditor.

192. Debit and Credit. The amount owed on a bill is called the *debit*, and any payments that have been made form the *credit.*

193. Bill. A bill usually begins about as follows:

Mr. A. B. Jefferson, 17 Monroe St.

Bought of ROE & DOE, Grocers
No. 4 Washington Street

It may also read: "To Roe & Doe, Dr." This means that Mr. Jefferson is debtor to Roe & Doe for the goods mentioned in the bill.

The following is an example of a bill filled out, footed, and receipted:

Indianapolis, Ind.,..........[*Date*]..........

Mrs. John Doe, 47 Tenth Ave.

To RICHARD ROE & CO., Dr.
Dry Goods Dealers, No. 10 Grand Street

.....[*Year*]..... ......[*Date*]......	$\frac{1}{2}$ yd. silk	.75		38
"	$\frac{3}{4}$ " "	1.25		94
"	$7\frac{1}{2}$ " velour	3.50	26	25
			27	57

Received payment

Richard Roe & Co.

To fill out the bill by inserting in the two columns at the right the amount of each of the various items is called "making the extensions."

194. Receipt. A receipt need not necessarily be on a bill for goods or services. If Robert owes George $1.25 for a catcher's mask that he bought from him last week, and pays the money to-day, he may ask for a receipt so that the fact of the payment shall not be forgotten. Such a receipt would read as follows:

$1.25 Davenport, Ia., April 6, 1916

Received from .... Robert Montgomery ....

One and $\frac{25}{100}$ ~~~~~~~~~~~~~~~~~~~~ Dollars

for a catcher's mask.

George Sinclair

A receipt should contain the place and date, the name of the one paying the money (the *payer*), the amount written both in figures and in words, a statement of the reason for paying the money (the nature of the debt), and the signature of the one to whom the money is paid (the *payee*).

WRITTEN EXERCISE

Write receipts for the following, inserting dates and names:

1. $1.15 for a pair of skates.
2. $1.20 for three baseball bats.
3. $0.75, balance due on a football.
4. $1.35, balance due on a mandolin.
5. $0.80, balance due on a tennis racket.
6. $1.60, balance due on a tennis racket and a tennis net.
7. $6.50 for two days' labor on sidewalk and fence.
8. $0.72 for 26 papers at 2¢ each, and 4 papers at 5¢ each.

WRITTEN EXERCISE

Date, fill, foot, and receipt the following bills, inserting the name and address of some purchaser and of some dealer whom you know:

1. Bill for furniture: 6 chairs @ $3.00; 2 armchairs @ $6.25; 1 rocking-chair @ $5.50; 1 table @ $22.75.

An item like "6 chairs @ $3.00" means $3.00 each.

2. Bill for dry goods: $3\frac{1}{2}$ yd. satin @ $1.50; 2 yd. flannel @ 50¢; $1\frac{1}{2}$ yd. canvas @ 20¢; $1\frac{1}{2}$ yd. ribbon @ 23¢.

3. Bill for meat: $2\frac{1}{2}$ lb. steak @ 32¢; $3\frac{1}{2}$ lb. fish @ 20¢; $4\frac{3}{4}$ lb. roast beef @ 36¢; 8 lb. turkey @ 31¢.

4. Bill for dry goods: $17\frac{1}{2}$ yd. embroidery @ 10¢; 3 yd. lawn @ 20¢; $7\frac{1}{2}$ yd. percale @ 25¢; $\frac{1}{8}$ yd. satin @ $1.50.

5. Bill for crockery: $\frac{1}{2}$ doz. cups and saucers @ $6.00; 3 doz. plates @ $4.25; $1\frac{1}{2}$ doz. tumblers @ 90¢; 2 doz. individual butter plates @ 70¢; $\frac{1}{2}$ doz. bowls @ $2.30.

An item like "$\frac{1}{2}$ doz. cups @ $6.00" means $6.00 a dozen.

6. Bill for groceries: 10 lb. sugar @ 8¢; 2 qt. berries @ 11¢; 4 doz. eggs @ 26¢; $5\frac{1}{2}$ lb. butter @ 32¢; $1\frac{1}{2}$ lb. cheese @ 22¢; $\frac{1}{2}$ doz. bars soap @ 44¢.

7. Bill for dry goods: 7 yd. muslin @ 18¢; 6 handkerchiefs @ 30¢; 3 pairs kid gloves @ $1.50; 12 yd. sheeting @ $12\frac{1}{2}$¢; 16 yd. pongee @ 70¢.

8. Bill for stationery: 500 envelopes @ $3.50 per M (that is, per 1000); 1 box writing paper @ $2.25; $\frac{1}{2}$ doz. pencils @ 35¢; 3 penholders @ 5¢; 1 bottle ink @ 30¢.

9. Bill for groceries: 5 lb. coffee @ 32¢; 2 lb. tea @ 45¢; 8 lb. ham @ 15¢; 7 lb. lard @ 8¢; $3\frac{1}{2}$ lb. butter @ 33¢; 3 qt. currants @ 7¢; 2 doz. eggs @ 31¢; $1\frac{3}{4}$ lb. cheese @ 16¢; $\frac{1}{2}$ gal. sirup @ 65¢.

195. Per Cents off. If you have a catcher's glove or a tennis racket that you have used for a year, you may be glad to sell it at half price so as to get part of the money that it cost and have this much toward buying a new one. In that case you would sell it at a *discount* of 50%.

Sometimes merchants have bargain sales to dispose of odd pieces or goods that are going out of style. At such times they usually sell goods at a certain "per cent off."

For example, goods which are marked $2.50, but are sold at 10% off, sell at $2.50 – 10% of $2.50, or $2.25.

$2.50	$2.50
.10	0.25
$0.25	$2.25

Merchants who sell at wholesale, that is, in large quantities to dealers, sell at a certain per cent off the price as stated in their catalogues, or off the *list price*, as it is called.

196. Discount. A reduction made on a price or amount is called a *discount*. Discounts are usually reckoned at some rate per cent of the amount from which the discount is made. If you should offer to sell for $2 a tennis racket that cost $3, you would be offering to sell it at a discount of $\frac{1}{3}$, or a discount of $33\frac{1}{3}$%.

197. Rate of Discount. The per cent of discount is called the *rate of discount*. The amount of a price or bill after the discount has been made is called the *net price* or the *net amount*.

What is the discount and the selling price, or net price, when the list price of a shipment of furniture is $245 and the rate of discount is 12%?

12% of $245 = $29.40, discount.
$245 – $29.40 = $215.60, selling price.

$245 list price
0.12 rate of discount
$29.40 discount

$245.
29.40
$215.60 selling price

198. Order for Goods. The following is a model order for goods:

WOOD & ROBERTS
Booksellers
Madison, Wis., Feb. 15, 1918

Messrs. Ginn and Company, Publishers
2301 Prairie Ave., Chicago

Dear Sirs: Please send at once, by express,
75 Wentworth-Smith Complete Arithmetics, Pt. I,
60 " " " Arithmetics, Pt. II.

Yours truly

WOOD & ROBERTS

199. Model Invoice. The following is a form of the invoice that Ginn and Company would send in reply:

Feb. 16, 1918

Messrs. Wood & Roberts
Madison, Wis.

Bought of GINN AND COMPANY
Educational Publishers
2301 PRAIRIE AVENUE, CHICAGO

Terms: *Cash*

75	W.-S. Complete Arithmetics, Pt. I	$.40	$30.00	
60	" " Arithmetics, Pt. II	.40	24.00	
			$54.00	
		Less 1/6	9.00	
				$45.00

It is customary to state per cents like $16\frac{2}{3}\%$, $12\frac{1}{2}\%$, 20%, and 25% in the common-fraction form, as above. In this case, $16\frac{2}{3}\%$ is stated as $\frac{1}{6}$. If the discount had been $12\frac{1}{2}\%$, it would have been stated as $\frac{1}{8}$.

ORAL EXERCISE

1. How much is $200 less $50? $100 less 25% of itself?

2. If from 1 we take $\frac{1}{4}$, how much is left? If from 1 we take 25%, how much is left?

3. How much is 1 less $\frac{1}{5}$? $\frac{100}{100}-\frac{20}{100}$? $1-0.20$? $1-20\%$?

4. If you buy a $1 book for 10% less than the list price, how much does it cost you?

Find the price that must be paid for goods marked as stated in each of these examples, and discounted at the rate given:

5. $2, 10%.
6. $6, 20%.
7. $5, 20%.
8. $4, 25%.
9. $50, 30%.
10. $60, $33\frac{1}{3}$%.
11. $1.60, 10%.
12. $100, 30%.
13. $6, $33\frac{1}{3}$%.
14. $9, $33\frac{1}{3}$%.
15. $10, 75%.
16. $1.50, 10%.

WRITTEN EXERCISE

Find the price that must be paid for goods marked as stated in each of these examples, and discounted at the rate given:

1. $635, 6%.
2. $350, 15%.
3. $275, 15%.
4. $425, 35%.
5. $21.25, 8%.
6. $45.50, 20%.
7. $375.25, 4%.
8. $175.50, 20%.
9. $225.75, 20%.

10. How much will a set of Scott's works, marked $14, cost at 20% discount? Check by taking 80% of $14.

11. If you can buy a $7 suit of clothes at 15% discount, how much will you save on the marked price?

12. At a bargain sale an $8 chair was sold at 30% discount. For how much did it sell?

13. If the list price of your books, pencils, and paper is $2.50 a year, how much will they cost at 12% discount?

Find the discount on goods of which the list price and rate of discount in each case are as follows:

14. $430, 15%.
15. $281, 20%.
16. $425, 10%.
17. $1235, 17%.
18. $2350, 14%.
19. $43.26, 33⅓%.
20. $42.64, 25%.
21. $6336, 6¼%.
22. $34.54, 16⅔%.

Find the selling price in each case, given the list price and rate of discount as follows:

23. $275, 20%.
24. $575, 5%.
25. $462, 3%.
26. $650, 12½%.
27. $425, 12%.
28. $320, 14%.
29. $12.75, 20%.
30. $6202, 14%.
31. $52.84, 16⅔%.
32. $26.32, 50%.
33. $4235, 22%.
34. $14.43, 33⅓%.

35. A $17.50 set of china is marked down 15%. What is the selling price?

36. If you buy $1.80 worth of fireworks at 15% discount, how much do they cost?

37. A book dealer buys a 65¢ arithmetic at 22% discount. How much does it cost him?

38. A $31 overcoat is marked "25% off" at a bargain sale. What is the selling price?

39. If you buy a $1.20 sled at 10% discount, how much does it cost? Suppose the discount is 20%; 25%.

40. How much will 2 dozen blackboard erasers cost at 45¢ a dozen, discount 10%? discount 20%?

41. A man paid $150 for a carriage, and after using it for a year sold it at a discount of 16⅔%. What did he receive for it?

42. A man bought a $900 automobile at 15% discount. After using it for a season he sold it for 15% less than he paid for it. How much did he receive for it?

ORAL EXERCISE

1. What is \$2.50 with 10% added? with 10% subtracted?

2. What is \$1200 with 20% added? with 25% subtracted?

3. A dealer buys some books for \$100 and sells them at a profit of 20%. How much does he gain?

4. A dealer buys some cloth for \$50 and sells it at a profit of 20%. How much does he gain? What is the selling price?

5. A grocer bought \$15 worth of berries, but left them in the sun so long that he had to sell them at a loss of 10%. How much did he lose?

200. Profit and Loss. The per cent of profit or loss is reckoned on the *cost* of the goods.

In practical business it is essential that the cost should be fully considered, including freight charges, storage, share of rent and sale charges, and all of what the business man calls "overhead charges." The principle is the same as in the simpler problems adapted to work in the school.

201. Illustrative Problems. 1. A man buys goods for \$40 and sells them at a profit of 20%. What is the selling price?

$$\begin{array}{l} \$40 \\ \underline{\ \ 0.20} \\ \$8 \ \ \text{profit} \end{array} \qquad\qquad \begin{array}{l} \$40 \\ \underline{\ \ \ \ 8} \\ \$48 \ \text{selling price.} \end{array}$$

2. A man buys goods for \$60 and sells them for \$40. He loses what per cent?

$$\begin{array}{l} \$60 \\ \underline{\ \ 40} \\ \$20 \ \text{loss} \end{array} \qquad\qquad \begin{array}{r} 6\cancel{0})\underline{2\cancel{0}\ \ \ \ } \\ 0.33\tfrac{1}{3} \end{array}$$

Since \$20 is lost on \$60, we wish to find what per cent \$20 is of \$60. Consequently we have the product (20) and one factor (60) given, to find the other factor.

$$\$20 \div \$60 = 0.33\tfrac{1}{3}.$$

Therefore the loss is $33\frac{1}{3}$%.

ORAL EXERCISE

1. A hat dealer buys hats at $48 a dozen and sells them at $5 each. What is his per cent of profit?

2. A grocer buys starch at 50¢ a box and sells it at 10% profit. What is the selling price?

3. A dealer buys stove polish at 3¢ a cake and sells it at 4¢. What is his per cent of profit?

4. A grocer buys matches at 48¢ a dozen boxes and sells them at 54¢. What is his per cent of profit?

5. A druggist buys some soap at the rate of $2.40 a dozen cakes and sells it at 25¢ a cake. Find his per cent of profit.

6. Some soap, bought at $1.50 a dozen cakes, is damaged by dampness and is sold at 10¢ a cake. Find the per cent of loss.

WRITTEN EXERCISE

1. A grocer pays $12 a gross for boxes of starch. He sells the starch at 10¢ a box. What is his per cent of profit?

2. He buys washing powder in 1-pound packages at $5.88 a gross and sells it at 5¢ a package. What is his per cent of profit?

3. Some of this washing powder was damaged and had to be sold at 4¢ a package. What was the per cent of loss on each?

4. A dealer bought 150 bags of salt, 56 lb. to the bag, for $63. He sold $\frac{2}{3}$ of the salt at the rate of 1¢ a pound, and the rest at the rate of 2 lb. for a cent. Did he gain or lose on the transaction, and what per cent?

5. Macaroni in packages costs a grocer $15 a gross. He sells it at 12¢ a package. What per cent does he gain?

6. Gelatin in packages costs a grocer $11.52 a gross. At what rate per package must he sell it to gain 25%?

ORAL EXERCISE

1. How much is $20 plus 10% of itself? plus 1% of itself?

2. How much is $3 plus 30% of itself? plus 6% of itself?

3. How much is $15 plus 1% of itself? plus 33⅓% of itself?

4. If you bought $10 worth of eggs for a dealer, receiving 5% for your work, how much would you receive?

5. If you bought $2 worth of eggs for a dealer and he paid you 10% of the cost, how much would you receive?

202. Commission. The pay received by one person for buying or selling for another is often reckoned at a certain per cent of the price, and is called *commission*.

For example, if a man buys $1000 worth of oats for another man, at 5% commission, he would receive 5% of $1000, or $50.

WRITTEN EXERCISE

1. Required the commission on an $850 sale, at 2½%.

2. Required the commission on a $1200 sale, at 5½%.

3. Required the commission on a $2000 purchase, at 4½%.

4. A commission merchant sells for a factory 7850 lb. of cheese at 13½¢ a pound, charging 5% commission. What sum should he remit to the factory?

5. A man sells at an auction $292 worth of property for another man and receives 10% commission. How much does he get? How much is left for the other man?

6. A dealer in fruit in a city sends a man to the fruit region to buy for him. He pays him 2% commission. The buyer purchases $2500 worth in one place, $3200 worth in another, and $4100 worth in another. What is his total commission?

ORAL EXERCISE

1. If you sold $2 worth of papers on a commission of 20%, how much would you make?

2. At 30%, how much is the commission on a $120 sale?

3. At 10% commission, how much does a collection agent receive for collecting a debt of $95.70?

4. An agent sells 100 school desks at $3.75 each and receives a commission of 10%. How much does he receive?

WRITTEN EXERCISE

1. How much is an architect's commission on a house costing $9472, at 5%?

2. How much is an auctioneer's commission for selling $358 worth of furniture, at 6%?

3. A real-estate dealer sells 168 acres of land at $40 an acre. How much is his commission at 2½%?

4. An agent sells our school $65 worth of maps, and his firm pays him 7½% commission. How much is this?

5. A farmer sends some butter and eggs to the city, and they are sold for $72.50. He is charged 8% by his agent. What is the net amount received by the farmer?

6. A real-estate dealer rents 21 city apartments for the owner, at $1500 each, and charges a commission of 2½%. How much is the commission?

7. A salesman's sales in a department store in a city amounted during the six days of one week to $257.70, $103.10, $175.02, $132.30, $302.25, $50.63. He receives a commission of 2½% on his sales. How much did he receive that week?

ORAL EXERCISE

1. If a man borrows money of another, how does he pay him for the loan, besides returning the money?

2. If you put money in a savings bank and leave it 6 mo. or 1 yr., will they pay you back only what you put in?

3. If any one where you live wished to borrow $100 for a year, what interest would he have to pay?

203. Interest. The money paid for the use of money is called *interest.* For example, if I borrow $50 for a year and pay $3 for the use of it, the $3 is interest.

204. Principal. The sum of money on which interest is charged is called the *principal.*

205. Reckoning Interest. Interest is ordinarily reckoned as a certain per cent of the principal. If the interest is 6% of the principal, and the principal is $200, the interest for 1 yr. is 6% of $200, or $12.

206. Rate of Interest. The per cent of interest is called the *rate.* That is, if the interest is 6% on the principal the rate is 6%.

WRITTEN EXERCISE

1. What is the interest on $375 for 1 yr. at 6%? at 4%?

2. What is the interest on $950 for 1 yr. at 5%? at 3%?

3. What is the interest on $235 for 1 yr. at 4%? at 5%?

4. What is the interest on $750 for 1 yr. at 6%? at 5%?

5. If you had $27 in a bank that paid 3% a year, how much interest would you receive for a year?

6. How much are the interest and principal together on $475 at the end of 1 yr., the rate of interest being 6%?

ORAL EXERCISE

Find the interest on the following sums for 1 yr.:

1. $100 at 4%.
2. $300 at 6%.
3. $60 at 3%.
4. $500 at 5%.
5. $900 at 3%.
6. $50 at 4%.
7. $400 at 6%.
8. $1200 at 3%.
9. $70 at 5%.
10. $1000 at 4%.
11. $3000 at 4%.
12. $90 at 6%.

Find the interest on the following sums for 6 mo., first finding the interest for 1 yr., and then dividing by 2:

13. $100 at 6%.
14. $200 at 5%.
15. $40 at 6%.
16. $400 at 3%.
17. $250 at 4%.
18. $60 at 4%.
19. $600 at 5%.
20. $400 at 5%.
21. $70 at 3%.
22. $800 at 4%.
23. $1000 at 3%.
24. $80 at 5%.

WRITTEN EXERCISE

Find the interest on the following sums for 1 yr.:

1. $652 at 4%.
2. $375 at 5%.
3. $27 at $3\frac{1}{2}$%.
4. $325 at 3%.
5. $$433\frac{1}{3}$ at 3%.
6. $42 at $4\frac{1}{2}$%.
7. $275 at 4%.
8. $450.25 at 4%.
9. $75 at $2\frac{1}{2}$%.
10. $725 at 6%.
11. $275.20 at 5%.
12. $84 at $4\frac{1}{2}$%.
13. $535 at 5%.
14. $625.20 at 5%.
15. $72 at $3\frac{1}{3}$%.

Find the interest on the following sums for 6 mo.:

16. $82 at 5%.
17. $175 at 4%.
18. $350 at $2\frac{1}{2}$%.
19. $75 at 6%.
20. $325 at 5%.
21. $675 at 3%.
22. $45 at 4%.
23. $450 at 6%.
24. $450 at $4\frac{1}{2}$%.
25. $35 at 2%.
26. $395 at 3%.
27. $864 at 3%.
28. $475 at 6%.
29. $750 at 5%.
30. $890 at 6%.

207. Finding Interest. We have found the interest on money, at various rates, for one year, and also for half a year. We shall now find interest for other periods of time.

What is the interest on $250 for 10 mo. at 6%?

The interest for 1 yr. is 6% of $250, and for 10 mo. it is $\frac{10}{12}$, or $\frac{5}{6}$, of this sum. We therefore have

$$\frac{5}{6}\text{ of }6\%\text{ of }\$250=\$12.50.$$

In general, use cancellation. Thus, in this example we have

$$\frac{10}{12}\text{ of }6\%\text{ of }\$250=\frac{\cancel{10}\times\cancel{6}\times\overset{25}{\$\cancel{250}}}{\underset{2}{\cancel{12}}\times\underset{\cancel{10}}{\cancel{100}}}=\$\frac{25}{2}=\$12.50.$$

In this particular case we see that $\frac{5}{6}$ of 6% is 5 × 1%, or 5%, and so we simply have 5% of $250 = 5 × $2.50 = $12.50. Advantage of such short methods should always be taken. In finding interest, consider 30 da. as a month.

WRITTEN EXERCISE

1. If the interest on a certain sum is $93.84 for 1 yr., how much is it for 3 mo.? for 6 mo.? for 9 mo.?

2. If the interest on a certain sum is $36.48 for 1 yr., how much is it for 3 mo.? for 6 mo.? for 9 mo.?

3. What is the interest on $436, at 6%, for 1 yr.? for 6 mo.? for 4 mo.? for 8 mo.? for 10 mo.?

4. What is the interest on $360, at 5%, for 1 yr.? for 6 mo.? for 10 mo.? for 1 mo.? for 9 mo.?

5. What is the interest on $540, at 4%, for 1 yr.? for 1 mo.? for $\frac{1}{2}$ mo.? for 5 da.? for 1 da.?

6. What is the interest on $175.50, at 4%, for 1 yr.? for 6 mo.? for 4 mo.? for 3 mo.? for 1 mo.? for 10 da.?

7. What is the interest on $1550, at 6%, for 1 yr.? for 6 mo.? for 3 mo.? for 9 mo.? for $1\frac{1}{2}$ mo.?

ORAL EXERCISE

Find the interest on the following:

1. $100, at 5%, for 1 yr.
2. $300, at 6%, for 1 yr.
3. $500, at 4%, for 1 yr.
4. $600, at 5%, for 1 yr.
5. $800, at 5%, for 1 yr.
6. 100, at 6%, for 6 mo.
7. $600, at 3%, for 6 mo.
8. $800, at 5%, for 6 mo.

208. Interest for Years and Months. We have seen that to find the interest for 2 yr. 6 mo., we find the interest for 1 yr., and multiply by $2\frac{1}{2}$. That is,

Find the interest for 1 year, and multiply by the number of years.

Thus, to find the interest on $125 for 2 yr. 3 mo. at 5% we proceed as follows:

2 yr. 3 mo. = $2\frac{1}{4}$ yr.
5% of $125 = $6.25, interest for 1 yr.
$2\frac{1}{4}$ × $6.25 = $14.06, interest for 2 yr. 3 mo.

$$\begin{array}{r}
\$125\\
.05\\
\hline
\$6.25\\
2\frac{1}{4}\\
\hline
1\ 56\frac{1}{4}\\
12\ 50\\
\hline
\$14.06
\end{array}$$

In all such cases cancellation will be found very convenient. Here, for example, since 2 yr. 3 mo. = $2\frac{1}{4}$ yr., we have

$$2\tfrac{1}{4} \times 5\% \text{ of } \$125 = \frac{9 \times 5 \times \$\overset{5}{\cancel{125}}}{4 \times \underset{4}{\cancel{100}}} = \$\tfrac{225}{16} = \$14.06.$$

WRITTEN EXERCISE

Find the interest on the following:

1. $250, at 6%, for 6 mo.
2. $300, at $5\frac{1}{2}$%, for 8 mo.
3. $8200, at 5%, for $2\frac{1}{2}$ yr.
4. $625, at 4%, for $3\frac{1}{2}$ yr.
5. $35, at 6%, for $1\frac{1}{2}$ yr.
6. $45.50, at 4%, for 2 yr. 3 mo.
7. $750, at 5%, for 2 yr. 4 mo.
8. $75.50, at 6%, for 2 yr. 6 mo.
9. $225, at 6%, for 3 yr. 6 mo.
10. $4250, at $4\frac{1}{2}$%, for 1 yr. 6 mo.

Find the interest on $250, at 6%, for:

11. 3 yr. 6 mo.
12. 2 yr. 4 mo.
13. 1 yr. 9 mo.
14. 3 yr. 3 mo.
15. 4 yr. 1 mo.
16. 3 yr. 7 mo.

Find the interest on $375, for 3 yr. 4 mo., at:

17. 6%.
18. 5%.
19. 4%.
20. 3%.
21. 2%.

Find the interest for 2 yr. 7 mo., at 6%, on:

22. $175.
23. $325.
24. $475.
25. $685.
26. $750.

27. A man borrows $575, agreeing to pay 6% interest. He pays the principal and interest in 1 yr. 8 mo. How much money does he pay?

28. A man buys 7 head of cattle at $85 a head, and agrees to pay for them in 8 mo., with interest at 6%. How much does he pay in all, principal and interest?

29. Mr. St. John hires Mr. Sinclair to paint his house, agreeing to pay him $65 for the work. Mr. Sinclair agrees to wait 3 mo. for his money if Mr. St. John will pay interest at the rate of 5%. How much is due at the end of the 3 mo.?

30. Higgins & Son are dealers in dry goods. They need $1250 with which to buy holiday goods. They borrow it Nov. 1 and agree to pay it back on Feb. 1 with interest at 5%. How much do they owe on Feb. 1?

31. McDowell and Perkins buy $725 worth of goods from a manufacturer, agreeing to pay for the goods in 5 mo., with interest at 6%. How much do they owe at the end of the 5 mo.?

32. On March 1, Mr. Roberts buys a secondhand automobile from Mr. Grant, agreeing to pay $475 for it on July 1, with interest at 6%. How much interest is due on July 1? What is the total amount due?

WRITTEN REVIEW

Write as common fractions, and also as per cents:

1. 0.1.
2. 0.12.
3. 0.62.
4. $0.62\frac{1}{2}$.
5. $0.03\frac{1}{3}$.
6. 0.33.
7. 0.2.
8. 0.02.
9. $0.16\frac{2}{3}$.
10. 0.75.

11. A man had $1640 in the bank and drew out 10% of his money. How much did he draw out? How much was left?

12. If Fred made 40 runs in this season's games, and Frank made 10% more, how many runs did Frank make?

13. Jack has won 35 games at tennis and has lost 20% less than this number. How many games has he lost?

14. A man bought a bicycle for $30 and sold it at a loss of 25%. How much did he lose?

15. A man bought 75 cows at $65 apiece and sold them at a profit of 18%. How much did he gain?

16. A dealer buys 45 sheep at $7.50 each and sells them at a profit of 10%. How much does he gain on them all?

17. A man bought an automobile for $750, and after using it two seasons sold it for 60% less than it cost. What was the selling price?

18. If there are 350 pupils in a school and 48% are boys, what decimal fraction of the pupils are boys? How many boys are there? How many girls are there?

19. If a city has a population of 26,200, and $22\frac{1}{2}$% are pupils in school, what decimal fraction of the population is in school? How many are in school?

20. A merchant has $16,000 invested in his business. The first year he made a profit of $9\frac{1}{2}$%, the second year $14\frac{1}{2}$%, and the third year 18%. How much did he make in the three years?

ORAL REVIEW

1. From these columns show that 20% = $\frac{1}{5}$, 40% = $\frac{2}{5}$, 60% = $\frac{3}{5}$, and 80% = $\frac{4}{5}$.

2. Show that 20% is $\frac{1}{2}$ of 40%, $\frac{1}{3}$ of 60%, $\frac{1}{4}$ of 80%, and $\frac{1}{5}$ of 100%.

3. Show that 40% is $\frac{2}{3}$ of 60%, $\frac{1}{2}$ of 80%, and $\frac{2}{5}$ of 100%.

4. Show that 60% is $1\frac{1}{2}$ times 40%, 3 times 20%, $\frac{3}{4}$ of 80%, and $\frac{3}{5}$ of 100%.

				20%
			20%	20%
		20%	20%	20%
	20%	20%	20%	20%
20%	20%	20%	20%	20%
20%	40%	60%	80%	100%

These columns should be written on the board. Columns of other per cents, such as $12\frac{1}{2}$%, $16\frac{2}{3}$%, 25%, and $33\frac{1}{3}$%, are also useful for rapid drill work.

5. A foot is what fraction of a yard? A foot is what per cent of a yard?

6. A pint is what fraction of a gallon? A pint is what per cent of a gallon?

7. Five cents is what fraction of a dollar? Five cents is what per cent of a dollar?

8. If you wish to find $33\frac{1}{3}$% of a number, what fractional part of the number do you wish to find?

9. If a man spends one fifth of his income for rent, what per cent does he spend for this purpose?

10. What per cent of the perimeter of a square is one side of the square? What per cent are two sides? three sides?

11. If there are 30 pupils in a class and 50% are boys, what fraction are boys? How many boys are there?

12. Find the interest on $500 for 1 yr. at 6%; at 5%; at 4%.

13. Find the interest on $800 for 2 yr. at 5%; at 4%; at 3%.

WRITTEN REVIEW

Write as common fractions, and also as per cents:

1. 0.01.
2. 0.37.
3. $0.37\frac{1}{2}$.
4. $0.06\frac{1}{4}$.
5. 0.31.
6. $0.31\frac{1}{4}$.
7. $0.33\frac{1}{3}$.
8. 0.66.
9. $0.66\frac{2}{3}$.
10. 0.87.
11. $0.87\frac{1}{2}$.
12. 0.16.
13. $0.07\frac{1}{2}$.
14. $0.08\frac{1}{3}$.
15. $0.03\frac{3}{4}$.

16. From a barrel of flour weighing 196 lb., 75% has been sold. How many pounds are left?

17. A man agreed to build a stone wall 225 ft. long. He has already built $33\frac{1}{3}\%$ of it. How many feet has he built, and how many feet has he still to build?

18. A trolley car has a run of 64 mi. a day. When it has run 75% of this distance, how many miles has it still to run?

Find the values of the following:

19. 25% of 168.
20. 75% of 184.
21. 20% of 135.
22. 30% of 140.
23. 5% of 34.
24. 5% of 48.
25. 6% of 250.
26. 6% of 320.
27. 0.25 × 484.
28. 25% of 536.
29. 0.75 × 368.
30. 75% of 460.

31. How much will $62\frac{1}{2}\%$ of a cubic foot of steel weigh if 1 cu. ft. of steel weighs 490 lb.?

32. If a steel car when full carries 96,000 lb. of coal, how much does it carry when it carries $87\frac{1}{2}\%$ of its capacity?

33. If out of 320 problems in arithmetic you solve 95% of them without errors, how many do you solve correctly?

34. If a locomotive weighing 118 tons can exert a pull equal to $22\frac{1}{2}\%$ of its weight, how many tons of pull can it exert?

35. A ball team has played 48 games this season and has won $62\frac{1}{2}\%$ of them. How many games has it lost?

ORAL REVIEW

State the values of the following:

1. $\frac{1}{2}$ of 48 0.5 of 48 0.50 of 48 50% of 48

2. $\frac{1}{4}$ of 36 0.25 of 36 25% of 36 25% of 80

3. $\frac{3}{4}$ of 20 0.75 of 20 75% of 20 75% of 12

4. $\frac{1}{4}$ of 40 25% of 40 75% of 40 75% of 80

With respect to the figures given below, answer the following:

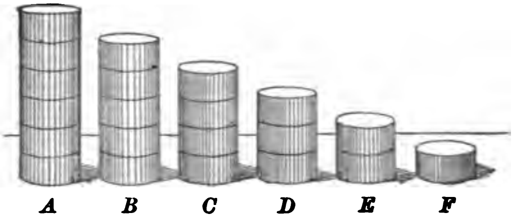
5. *F* is what part as large as *E*? what per cent as large?

6. *F* is what part as large as *D*? what per cent as large?

7. *F* is what part as large as *C*? what per cent as large?

8. *F* is what part as large as *B*? what per cent as large?

9. *F* is what part as large as *A*? what per cent as large?



In each of the following pairs tell what per cent the first is of the second:

10. *E, C.*
11. *E, A.*
12. *E, D.*
13. *E, B.*
14. *D, C.*
15. *C, B.*
16. *D, A.*
17. *D, B.*
18. *B, A.*
19. *C, A.*

20. How much is the commission, at 2%, on a sale amounting to $500? on a sale amounting to $5000?

21. How much is the interest on $1000 for 2 yr. at 4%? at 5%? at 6%? at 3%?

WRITTEN REVIEW

1. The diameter of a circle is about $31\frac{4}{5}\%$ as long as the circumference. What is the diameter of a circle whose circumference is 30 ft.?

2. A meter is $9\frac{13}{36}\%$ longer than a yard. How many inches are there in a meter?

3. A knot, used in measuring distances at sea, is $15\frac{1}{5}\%$ longer than a mile. How many feet are there in a knot?

Find the values of the following:

4. 40% of 160.
5. 60% of 180.
6. 70% of 150.
7. 80% of 120.
8. 4% of 120.
9. 4% of 230.
10. 3% of 125.
11. 3% of 235.
12. $\frac{1}{5}$ of 1275.
13. 20% of 95.
14. $\frac{3}{4}$ of 724.
15. 75% of 724.

16. A man makes a profit of $12\frac{1}{2}\%$ on a sale of property which cost him $1600. How much was the profit?

Find the values of the following:

17. 50% of $348.
18. 25% of $464.
19. $12\frac{1}{2}\%$ of $498.
20. $6\frac{1}{4}\%$ of $3376.
21. $37\frac{1}{2}\%$ of $49.76.
22. $62\frac{1}{2}\%$ of $91.92.
23. $87\frac{1}{2}\%$ of $42.48.
24. $3\frac{1}{8}\%$ of $739.20.
25. $66\frac{2}{3}\%$ of $372.60.
26. $83\frac{1}{3}\%$ of $42.36.
27. 20% of $926.45.
28. 40% of $732.25.
29. 60% of $821.72.
30. $82\frac{1}{2}\%$ of $27,500.
31. 35% of $17,500.
32. 63% of $26,200.

33. Find $1\frac{1}{4}$ times 480; 1.25 times 480; 125% of 480.

34. Find $1\frac{3}{8}$ times 960; $1.37\frac{1}{2}$ times 960; $137\frac{1}{2}\%$ of 960.

WRITTEN REVIEW

Find the values of the following:

1. 50% of 167.
2. 20% of 241.
3. 90% of 340.
4. 2% of $426.
5. 7% of $341.
6. 8% of $468.
7. $\frac{3}{4}$ of 236.
8. 75% of 236.
9. 0.75×236.

Find the net prices of goods billed as follows, with discounts as stated:

10. $450, 20%.
11. $375, 15%.
12. $480, 25%.
13. $675, 8%.
14. $840, $12\frac{1}{2}$%.
15. $925, 28%.
16. $960, $16\frac{2}{3}$%.
17. $1760, $12\frac{1}{2}$%.
18. $1575, $33\frac{1}{3}$%.

19. A typewriter was marked $80 but was sold at 18% discount. What was the net price?

20. An automobile was marked $950 but was sold at 12% discount. What was the net price?

21. A merchant bought some goods for $1687.50, less 20%. What was the net price?

22. On which will the net amount after deducting the commission be greater, a sale of $1750 less 2% commission or a sale of $1780 less 3% commission? How much greater?

23. A clerk receives $6 a week and 3% commission on goods sold. If he sells $500 worth of goods in a week, what is his income for the week?

24. How much is the interest on $1800 at 6% for 1 yr.? for 2 yr.? for $\frac{1}{4}$ yr.? for $1\frac{1}{4}$ yr.? for 2 yr. 3 mo.?

25. A dealer bought a bill of goods amounting to $2750. By paying promptly he was allowed a discount of 6% on this bill. How much was the discount?

LITTLE EXAMINATIONS

I. 1. 75% of 9.6.
2. $12\frac{1}{2}\%$ of $1.76.
3. $33\frac{1}{3}\%$ of $2.46.
4. $66\frac{2}{3}\%$ of $9.75.
5. $16\frac{2}{3}\%$ of $8.46.
6. $2\frac{1}{2}\%$ of $17.50.
7. 104% of $64\frac{1}{2}$ ft.
8. 3% of $4876.
9. 1.7% of $275.
10. 512% of $290.

II. 1. 25% of $75.
2. 75% of $25.
3. $12\frac{1}{2}\%$ of $37.50.
4. $37\frac{1}{2}\%$ of $12.50.
5. $16\frac{2}{3}\%$ of $18.60.
6. $3\frac{3}{4}\%$ of 2 ft. 3 in.
7. $2\frac{7}{8}\%$ of 4 yd. 3 in.
8. 8% of $75.50.
9. 1.6% of $4.80.
10. 275% of $500.

III. 1. $12\frac{1}{2}\%$ of $16.40.
2. $37\frac{1}{2}\%$ of $17.60.
3. $62\frac{1}{2}\%$ of $33.60.
4. $87\frac{1}{2}\%$ of $42.40.
5. $66\frac{2}{3}\%$ of $48.90.
6. $2\frac{1}{2}\%$ of $75.
7. $3\frac{1}{8}\%$ of $1.20.
8. 7% of $24.40.
9. $2.3\frac{1}{3}\%$ of $66.
10. 250% of 75 ft.

IV. 1. How much is $175 less 8% of itself?
2. How much is $250 increased by 15% of itself?
3. Make out a bill of six items and receipt it.
4. How much is the commission at 8% on a sale of goods amounting to $2750? at $8\frac{1}{2}\%$? at $12\frac{1}{2}\%$?
5. Find the interest on $2450 for 2 yr. 6 mo. at 5%.

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. Write 1921 and 1927 in Roman numerals, and LXIX and LXXXVIII in ordinary numerals.

Find the sum, difference, product, and quotient of each of the following pairs of numbers, dividing the greater by the less:

2. 6531, 2177.
3. 18,846, 3141.
4. 29,547, 4221.
5. 30,195, 6039.
6. 56,080, 7010.
7. 28,404, 7101.
8. 31,104, 3456.
9. 24,258, 4043.
10. 69,993, 7777.

Find the sum, difference, product, and both quotients of each of the following pairs of fractions:

By "both quotients" is meant that we should divide $\frac{2}{3}$ by $\frac{3}{4}$ and $\frac{3}{4}$ by $\frac{2}{3}$.

11. $\frac{2}{3}$, $\frac{3}{4}$.
12. $\frac{1}{3}$, $\frac{3}{8}$.
13. $\frac{2}{3}$, $\frac{5}{8}$.
14. $\frac{2}{3}$, $\frac{1}{8}$.
15. $\frac{3}{4}$, $\frac{1}{8}$.
16. $\frac{3}{4}$, $\frac{5}{8}$.
17. $\frac{1}{2}$, $\frac{3}{16}$.
18. $\frac{1}{3}$, $\frac{5}{16}$.

Find the values of the following:

19. 7% of 48.
20. 9% of 75.
21. 6% of 97.
22. 9% of 7.4.
23. 8% of 6.8.
24. $2\frac{1}{2}$% of 38.
25. $3\frac{1}{3}$% of 69.
26. $1\frac{1}{4}$% of 64.
27. $6\frac{1}{4}$% of 7.2.
28. $3\frac{1}{3}$% of 7.2.
29. 50% of 9 ft. 8 in.
30. $33\frac{1}{3}$% of 10 ft. 9 in.
31. 25% of 8 ft. 6 in.
32. 75% of 3 lb. 4 oz.
33. $37\frac{1}{2}$% of 9 lb. 8 oz.

ORAL EXERCISE

Find 125% of the following:

1. 36.
2. 64.
3. 80 ft.
4. $160.
5. 28.
6. 32.
7. 72 ft.
8. $360.
9. 80.
10. 88.
11. 60 ft.
12. $2.40.

Find 150% of the following:

13. 40.
14. 30.
15. 60 in.
16. $220.
17. 70.
18. 25.
19. 90 yd.
20. $150.
21. 38.
22. 52.
23. 62 rd.
24. $1.20.

25. If a man sells $2400 worth of goods at 25% profit, what is the selling price?

26. If a manufacturer spent $50,000 in advertising and increased this 50%, how much did he then spend?

27. Find $\frac{3}{4}$ of 16; of $160; of 1600 lb.; of 1.6 A.

28. Find $\frac{3}{8}$ of 24; of $240; of 2400 lb.; of 2.4 A.

Find $133\frac{1}{3}$% of the following:

29. 36.
30. 48.
31. 60 A.
32. $150.
33. 51.
34. 57.
35. 90 min.
36. $2.70.

Find $116\frac{2}{3}$% of the following:

37. 36.
38. 30 yd.
39. 54.
40. 84 rd.
41. 48.
42. $360.
43. 72.
44. $2.40.

45. Goods costing $2400 are sold at $16\frac{2}{3}\%$ above cost. What is the selling price?

46. Goods costing $2700 are damaged to the extent of $33\frac{1}{3}\%$. How much is the loss?

47. Multiply 240 by 100; by 50; by 25.

WRITTEN EXERCISE

Add the following:

1.	2.	3.	4.
$247.60	$382.68	$129.36	$287.62
329.48	592.39	427.96	469.83
272.56	78.62	380.09	48.79
752.40	400.00	870.64	602.76
670.52	32.48	572.04	85.55
727.44	162.96	619.91	328.76

Subtract the following:

5.	6.	7.	8.
$2763.01	$8101.23	$9124.51	$8170.01
1987.62	6472.85	7235.62	5296.83

Multiply as indicated:

9. $\frac{2}{3} \times 468$.
10. $\frac{3}{4} \times 564$.
11. $\frac{7}{8} \times 336$.
12. $2\frac{1}{2} \times 5.28$.
13. $3\frac{3}{4} \times 5.08$.
14. $4\frac{5}{8} \times 7.36$.
15. 294×326.
16. 34.7×289.
17. 42.9×57.6.
18. 5.38×27.9.
19. 7.29×8.26.
20. 6.87×0.79.
21. 0.7×17.09.
22. 0.6×0.072.
23. 2.8×0.009.
24. 7.3×6.007.
25. 5.8×72.01.
26. $6.6\frac{2}{3} \times 8.46$.

Divide, carrying the quotients to the nearest hundredth:

27. $4.5 \div 7.2$.
28. $9.8 \div 0.09$.
29. $0.7 \div 3.4$.
30. $84 \div 67$.
31. $1.42 \div 3.7$.
32. $1.56 \div 0.7$.
33. $28.1 \div 0.06$.
34. $597 \div 0.32$.
35. $8 \div 72.6$.

Perform the operations indicated:

36. $4\frac{3}{8} + 9\frac{5}{16}$.
37. $5\frac{2}{3} \div 3\frac{7}{8}$.
38. $1.7\frac{2}{3} \times 3.1\frac{1}{2}$.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. If you have a given per cent, how do you express it as a common fraction?

2. If you are to find a certain fractional part of a number, how do you express this as a per cent of the number?

3. If you have a certain rate per cent expressed with the per cent sign, how do you express it as a decimal?

4. If you have a decimal fraction, how do you express the same number with a per cent sign?

5. If you know what a given per cent of a number is, how do you find the number?

6. A man's salary two years ago was increased a certain per cent. Last year it was decreased the same per cent. Is his salary at present greater than or less than it was two years ago?

7. A merchant marks his goods a certain per cent above cost. Owing to hard times he sells them at the same per cent below the marked price. Does he sell them for more than or less than cost?

8. If you know the amount paid for a certain property, and the rate of commission, how do you find the commission?

9. If you know the net amount left after the commission has been deducted, and the price at which the property was sold, how do you find the rate of commission?

10. If you know the time that a sum of money is on interest, the rate, and the sum of money, how do you find the interest?

11. If you know what a dealer paid for some goods, and his selling price, how do you find the per cent of profit or loss?

The boys in class should be asked how to find a baseball player's fielding average, batting average, and so on.

III. INDUSTRIAL PROBLEMS

SHALL IT BE A CAR OR A HORSE?

1. A physician finds that it costs him $4.50 a week to keep his horse at a livery stable. He pays $2.50 twice in every three months for shoeing his horse. Veterinary attention averages $4 a year. What is the total expense of a horse and buggy per year?

2. The horse and buggy cost $275. At 6% interest, how much must be added to the above to obtain the total outlay for a year?

3. Deterioration is to be considered. Figuring that in 15 yr. he could sell the horse and buggy for $50, how much must the physician add to his annual outgo because of deterioration?

4. He decides to see whether it might not be better to buy an automobile. He finds that he can get a serviceable one for $765. At 6%, what would be his annual loss in interest on his money?

5. He finds that every 3 yr. he can, by paying $250 additional, exchange his old car for a new one. How does the deterioration per year compare with that on the horse and buggy (Ex. 3)?

6. Allowing $12.50 a month, what will the tires cost a year?

7. If 15 gal. of gasoline costing 20¢ a gallon are used a week, besides 80¢ worth of lubricating oil, and the garage costs $3 a week, what will this amount to in a year?

8. The automobile will save him 3 hr. a day. If he counts the time at 75¢ an hour, how much will this amount to in 361 da.?

9. He thinks it would be worth $200 a year in comfort to have a car. Now consider the whole question. What are the total expenses of the horse and buggy? What are the total expenses of the car, less the comfort and the saving in time? Which is the better for him to have?

THRIFT IN THE HOME

1. Mrs. Brown finds that she can save at least 50¢ a week by going to the market and buying in person. By doing this, how much a year, at least, will she save?

2. Frank gets interested in the idea of helping to cut down the high cost of living. He says he will raise vegetables. Allowing $1.25 for plowing, $3.25 for fertilizer, $1.30 for seed, and 75¢ a week for 22 wk., which wages his mother insists that he shall have, what is the cost of the season's vegetables?

3. Mrs. Brown used to spend $54 for these same vegetables at the store. What was the saving in having their own garden?

4. Since Frank's mother no longer buys over the telephone, she decides to cut off this expense. The telephone costs $2.25 a month. How much does she save in a year by not having one?

5. By buying at a bargain sale an $18 suit at 15% off, three $1.50 shirts at 10% off, a dozen 25-cent handkerchiefs at $16\frac{2}{3}\%$ off, a pair of $4.75 shoes at 25% off, and a hat and some ties amounting to $5.35 at 20% off, how much does Frank save in all?

6. How much have Frank and his mother together saved on the purchases and telephone in Exs. 1–5, as stated above?

7. They find that they can invest this money at 5% interest. What will it amount to in 10 yr. at simple interest?

8. A year after the investment mentioned in Ex. 7 was made they invested an equal amount again at 5% interest, and they did this every year until the end of the 10 yr. How much did they have at the end of that time, including the interest?

9. Frank will then be old enough to go to college. His mother says that he will need $1500 if he can earn a free scholarship. Will they have money enough saved to send Frank to college?

IV. EXAMINATION REVIEWS

ORAL EXAMINATION

1. State rapidly the following sums:

87	46	89	67	49	85	74
$\underline{95}$	$\underline{28}$	$\underline{98}$	$\underline{85}$	$\underline{86}$	$\underline{49}$	$\underline{38}$

2. Make change from $10, the purchases amounting to:

$6.75 $4.80 $6.92 $3.78 $5.69 $8.17

3. State the results of the following:

3×34 6×33 $294 \div 14$ $198 \div 18$
7×61 8×32 $330 \div 15$ $264 \div 12$

4. Express the following as common fractions:

20% 50% 90% $12\frac{1}{2}\%$ $33\frac{1}{3}\%$

5. State the results of the following:

$12\frac{1}{2}\%$ of 88 $37\frac{1}{2}\%$ of 88 $16\frac{2}{3}\%$ of 24 $66\frac{2}{3}\%$ of 24

6. How many rods in a mile? square rods in an acre?

7. How many days from March 7 to April 7?

8. If $16\frac{2}{3}\%$ of the pupils in a school of 180 pupils were not in school last year, how many new pupils are there this year?

9. A rectangle 6 in. by 8 in. was increased 50% in length and $33\frac{1}{3}\%$ in width. What is the area after the increase?

10. In a school of 220 pupils, if there should be an increase of 5% next year, how many pupils would there then be?

Arranged as they are on the cumulative plan and with exactly ten questions in each, these examinations furnish a series of excellent reviews and at the same time allow for easy rating on the scale of ten or a hundred.

WRITTEN EXAMINATION

1. Add the following:

$4\frac{7}{8} + 1\frac{2}{3}$ $6\frac{5}{8} + 7\frac{3}{4}$ $9\frac{3}{4} + 5\frac{2}{3}$ $16\frac{7}{12} + 15\frac{3}{4}$

2. Subtract the following:

$4\frac{1}{24} - 1\frac{9}{16}$ $9\frac{1}{48} - 3\frac{7}{8}$ $8\frac{5}{24} - 7\frac{1}{4}$ $18\frac{11}{12} - 12\frac{7}{8}$

3. Perform the operations indicated:

$5\frac{3}{4} \times 2\frac{2}{3}$ $7\frac{5}{8} \times 8\frac{3}{4}$ $6\frac{3}{4} \times 7\frac{1}{2}$ $28 \times 75\frac{3}{4}$
$2\frac{1}{2} \div 1\frac{1}{4}$ $13\frac{3}{4} \div 2\frac{3}{4}$ $\frac{9}{16} \div \frac{3}{8}$ $15\frac{1}{4} \div 18\frac{3}{8}$

4. Copy and complete the following:

$2\frac{1}{2}$ mi. = (?) rd. 49 ft. = (?) in.
$82\frac{1}{2}$ ft. = (?) rd. $22\frac{1}{2}$ sq. ft. = (?) sq. yd.
4800 lb. = (?) T. 384 cu. ft. = (?) cd.

5. Find the values of the following:

25% of 844 0.75 × 368 90% of 480
75% of 724 $\frac{3}{4}$ of 5040 69% of 539

6. Find the net prices of goods marked and discounted as follows:

\$850, 10% \$240, $12\frac{1}{2}$% \$450, $66\frac{2}{3}$%
\$900, 25% \$360, $16\frac{2}{3}$% \$270, $33\frac{1}{3}$%

7. Increase each of the following by the per cent indicated:

\$120, 40% \$640, $37\frac{1}{2}$% \$1296, 25%
\$696, $16\frac{2}{3}$% \$1250, 10% \$18.75, $33\frac{1}{3}$%

8. Find the number of which

10 is 10% 12 is 6% 75 is 75%

9. Compare 25% of 50% of \$1000 with 50% of 25% of \$1000, and each with $12\frac{1}{2}$% of 50% of \$2000.

10. The number 85 is 15% less than what number?

WRITTEN EXAMINATION

1. Write 1921 in three different ways.

2. A man has two pieces of land: one of 240 A., which he values at $125 per acre; and one of 65 A., which he values at $80 per acre. What is the average value of all the land per acre?

3. A state road 471 mi. long cost $2,325,798. What was the average cost per mile?

4. A farmer has 160 A. of land worth $150 per acre, 10 cows worth $65 each, 30 T. of hay worth $9.50 per ton, 800 bu. of corn in the crib worth 56¢ per bushel, $125 worth of poultry, farm machinery worth $450, an automobile worth $750, 20 hogs worth $7.50 each, and other property worth $1125. Make an inventory of his property.

5. A careless farmer failed to test his seed corn, and only four grains out of five that were planted grew. The farmer's yield on 40 A. was 1800 bu. If the corn was worth 48¢ per bushel, how much did he lose by his neglect?

6. If corn shrinks 10% in weight when carried over to the next summer, what is the loss on 80 A. of corn yielding 45 bu. (56 lb. each) per acre, the price remaining 50¢ a bushel?

7. Draw a plan of a village lot 200 ft. long and 100 ft. wide, using the scale $\frac{1}{200}$.

8. Draw a plan for a living room 18′ × 24′, with a bedroom 10′ × 12′ adjoining, using the scale $\frac{1}{48}$.

9. In a school of 492 pupils $16\frac{2}{3}$% were not there last year. How many of the present number were in the school last year?

10. Using short methods, find the cost of the following bill of goods: 14 pr. gloves @ $1.25, $12\frac{1}{2}$ yd. cloth @ 64¢, 16 rugs @ $12.50, and 4 doz. spools thread @ 15¢ per dozen.

WRITTEN EXAMINATION

1. I bought a hat for $2.40, which was 20% below the marked price. Find the marked price.

2. A farmer had 5½ A. that yielded 12 T. of ensilage to the acre. He put this into a silo, and fed 12 cows an average of 30 lb. a day from November 1 to June 1. About how many tons did he have left in the silo on June 1?

3. By selection of the seed the yield from 80 A. of corn was increased 20% over 45 bu. per acre. Find the total yield.

4. A new road was built along one side of this farm at a cost of $3600 per mile. If the corn is worth 50¢ a bushel, the yield will pay for what part of a mile of this road?

5. A poultry raiser has a lot 10 rd. square which he wishes to fence into four equal square lots, placing the fence posts 1 rd. apart. How many posts will he need? Draw a diagram.

6. A man paid $1200 for an automobile. He sold it after 2 yr. for a price which showed a discount of 25% at the end of 1 yr., and a further discount the next year of 25% on the remainder. What price did he get for his car?

7. A salesman is offered a salary of $1250 a year, or half that amount and 2% commission on his sales. If the sales amount to $30,000 a year, which is the better plan for him?

8. Find the interest on $1250 for 2 yr. 4 mo. at 6%.

9. A teacher who received a salary of $75 a month for 9 mo. lent her brother at the end of the year 20% of her salary. He kept the money 4 yr. 6 mo., and then returned it with 6% interest. How much did he pay her?

10. If an automobile travels at the rate of 24 mi. an hour, how long does it take to go 0.8 mi.?

WRITTEN EXAMINATION

1. Write the following as decimal fractions:

$$\frac{1}{10} \qquad \frac{4}{10} \qquad \frac{232}{1000} \qquad \frac{723}{1000} \qquad \frac{3}{10}$$

2. Write as whole numbers or mixed decimals:

$$\frac{13}{10} \qquad \frac{15}{10} \qquad \frac{225}{100} \qquad \frac{3007}{1000} \qquad \frac{2800}{1000}$$

3. Add, and also subtract the following:

$$\begin{array}{ccccc} 127.42 & 403.61 & 512.71 & 623.42 & 803.40 \\ \underline{109.64} & \underline{362.83} & \underline{329.86} & \underline{435.64} & \underline{637.68} \end{array}$$

4. Divide the following as indicated:

$$\begin{array}{lll} 41.6 \div 0.4 & 2.22 \div 0.02 & 36.08 \div 3.28 \\ 0.25 \div 0.5 & 3.33 \div 0.03 & 1.036 \div 0.56 \end{array}$$

5. A man pays $113 for coal at $5.65 a ton. How many tons does he buy?

6. The product of two numbers is 156.25, and one of the numbers is 5000. What is the other number?

7. If wind blows in a hurricane at the rate of 8796.6 ft. per minute, what is the rate per second?

8. A contractor bought a shipment of bricks for $176.40, paying $7.20 a thousand. How many bricks did he buy?

9. In a recent year there were 614 coöperative creameries in Minnesota, and they made 74,934,940 lb. of butter in that year. Find to the nearest pound the average amount per creamery.

10. A farmer found that he could average 2.76 T. of alfalfa per acre on his 40-acre field. At $7 per ton, what was this crop worth? How did the value of the crop compare with that of clover in another 40-acre field averaging 1.85 T. per acre and worth $5.40 per ton?

WRITTEN EXAMINATION

1. Multiply the following:

1.3×48
2.9×62
4.25×64
6.75×84
3.42×286
4.25×66.4

2. Multiply the following:

8.2×6.8
7.7×8.8
8.36×4.8
9.82×6.7
5.33×2.96
4.56×8.21

3. Divide, carrying the quotients to two decimal places:

$6.25 \div 25$
$6.75 \div 75$
$3.49 \div 26$
$47.2 \div 34$
$8.34 \div 26$
$342.9 \div 0.16$

4. Divide, carrying the quotients to three decimal places:

$8.48 \div 1.7$
$39.4 \div 2.8$
$426.9 \div 0.31$

5. If a locomotive costs \$23,000, a steel sleeping car \$26,000, and a steel day coach \$8000, how much will a railway company have to pay for 36 day coaches, 5 sleeping cars, and 2 locomotives?

6. In a year when the Montana mines produced 285,719,918 lb. of copper and Michigan produced 155,715,286 lb., Montana produced how much more than Michigan?

7. A contractor bought a lot of shingles for \$717.75, paying $\$4.12\frac{1}{2}$ a thousand. How many thousand shingles did he buy?

8. In a year when this country produced 304,043,400 lb. of wool, the total value being \$75,819,251, what was the average value per pound, to the nearest cent?

9. In a year when Iowa produced 378,766,000 bu. of corn worth 50¢ a bushel, what was the value of the crop?

10. How many times will a roller 4.25 ft. in circumference turn in rolling the length of a lawn 136 ft. long?

TABLES FOR REFERENCE

Length

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)
2150.42 cu. in. (approximately $1\frac{1}{4}$ cu. ft.) = 1 bushel

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) = 1 year (yr.)
365 days = 1 common year
366 days = 1 leap year

Value

10 mills = 1 cent (¢ or ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
10 dollars = 1 eagle (E.)

Angles and Arcs

60 seconds (60″) = 1 minute (1′)
60 minutes = 1 degree (1°)

Surveyors' Table of Length

7.92 inches = 1 link (li.)
100 links = 4 rods = 1 chain (ch.)
80 chains = 5280 feet = 1 mile

Counting

12 units = 1 dozen (doz.)
12 dozen, or 144 = 1 gross (gro.)

Paper

24 sheets = 1 quire
20 quires, or 480 sheets = 1 ream

For convenience in counting, 500 sheets are now more often called a *ream*; and the word "quire" is used only for folded note paper, other paper being usually sold by the pound.

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Page 4. 1. 11. 2. 64. 3. 80. 4. 26. 5. 66. 6. 710. 7. 72. 8. 97. 9. 1610. 10. 75. 11. 34. 12. 1920. 13. XXXI. 14. CLXXV. 15. XLII. 16. MDCCCCXVI or MCMXVI. 17. CXXV. 18. MDCCCCXIX or MCMXIX. 19. CL. 20. MDCCCCXXVIII or MCMXXVIII.

Page 10. 1. $375.50. 2. $1000.50. 3. $1160.10. 4. $2000. 5. $2400.92. 6. $2600. 7. $1751.11. 8. $2700.54. 9. $30,000. 10. $30,000. 11. $20,451.80. 12. $19,112.20. 13. $11,583.40. 14. $18,877.91. 15. $17,499.73. 16. $16,463.48.

Page 12. 1. 1412. 2. 1402. 3. 1398. 4. 1347. 5. 1159. 6. 2362. 7. 2419. 8. 2389. 9. 1378. 10. 256. 11. 4069. 12. 1535. 13. 1165. 14. 2011. 15. 1949. 16. 2659. 17. 375. 18. 1452. 19. 2779. 20. 1779. 21. 1639. 22. 3486. 23. 2587. 24. 1799. 25. 1548.

Page 14. 1. 21,175. 2. 4775. 3. 14,835. 4. 20,128. 5. 21,565. 6. 7999. 7. 11,238. 8. 13,993. 9. $93.64. 10. $61.89. 11. $109.31. 12. $330.28. 13. $100.09. 14. $278.78. 15. $121.49. 16. $239.83. 17. $732.84. 18. $106.26. 19. $572.93. 20. $573.99. 21. $386.49. 22. $588.89. 23. $12.40. 24. $196.42. 25. $279.89. 26. $178.87. 27. $188.33. 28. $278.40.

Page 15. 1. $2.45. 2. $178.45. 3. $1.55. 4. $0.58. 5. $1.30. 6. $0.18. 7. $77.74. 8. $43. 9. $180.74. 10. $18.95. 11. $10,493.45.

Page 21. 1. 7800. 2. 16,832. 3. 23,698. 4. 25,546. 5. 38,391. 6. 31,902. 7. 24,653. 8. 46,368. 9. 435,136. 10. 262,704. 11. 217,588. 12. 341,885. 13. 245,904. 14. 479,906. 15. 809,009. 16. 209,235. 17. 257,856. 18. 270,803. 19. 270,096. 20. 423,852. 21. 726,612. 22. 451,136. 23. 725,841. 24. 294,120. 25. 146,495. 26. 163,017. 27. 94,898. 28. 190,182. 29. 257,103. 30. 380,044. 31. 311,026. 32. 363,752. 33. 557,376. 34. 164,373. 35. 111,100. 36. 213,032. 37. 586,810. 38. 305,564. 39. 679,936. 40. 419,148. 41. 291,132. 42. 430,837. 43. 439,956. 44. 316,043. 45. 78,982. 46. 274,164. 47. 574,126. 48. 1,237,872. 49. 935,723. 50. 1,645,056. 51. 2,482,806. 52. 5,120,576. 53. 173,061. 54. 741,748. 55. 3,343,026. 56. 2,809,312. 57. 6,372,489. 58. 2,294,592. 59. 1,337,640. 60. 1,540,608. 61. 5,597,991. 62. 994,536. 63. 4,397,226. 64. 5,131,323. 65. 7,787,430. 66. 2,015,496. 67. 3,235,680. 68. 1,645,920. 69. 5,250,078. 70. 4,764,080. 71. $204,223.75. 72. $213,257.70.

Page 22. 1. $72,660. 2. 745,600 ft. 3. $41,280,000. 4. $107,940. 5. $81,500. 6. $1,348,800. 7. 604,800 rd. 8. $35,475,000. 9. $195,750. 10. $85,305. 11. 960 mi. 12. $4425.

Page 23. 1. 286; 572; 818; 1054; 1366. 2860; 5720; 8180; 10,540; 13,660. 28,600; 57,200; 81,800; 105,400; 136,600. 2. $2.86; $5.72; $81.80; $105.40; $13.66. $28.60; $57.20; $818; $1054; $136.60. $286; $572; $8180; $10,540; $1366. 3. 1446; 1575; 1899; 2520; 2700. 14,460; 15,750; 18,990; 25,200; 27,000. 144,600; 157,500; 189,900; 252,000; 270,000. 4. $14.46; $157.50; $189.90; $25.20; $27. $144.60; $1575; $1899; $252; $270. $1446; $15,750; $18,990; $2520; $2700. 5. 29,440; 32,840; 38,520; 29,680; 35,720. 294,400; 328,409; 385,200; 296,800; 357,200. 36,800; 41,050; 48,150; 37,100; 44,650. 368,000; 410,500; 481,500; 371,000; 446,500. 6. $214; $357.20; $293.60; $272.40; $390. $2140; $3572; $2936; $2724; $3900. $267.50; $446.50; $367; $340.50; $487.50. $2675; $4465; $3670; $3405; $4875. 7. $2111.85; $21,118.50; $211,185; $230,426.30. 8. $2203.20; $22,032; $220,320; $240,699.60. 9. $2271.15; $11,355.75; $98,957.25. 10. $5079.60; $50,796; $2539.80; $253,980. 11. $5100. 12. $3850. 13. $10,800. 14. $51,750.

Page 25. 1. 11,850; 21,250 ft.; $3425; $49,370. 2. 3350; 20,050 ft.; $1240; $9610. 3. 8000; 32,000; 363,000; $21,400; $62,200. 4. 240; 370; 440; 410; 2240; 6320; 18,210. 5. 320; 190; 390; 480; 4130; 12,440; 40,885. 6. 3300; 1900; 4800; 3800; 11,200; 108,600; 212,100. 7. 4400; 2300; 4900; 2100; 16,900; 155,400; 327,150. 8. 800; 900; 1600; 1400; 5600; 121,100; 100,900. 9. $20; $100; $80; $120; $180; $240; $1400. $10; $50; $40; $60; $90; $120; $700. $8; $40; $32; $48; $72; $96; $560. $4; $20; $16; $24; $36; $48; $280.

Page 26. 1. $1800; $18,000. 2. $1375; $2750; $275. 3. $35,625; $71,250. 4. $386. 5. $41.04. 6. $4.50; $7.50; $15. 7. 1960. 8. $4875. 9. $41,625. 10. 15,504. 11. $41,625.

Page 29. 1. 4121. 2. 1374. 3. 1751. 4. 803. 5. 672. 6. 439. 7. 1046. 8. $446\frac{5}{9}$. 9. $3777\frac{1}{2}$. 10. 3005. 11. $9.68. 12. $18.55. 13. \$$6.88\frac{2}{6}$. 14. $14.21. 15. $8.21. 16. $9.29. 17. 20,251. 18. $7801\frac{3}{5}$. 19. $6878\frac{1}{6}$. 20. $11,435\frac{2}{7}$. 21. $5293\frac{1}{8}$. 22. $8467\frac{6}{9}$. 23. \$$17.60\frac{1}{2}$. 24. \$$24.14\frac{1}{3}$. 25. \$$15.30\frac{3}{4}$. 26. \$$14.47\frac{1}{5}$. 27. $13.67. 28. $6.01. 29. $9.40. 30. \$$8.13\frac{1}{9}$.

Page 30. 1. 86. 2. 47. 3. 21. 4. 21. 5. 19. 6. 12. 7. 12. 8. 12. 9. 5. 10. 9. 11. $37\frac{5}{10}$. 12. $31\frac{5}{20}$. 13. $20\frac{25}{30}$. 14. $11\frac{35}{40}$. 15. $6\frac{36}{50}$. 16. $7\frac{37}{70}$. 17. $9\frac{12}{70}$. 18. $6\frac{15}{80}$. 19. $9\frac{63}{90}$. 20. $9\frac{78}{90}$. 21. 413. 22. 189. 23. 181. 24. 231. 25. $145\frac{1}{500}$. 26. $113\frac{3}{600}$. 27. $70\frac{521}{700}$. 28. $40\frac{448}{800}$. 29. $420\frac{367}{900}$. 30. 146. 31. 57. 32. 149. 33. $110\frac{3600}{6000}$. 34. $81\frac{2800}{7000}$. 35. $106\frac{623}{8000}$. 36. $81\frac{542}{9000}$. 37. 18. 38. 28. 39. 31. 40. 95. 41. 63. 42. 5; $250. 43. 93; $400. 44. 298; 1350.

Page 33. 1. 42. 2. 51. 3. 63. 4. 89. 5. 79. 6. 88. 7. 68. 8. 84. 9. 13. 10. 14. 11. 22. 12. 33. 13. 38. 14. 39. 15. 47. 16. 29. 17. 19. 18. 16. 19. 18. 20. 33. 21. 23. 22. 33. 23. 37. 24. 111. 25. 123. 26. 112. 27. 101. 28. 219. 29. 408. 30. 107. 31. 511. 32. 462. 33. 531. 34. 617. 35. 609. 36. 701. 37. 804. 38. 810. 39. 1456. 40. 862. 41. 877. 42. 888. 43. 880. 44. 990. 45. 999. 46. 987. 47. 998. 48. 777. 49. 888. 50. 999. 51. 567. 52. 909. 53. 806. 54. 809.

Page 34. 1. 74. 2. 93. 3. 106. 4. 63. 5. 205. 6. 65. 7. 49. 8. 78. 9. 130. 10. 130. 11. 333. 12. 426. 13. 916. 14. 861. 15. 899. 16. 710. 17. 4706. 18. 9009.

Page 35. 1. $129. 2. $146.25. 3. 17 A. 4. 13. 5. 292. 6. 20 da. 7. 10 sec. 8. 2009. 9. 312. 10. 625. 11. 4025. 12. 329 mi. 13. 3168; 8800.

Page 37. 1. 85. 2. 83. 3. 77. 4. 727. 5. 1950. 6. 8761. 7. 504. 8. 1060. 9. 626. 10. 1050. 11. 365. 12. 467. 13. 675. 14. 4268. 15. 468. 16. 207. 17. 783. 18. 858. 19. 367. 20. 780. 21. 1332. 22. 1875. 23. 2397. 24. 2086. 25. $1133\frac{180}{832}$. 26. $112\frac{25}{69}$. 27. $113\frac{10}{76}$. 28. $373\frac{6}{23}$. 29. $1365\frac{12}{56}$. 30. $293\frac{1}{64}$. 31. $1259\frac{34}{57}$. 32. $416\frac{4}{46}$. 33. $564\frac{15}{47}$. 34. $467\frac{6}{59}$. 35. $682\frac{20}{69}$. 36. $617\frac{8}{82}$. 37. $436\frac{8}{232}$. 38. $890\frac{10}{124}$. 39. $612\frac{6}{412}$. 40. $185\frac{25}{653}$. 41. $617\frac{6}{422}$. 42. $831\frac{100}{232}$. 43. 36. 44. 54. 45. 37. 46. 75. 47. 77. 48. 98. 49. $564\frac{56}{2836}$. 50. $384\frac{40}{6237}$.

Page 38. 1. $135; $540. 2. $28; $196. 3. $580. 4. $540; $945. 5. $294; $490; $9800. 6. 1412. 7. 1351. 8. 3919. 9. $25. 10. $4938. 11. $18,246. 12. $780. 13. $90.

Page 39. 1. $8.77. 2. $2.88. 3. $39.35. 4. $625.72. 5. $572.30. 6. $4213.01. 7. $682.02. 8. $617.20. 9. $527.30. 10. $492.76. 11. $223.13. 12. $33.87. 13. $879.17. 14. $932.60. 15. $49.50. 16. $79. 17. $77. 18. $43. 19. $2.50.

Page 42. 1. 44,528. 2. 34,342. 3. 578,154. 4. 66,717. 5. 254,347. 6. 201,501. 7. 204,100. 8. 781,312. 9. 4,402,496. 10. 12,975,035. 11. 390,208. 12. 12,899,583. 13. 144. 14. $2169\frac{98}{102}$. 15. 67,910. 16. 408. 17. 912. 18. 37,864.

Page 44. 1. $5. 2. $75. 3. $48.

Page 45. 1. 35.70\frac{2}{3}$; 37.66\frac{1}{3}$; 35.76\frac{1}{2}$; 19.97\frac{2}{3}$. 2. 0.05\frac{5}{6}$; 17.68\frac{2}{3}$.

Page 46. 1. July 4. 2. July 22. 3. July 5. 4. $4.50. 5. $2.50. 6. $7; $0.75; $6.25.

Page 49. 1. $\frac{7}{16}$. 2. $\frac{9}{16}$. 3. $\frac{12}{32}$. 4. $\frac{8}{9}$. 5. $\frac{7}{32}$. 6. $\frac{17}{18}$. 7. $\frac{9}{64}$. 8. $\frac{5}{32}$.

Page 51. 1. $\frac{1}{2}$; $\frac{1}{2}$; $\frac{2}{2}$; $\frac{3}{2}$; $\frac{4}{2}$; $\frac{6}{2}$; $\frac{8}{2}$; $\frac{10}{2}$. 2. $\frac{2}{4}$; $\frac{1}{4}$; $\frac{2}{4}$; $\frac{8}{4}$; $\frac{5}{4}$; $\frac{10}{4}$; $\frac{9}{4}$; $\frac{11}{4}$. 3. $\frac{4}{8}$; $\frac{2}{8}$; $\frac{6}{8}$; $\frac{8}{8}$; $\frac{12}{8}$; $\frac{28}{8}$; $\frac{1}{8}$; $\frac{2}{8}$. 4. $\frac{6}{12}$; $\frac{3}{12}$; $\frac{9}{12}$; $\frac{4}{12}$; $\frac{8}{12}$; $\frac{2}{12}$; $\frac{10}{12}$; $\frac{1}{12}$. 5. $\frac{1}{2}$; $\frac{1}{2}$; $\frac{5}{4}$; $\frac{4}{5}$; $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{1}{2}$. 6. $\frac{2}{6}$ in.; $\frac{3}{6}$ in.; $\frac{1}{2}$ in. is greater. 7. $\frac{4}{12}$ in.; $\frac{3}{12}$ in.; $\frac{1}{3}$ in. is greater.

Page 52. 1. $\frac{4}{4}$; $\frac{5}{4}$; $\frac{7}{4}$; $\frac{8}{4}$. 2. $\frac{5}{5}$; $\frac{6}{5}$; $\frac{7}{5}$; $\frac{8}{5}$; $\frac{9}{5}$; $\frac{10}{5}$. 3. $\frac{6}{6}$; $\frac{7}{6}$; $\frac{11}{6}$; $\frac{12}{6}$; $\frac{13}{6}$; $\frac{17}{6}$; $\frac{18}{6}$. 4. $\frac{7}{7}$; $\frac{8}{7}$; $\frac{9}{7}$; $\frac{10}{7}$; $\frac{11}{7}$; $\frac{12}{7}$; $\frac{13}{7}$; $\frac{14}{7}$. 5. $\frac{8}{8}$; $\frac{16}{8}$; $\frac{17}{8}$; $\frac{19}{8}$; $\frac{21}{8}$; $\frac{23}{8}$; $\frac{24}{8}$. 6. $\frac{9}{9}$; $\frac{18}{9}$; $\frac{27}{9}$; $\frac{36}{9}$; $\frac{10}{9}$; $\frac{20}{9}$; $\frac{27}{9}$. 7. $\frac{10}{10}$; $\frac{20}{10}$; $\frac{30}{10}$; $\frac{31}{10}$; $\frac{33}{10}$; $\frac{37}{10}$; $\frac{40}{10}$. 8. $\frac{12}{12}$; $\frac{24}{12}$; $\frac{36}{12}$; $\frac{48}{12}$; $\frac{25}{12}$; $\frac{41}{12}$; $\frac{55}{12}$; $\frac{59}{12}$.

Page 53. 1. $\frac{1}{2}$; $\frac{3}{8}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{2}{3}$; $\frac{7}{16}$; $\frac{1}{2}$; $\frac{3}{4}$. 2. $\frac{6}{16}$; $\frac{12}{16}$; $\frac{14}{16}$; $\frac{12}{16}$; $\frac{15}{16}$; $\frac{4}{16}$; $\frac{5}{16}$; $\frac{10}{16}$. 3. $\frac{8}{32}$; $\frac{24}{32}$; $\frac{20}{32}$; $\frac{28}{32}$; $\frac{30}{32}$; $\frac{9}{32}$; $\frac{16}{32}$; $\frac{12}{32}$. 4. $\frac{24}{64}$; $\frac{40}{64}$; $\frac{28}{64}$; $\frac{36}{64}$; $\frac{18}{64}$; $\frac{30}{64}$; $\frac{32}{64}$; $\frac{6}{64}$. 5. $\frac{6}{12}$; $\frac{4}{12}$; $\frac{3}{12}$; $\frac{8}{12}$; $\frac{9}{12}$; $\frac{2}{12}$; $\frac{6}{12}$; $\frac{5}{12}$. 6. $\frac{9}{24}$; $\frac{12}{24}$; $\frac{14}{24}$; $\frac{8}{24}$; $\frac{1}{24}$; $\frac{3}{24}$; $\frac{10}{24}$; $\frac{22}{24}$.

Page 54. 1. $\frac{48}{8}$. 2. $\frac{27}{3}$. 3. $\frac{17}{2}$. 4. $\frac{32}{5}$. 5. $\frac{26}{3}$. 6. $\frac{41}{6}$. 7. $\frac{64}{9}$. 8. $\frac{43}{5}$. 9. $\frac{22}{3}$. 10. $\frac{33}{4}$. 11. $\frac{49}{9}$. 12. $\frac{48}{5}$. 13. $\frac{79}{8}$. 14. $\frac{77}{9}$. 15. $\frac{56}{9}$. 16. $\frac{410}{10}$. 17. $\frac{37}{5}$. 18. $\frac{14}{3}$. 19. $\frac{39}{4}$. 20. $\frac{57}{8}$. 21. $\frac{27}{8}$.

Page 55. 1. 11; $5\frac{5}{24}$; $19\frac{3}{4}$; $31\frac{3}{4}$; $10\frac{1}{8}$; $13\frac{3}{8}$. 2. $7\frac{5}{6}$; $18\frac{5}{8}$; $15\frac{5}{6}$; $7\frac{1}{4}$; $7\frac{2}{3}$; $9\frac{2}{3}$. 3. 7; $7\frac{3}{8}$; $13\frac{9}{32}$; $2\frac{29}{32}$; $2\frac{3}{8}$; 5. 4. $15\frac{3}{5}$; $10\frac{29}{32}$; $17\frac{1}{4}$; $12\frac{3}{8}$; $22\frac{21}{32}$; $16\frac{13}{32}$. 5. $8\frac{1}{8}$; $35\frac{7}{12}$; $26\frac{5}{24}$; $20\frac{3}{4}$; $28\frac{1}{2}$; $29\frac{5}{12}$. 6. $8\frac{5}{8}$; $52\frac{1}{12}$; $18\frac{5}{12}$; $26\frac{1}{24}$; 56; $16\frac{1}{16}$. 7. $9\frac{2}{3}$; $5\frac{15}{32}$; $14\frac{1}{8}$; $15\frac{5}{8}$; 147; 183.

Page 57. 1. $\frac{3}{6}$; $\frac{5}{6}$. 2. $\frac{3}{12}$; $\frac{10}{12}$. 3. $\frac{9}{12}$; $\frac{2}{12}$. 4. $\frac{3}{24}$; $\frac{20}{24}$. 5. $\frac{5}{8}$; $\frac{6}{8}$. 6. $\frac{21}{24}$; $\frac{8}{24}$. 7. $\frac{5}{6}$; $\frac{4}{6}$. 8. $\frac{21}{24}$; $\frac{16}{24}$. 9. $\frac{9}{15}$; $\frac{10}{15}$. 10. $\frac{32}{40}$; $\frac{15}{40}$. 11. $\frac{10}{24}$; $\frac{7}{24}$. 12. $\frac{9}{48}$; $\frac{10}{48}$. 13. $\frac{21}{24}$; $\frac{10}{24}$. 14. $\frac{15}{24}$; $\frac{11}{24}$. 15. $\frac{21}{24}$; $\frac{13}{24}$. 16. $\frac{28}{32}$; $\frac{5}{32}$. 17. $\frac{20}{32}$; $\frac{11}{32}$. 18. $\frac{14}{32}$; $\frac{5}{32}$. 19. $\frac{30}{32}$; $\frac{31}{32}$. 20. $\frac{26}{32}$; $\frac{29}{32}$. 21. $\frac{15}{24}$; $\frac{7}{24}$. 22. $\frac{7}{8}$; $\frac{7}{8}$. 23. $\frac{40}{96}$; $\frac{9}{96}$. 24. $\frac{56}{96}$; $\frac{15}{96}$. 25. $\frac{88}{96}$; $\frac{45}{96}$. 26. $\frac{4}{96}$; $\frac{3}{96}$. 27. $\frac{20}{96}$; $\frac{9}{96}$. 28. $\frac{28}{96}$; $\frac{27}{96}$. 29. $\frac{14}{48}$; $\frac{9}{48}$. 30. $\frac{22}{48}$; $\frac{13}{48}$. 31. $\frac{56}{64}$; $\frac{9}{64}$. 32. $\frac{20}{64}$; $\frac{11}{64}$. 33. $\frac{22}{64}$; $\frac{15}{64}$. 34. $\frac{16}{192}$; $\frac{9}{192}$. 35. $\frac{80}{192}$; $\frac{21}{192}$. 36. $\frac{8}{192}$; $\frac{3}{192}$. 37. $\frac{56}{192}$; $\frac{75}{192}$. 38. $\frac{18}{48}$; $\frac{15}{48}$; $\frac{14}{48}$. 39. $\frac{21}{24}$; $\frac{10}{24}$; $\frac{11}{24}$. 40. $\frac{60}{96}$; $\frac{56}{96}$; $\frac{45}{96}$. 41. $4\frac{28}{48}$; $6\frac{27}{48}$. 42. $5\frac{9}{48}$; $7\frac{10}{48}$. 43. $9\frac{44}{96}$; $7\frac{45}{96}$.

Page 60. 1. $\frac{5}{6}$. 2. $1\frac{1}{6}$. 3. $\frac{7}{10}$. 4. $\frac{9}{10}$. 5. $1\frac{1}{10}$. 6. $1\frac{3}{10}$. 7. $1\frac{1}{4}$. 8. $\frac{7}{8}$. 9. $1\frac{3}{8}$. 10. $1\frac{3}{8}$. 11. $1\frac{5}{8}$. 12. $\frac{37}{40}$. 13. $\frac{39}{40}$. 14. $1\frac{11}{40}$. 15. $1\frac{27}{40}$. 16. $1\frac{1}{3}$. 17. $1\frac{7}{12}$. 18. $\frac{7}{24}$. 19. $\frac{13}{24}$. 20. $1\frac{17}{24}$. 21. $\frac{41}{64}$ in. 22. $\frac{7}{8}$. 23. $1\frac{3}{8}$. 24. $1\frac{7}{8}$. 25. $\frac{2}{3}$. 26. $1\frac{1}{6}$. 27. 2. 28. $\frac{13}{24}$. 29. $\frac{7}{8}$. 30. $1\frac{7}{8}$. 31. $1\frac{1}{3}$ in. 32. $1\frac{3}{8}$ in. 33. $\frac{9}{16}$ in. 34. $\frac{5}{32}$ in. 35. $\frac{15}{16}$ in. 36. $\frac{11}{24}$. 37. $1\frac{5}{24}$. 38. $\frac{2}{3}$. 39. $1\frac{1}{12}$. 40. $\frac{11}{48}$. 41. $\frac{25}{48}$. 42. $\frac{43}{48}$. 43. $1\frac{5}{48}$. 44. $\frac{25}{48}$. 45. $\frac{37}{48}$. 46. $1\frac{2}{7}$. 47. $\frac{31}{48}$. 43. $\frac{47}{48}$. 49. $1\frac{17}{48}$. 50. $\frac{5}{12}$. 51. $1\frac{1}{12}$.

Page 61. 1. $52\frac{1}{2}$ ft. 2. 2 lb. 3. $43\frac{1}{2}$. 4. $58\frac{1}{3}$. 5. 21. 6. 47. 7. $58\frac{2}{3}$. 8. 31. 9. 144. 10. $107\frac{1}{2}$. 11. 126. 12. 87. 13. 118. 14. 47.

Page 62. 1. $14\frac{5}{6}$ ft. 2. $26\frac{1}{4}$ ft. 3. $12\frac{23}{48}$ ft. 4. $79\frac{1}{8}$ lb. 5. $1\frac{7}{8}$. 6. 3. 7. $7\frac{7}{8}$. 8. 5. 9. $10\frac{7}{16}$. 10. $8\frac{11}{16}$.

Page 63. 1. $\frac{3}{8}$ yd. 2. $\frac{1}{8}$ mi. 3. $\frac{1}{12}$. 4. $\frac{13}{16}$. 5. $\frac{1}{8}$. 6. $\frac{1}{4}$. 7. $\frac{9}{16}$. 8. $\frac{1}{8}$. 9. $\frac{2}{3}$. 10. $\frac{1}{6}$. 11. $\frac{1}{6}$. 12. $\frac{1}{6}$. 13. $\frac{5}{16}$. 14. $\frac{5}{12}$. 15. $\frac{1}{12}$. 16. $\frac{1}{30}$. 17. $\frac{1}{10}$. 18. $\frac{1}{24}$. 19. $\frac{3}{10}$. 20. $\frac{13}{24}$. 21. $\frac{5}{8}$. 22. $\frac{5}{24}$. 23. $\frac{1}{6}$. 24. $\frac{3}{8}$. 25. $\frac{17}{32}$. 26. $\frac{2}{5}$.

Page 64. 1. $11\frac{1}{4}$ in. 2. $7\frac{1}{2}$ in. 3. $30\frac{3}{4}$ mi. 4. $5\frac{1}{3}$. 5. $1\frac{5}{8}$. 6. $4\frac{3}{5}$. 7. $\frac{1}{8}$. 8. $3\frac{2}{5}$. 9. $11\frac{5}{8}$. 10. $1\frac{2}{5}$. 11. $11\frac{11}{16}$. 12. $14\frac{9}{16}$. 13. $14\frac{7}{16}$. 14. $1\frac{1}{4}$ in. 15. $\frac{29}{32}$ in. 16. $4\frac{25}{32}$ in. 17. $5\frac{21}{32}$ in. 18. $5\frac{63}{64}$ in.

Page 65. 1. $72\frac{7}{8}$. 2. $2\frac{3}{8}$. 3. $2\frac{5}{6}$. 4. $56\frac{5}{8}$. 5. $4\frac{7}{8}$. 6. $2\frac{3}{8}$. 7. $51\frac{3}{4}$. 8. $6\frac{17}{24}$. 9. $6\frac{1}{40}$. 10. $25\frac{5}{6}$. 11. $7\frac{13}{24}$. 12. $8\frac{7}{12}$. 13. $104\frac{7}{12}$. 14. $5\frac{13}{15}$. 15. $8\frac{13}{40}$. 16. $\frac{5}{16}$ in. 17. $1\frac{3}{8}$ in. 18. $5\frac{1}{2}$ yd. 19. $9\frac{1}{8}$ in. 20. $19\frac{5}{8}$ in.

Page 66. 1. $10\frac{1}{3}$. 2. $11\frac{7}{8}$. 3. $14\frac{1}{2}$. 4. $8\frac{1}{24}$. 5. 17. 6. $7\frac{17}{24}$. 7. $21\frac{5}{8}$. 8. $11\frac{1}{2}$. 9. $18\frac{1}{8}$. 10. $6\frac{3}{4}$. 11. $24\frac{11}{24}$. 12. $7\frac{7}{24}$. 13. $28\frac{5}{12}$. 14. $11\frac{7}{12}$. 15. $39\frac{3}{4}$. 16. $16\frac{7}{8}$. 17. $49\frac{41}{120}$. 18. $12\frac{1}{12}$. 19. $51\frac{1}{2}$. 20. $17\frac{3}{4}$. 21. $48\frac{5}{8}$. 22. $26\frac{5}{8}$. 23. $38\frac{1}{4}$. 24. $34\frac{11}{12}$. 25. $27\frac{3}{8}$ mi. 26. $53\frac{3}{8}$ mi. 27. $61\frac{1}{4}$ ft. 28. $10\frac{1}{4}$ ft. 29. $8\frac{7}{8}$ lb. 30. 2.

Page 67. 1. $3\frac{19}{24}$ ft. 2. $10\frac{11}{12}$ yr. 3. $3\frac{3}{16}$ in. 4. $78\frac{3}{8}$ lb. 5. $196\frac{19}{24}$ yd. 6. $22\frac{1}{2}$ in. by $36\frac{3}{8}$ in. 7. 6 in. 8. 26 ft. $9\frac{3}{8}$ in. 9. $1\frac{3}{8}$ in.; 2 in. 10. $11\frac{85}{96}$ qt.

Page 70. 1. $5\frac{3}{8}$. 2. $8\frac{3}{4}$. 3. $1\frac{5}{8}$. 4. $10\frac{1}{2}$. 5. $6\frac{1}{8}$. 6. 55. 7. $6\frac{3}{16}$. 8. $10\frac{1}{2}$. 9. $5\frac{1}{4}$. 10. $6\frac{2}{3}$. 11. $13\frac{1}{2}$. 12. 33.

Page 71. 1. 92; 276. 2. 144; 720. 3. 663; 1105; 1610. 4. 2254; 290; 1460. 5. 1470; 1480; 211. 6. 1266; 2120; 1491. 7. 4220; 4240; 1065. 8. 52. 9. 66; 110. 10. 126; 210.

Page 72. 1. $\frac{2}{3}$. 2. $\frac{3}{8}$. 3. $\frac{5}{8}$. 4. $\frac{11}{36}$. 5. $\frac{25}{48}$. 6. $\frac{5}{32}$. 7. $\frac{1}{10}$. 8. $\frac{21}{64}$. 9. $\frac{1}{2}$. 10. $\frac{3}{16}$. 11. $\frac{1}{6}$. 12. $\frac{1}{6}$. 13. $\frac{1}{40}$. 14. $\frac{1}{50}$. 15. $\frac{1}{9}$. 16. $\frac{2}{5}$. 17. $\frac{1}{8}$. 18. $\frac{1}{4}$. 19. $\frac{1}{2}$. 20. $\frac{1}{6}$. 21. $\frac{1}{16}$. 22. $\frac{5}{12}$. 23. $\frac{9}{20}$. 24. $\frac{3}{5}$. 25. $\frac{2}{7}$. 26. $\frac{4}{5}$. 27. $\frac{1}{4}$. 28. $\frac{5}{16}$. 29. $\frac{2}{9}$. 30. $\frac{13}{24}$. 31. $\frac{3}{8}$. 32. $\frac{7}{27}$. 33. $\frac{1}{4}$. 34. $\frac{11}{15}$. 35. $\frac{1}{4}$. 36. $\frac{2}{5}$. 37. $\frac{1}{4}$. 38. $\frac{1}{3}$. 39. $\frac{9}{160}$. 40. $\frac{9}{40}$. 41. $\frac{105}{512}$. 42. $\frac{21}{32}$ min. 43. $\frac{1}{4}$. 44. $\frac{15}{128}$. 45. $\frac{9}{20}$. 46. $\frac{1}{10}$ mi. 47. $\frac{3}{8}$ yd. 48. $\frac{42}{55}$ min. 49. $\frac{7}{24}$ mi.; $\frac{7}{48}$ mi. 50. $1\frac{9}{40}$ yd.

Page 74. 1. $43\frac{1}{2}$. 2. 1122. 3. 4011. 4. 5067. 5. $1790\frac{2}{3}$. 6. 856. 7. 1701. 8. 1060. 9. $3391\frac{1}{2}$. 10. 462. 11. $3061\frac{1}{2}$. 12. $1669\frac{1}{2}$. 13. $1291\frac{1}{3}$. 14. $54\frac{9}{64}$. 15. $148\frac{1}{6}$. 16. $245\frac{2}{3}$. 17. $224\frac{4}{9}$. 18. $247\frac{4}{5}$. 19. $443\frac{11}{28}$. 20. 506. 21. 448.

Page 75. 1. $13\frac{1}{3}$. 2. $9\frac{5}{8}$. 3. 14. 4. $15\frac{21}{64}$. 5. $45\frac{29}{60}$. 6. $97\frac{45}{64}$. 7. $10\frac{1}{8}$ in. 8. $20\frac{27}{32}$. 9. $22\frac{29}{32}$. 10. $4\frac{1}{12}$. 11. $7\frac{11}{12}$. 12. $24\frac{1}{4}$. 13. $34\frac{5}{12}$. 14. $8\frac{1}{4}$ ft. 15. $\frac{3}{8}$. 16. $\frac{3}{8}$. 17. $\frac{1}{2}$. 18. $\frac{1}{7}$. 19. $\frac{13}{24}$. 20. $\frac{3}{8}$. 21. $\frac{2}{5}$. 22. $\frac{1}{4}$. 23. $\frac{1}{20}$ in. 24. $\frac{1}{15}$ in. 25. $1\frac{1}{4}$ in. 26. 9 in. 27. 20. 28. 7 cups. 29. $44\frac{1}{4}$ sec. 30. $\frac{3}{10}$ A.; $\frac{9}{50}$ A. 31. $34\frac{1}{2}$. 32. $\frac{5}{8}$. 33. $\frac{3}{4}$; $\frac{3}{8}$; $\frac{9}{16}$. 34. $\frac{3}{8}$ in.

Page 77. 1. \$1.56. 2. \$1.82. 3. \$5.21. 4. \$3.10. 5. \$2.38. 6. \$16.26.

Page 80. 1. 660; 1356; 37,268. 2. 9636; 8124; 3672. 3. 1235; 9100; 3445; 31,085.

Page 83. 1. $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{3}{16}$. 2. $\frac{1}{6}$; $\frac{2}{9}$; $\frac{1}{12}$; $\frac{1}{4}$; $\frac{1}{8}$. 3. $\frac{2}{15}$; $\frac{4}{15}$; $\frac{1}{16}$; $\frac{2}{5}$; $\frac{2}{3}$. 4. $\frac{1}{6}$; $\frac{5}{16}$; $\frac{5}{24}$; $\frac{5}{32}$; $\frac{1}{8}$. 5. $\frac{3}{7}$; $\frac{2}{7}$; $\frac{6}{35}$; $\frac{1}{7}$; $\frac{5}{48}$. 6. $\frac{1}{8}$; $\frac{3}{4}$; $2\frac{1}{2}$; $\frac{3}{5}$; $\frac{2}{3}$. 7. $4\frac{1}{4}$; $7\frac{1}{6}$; $12\frac{1}{5}$; $6\frac{1}{5}$; $7\frac{3}{16}$. 8. $3\frac{1}{3}$; $3\frac{3}{16}$; $3\frac{1}{8}$; $5\frac{3}{4}$; $26\frac{9}{10}$.

Page 85. 1. 75. 2. 128. 3. 185. 4. 258. 5. 102. 6. 504. 7. 54. 8. 432. 9. 816. 10. 648. 11. 64. 12. 92. 13. 192. 14. 216. 15. 87. 16. 240. 17. 504. 18. 162. 19. 204. 20. 1164. 21. 780. 22. 56. 23. 236. 24. 604. 25. $1\frac{1}{3}$. 26. 8. 27. 12. 28. 6. 29. 10. 30. 5. 31. 18. 32. 500. 33. 284. 34. 213.

Page 86. 1. 16. 2. 13. 3. 18. 4. 21. 5. 12. 6. $6\frac{2}{3}$. 7. 5. 8. $\frac{25}{32}$. 9. $1\frac{2}{5}$. 10. $10\frac{1}{4}$.

Page 87. 1. $1\frac{5}{16}$. 2. $1\frac{1}{9}$. 3. $1\frac{1}{8}$. 4. $\frac{10}{21}$. 5. $\frac{9}{10}$. 6. $4\frac{2}{3}$. 7. $\frac{5}{14}$. 8. $1\frac{1}{5}$. 9. $\frac{5}{16}$. 10. $\frac{1}{4}$. 11. $\frac{12}{17}$. 12. $\frac{39}{50}$. 13. $1\frac{2}{3}$. 14. $\frac{2}{7}$. 15. $1\frac{1}{2}$. 16. $\frac{1}{3}$. 17. $3\frac{1}{2}$. 18. $\frac{21}{55}$.

Page 88. 1. $\frac{8}{9}$. 2. $1\frac{1}{8}$. 3. $\frac{5}{6}$. 4. $1\frac{1}{5}$. 5. $\frac{1}{2}$. 6. 2. 7. $\frac{25}{32}$. 8. $1\frac{7}{25}$. 9. $1\frac{1}{6}$. 10. $\frac{6}{7}$. 11. $\frac{32}{35}$. 12. $1\frac{3}{32}$. 13. $\frac{5}{6}$. 14. $1\frac{1}{5}$. 15. $\frac{25}{36}$. 16. $1\frac{11}{25}$. 17. $\frac{9}{10}$. 18. $1\frac{1}{9}$. 19. $\frac{5}{8}$. 20. $1\frac{3}{5}$. 21. 8. 22. $8\frac{2}{5}$. 23. $4\frac{2}{7}$. 24. 10. 25. $4\frac{5}{6}$. 26. $6\frac{1}{24}$. 27. 13. 28. $2\frac{1}{3}$. 29. 12. 30. 18.

Page 89. 1. 4. 2. $8\frac{1}{4}$. 3. 12. 4. $6\frac{2}{9}$. 5. $4\frac{1}{2}$. 6. $5\frac{1}{3}$. 7. 7. 8. 8. 9. $3\frac{2}{3}$. 10. $4\frac{3}{13}$. 11. $2\frac{9}{19}$. 12. $6\frac{17}{20}$. 13. $3\frac{4}{7}$. 14. $4\frac{13}{20}$. 15. $9\frac{1}{7}$. 16. 4. 17. $9\frac{4}{5}$. 18. 45. 19. $4\frac{4}{5}$. 20. $6\frac{2}{5}$. 21. $12\frac{6}{7}$. 22. 12. 23. $\frac{33}{40}$. 24. $6\frac{33}{76}$. 25. $13\frac{1}{2}$. 26. $\frac{1}{2}$. 27. $1\frac{5}{16}$. 28. 2. 29. $3\frac{7}{44}$. 30. 10. 31. $6\frac{1200}{1653}$. 32. 14. 33. 10. 34. 7. 35. 7. 36. 21; 57; 81. 37. 25. 38. 34. 39. 98. 40. 16.

Page 92. 1. \$18.38. 2. \$18.83. 3. \$33.67. 4. \$220. 5. \$165.67. 6. \$126. 7. \$156. 8. \$6.

Page 93. 1. $\frac{5}{6}$; $140\frac{5}{6}$; $155\frac{5}{12}$; $76\frac{19}{20}$. 2. $\frac{8}{15}$; $97\frac{8}{15}$; $102\frac{7}{15}$; $107\frac{1}{2}$. 3. $\frac{2}{3}$; $96\frac{2}{3}$; $112\frac{7}{8}$; $117\frac{3}{10}$. 4. $1\frac{11}{20}$; $68\frac{11}{20}$; $150\frac{3}{10}$; $55\frac{9}{10}$. 5. $\frac{3}{10}$; $57\frac{3}{10}$; $49\frac{1}{6}$; $24\frac{7}{8}$. 6. $\frac{7}{20}$; $55\frac{7}{20}$; $10\frac{19}{24}$; $13\frac{5}{8}$. 7. $\frac{11}{40}$; $16\frac{11}{40}$; $24\frac{13}{24}$; $225\frac{1}{3}$. 8. 396; $231\frac{1}{4}$; $8\frac{1}{8}$; 4836. 9. $1741\frac{1}{2}$; 374; $12\frac{3}{16}$; 3872. 10. 2754; 168; $28\frac{11}{16}$; $776\frac{9}{16}$. 11. 125; $45\frac{3}{5}$; $18\frac{2}{3}$; $115\frac{3}{5}$. 12. 637; $1022\frac{1}{4}$; 35; $17{,}368\frac{1}{3}$. 13. $1\frac{7}{9}$; $\frac{9}{16}$; $1\frac{3}{32}$; $3\frac{1}{5}$. 14. $\frac{5}{6}$; $1\frac{1}{5}$; $\frac{6}{7}$; $\frac{15}{22}$. 15. $\frac{25}{88}$; $\frac{15}{16}$; $\frac{1}{7}$; $4\frac{3}{14}$. 16. $\frac{1}{5}$; $\frac{1}{2}$; $\frac{8}{35}$; 5. 17. $\frac{3}{5}$; $\frac{5}{18}$; $\frac{27}{70}$; $\frac{4}{5}$. 18. $\frac{6}{25}$ cu. ft.; $\frac{1}{3}$ cu. in. 19. 20 sq. ft.; 432 sq. ft. 20. 56 sq. in.; 78 sq. in.

Page 94. 1. $\frac{4}{7}$. 2. $\frac{7}{9}$. 3. $\frac{16}{21}$. 4. $\frac{21}{121}$. 5. $\frac{1}{2}$. 6. $\frac{1}{2}$. 7. $\frac{1}{9}$. 8. $\frac{1}{8}$. 9. $\frac{1}{4}$. 10. $\frac{1}{9}$. 11. $\frac{3}{8}$. 12. $\frac{1}{6}$. 13. $\frac{1}{7}$. 14. $\frac{1}{3}$. 15. $\frac{9}{71}$. 16. $\frac{1}{2}$. 17. $\frac{1}{6}$. 18. $\frac{20}{27}$. 19. $\frac{8}{15}$. 20. $\frac{8}{9}$. 21. $\frac{1}{7}$. 22. $\frac{1}{2}$. 23. $\frac{1}{9}$. 24. $\frac{1}{11}$. 25. $\frac{2}{3}$. 26. $\frac{1}{3}$. 27. $\frac{1}{2}$; $\frac{3}{4}$.

Page 95. 1. 69. 2. 57. 3. 72. 4. 200. 5. 50. 6. 250. 7. 259. 8. 160. 9. 256. 10. \$250. 11. \$3.60. 12. \$18.45. 13. 153. 14. 1394. 15. \$2250. 16. \$5350.

Page 96. 1. 1. 2. $4\frac{4}{9}$. 3. $\frac{3}{5}$. 4. 1. 5. 7245. 6. 24,990. 7. 10,680. 8. $8812\frac{1}{2}$. 9. 211,200. 10. 1824. 11. $6021\frac{3}{4}$. 12. $3509\frac{1}{3}$. 13. $13\frac{1}{2}$. 14. $11\frac{1}{4}$. 15. 28. 16. 18. 17. $32\frac{2}{3}$. 18. $9\frac{3}{4}$. 19. $29\frac{3}{4}$. 20. 21. 21. $19\frac{1}{2}$. 22. $27\frac{1}{2}$. 23. $\frac{1}{6}$. 24. $\frac{21}{32}$. 25. $\frac{1}{6}$. 26. $\frac{1}{4}$. 27. $\frac{1}{2}$. 28. $\frac{1}{4}$. 29. $\frac{5}{18}$. 30. $\frac{9}{16}$. 31. $\frac{3}{4}$. 32. $\frac{1}{6}$. 33. $\frac{2}{3}$. 34. $1\frac{1}{3}$. 35. $\frac{5}{32}$. 36. $\frac{3}{8}$. 37. $\frac{1}{6}$. 38. $\frac{1}{3}$. 39. 4. 40. $4\frac{1}{5}$. 41. $3\frac{31}{48}$. 42. 6. 43. $\frac{1}{5}$. 44. $\frac{3}{4}$. 45. $\frac{1}{15}$. 46. $\frac{1}{8}$. 47. $1\frac{1}{2}$. 48. $\frac{1}{2}$. 49. 70. 50. $56\frac{7}{8}$. 51. $64\frac{2}{5}$. 52. $445\frac{1}{2}$. 53. 16. 54. $26\frac{2}{3}$. 55. $\frac{15}{32}$. 56. $\frac{16}{27}$. 57. $\frac{5}{6}$. 58. $8\frac{5}{6}$. 59. $3\frac{1}{3}$. 60. $1\frac{1}{5}$. 61. $\frac{2}{3}$. 62. $2\frac{4}{5}$. 63. $1\frac{1}{14}$. 64. $\frac{5}{12}$. 65. $\frac{1}{40}$. 66. 40. 67. $\frac{2}{5}$. 68. $1\frac{1}{4}$. 69. 4. 70. 4. 71. 4. 72. 3. 73. $2\frac{1}{3}$.

Page 99. 2. 7,000,007. 3. 111,205. 4. \$252.28. 5. 304,475. 6. \$645.46. 7. 228,440,248. 8. \$8084.50. 9. 269. 10. 1566; 1566. 11. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{4}$; $\frac{1}{2}$; 1. 12. 12; 13; 23. 13. $\frac{63}{4}$; $\frac{126}{8}$. 14. $12\frac{1}{2}$; $9\frac{1}{4}$; $3\frac{7}{8}$. 15. $12\frac{3}{4}$. 16. $2\frac{7}{8}$ in. 17. $5\frac{5}{8}$ in.; $10\frac{5}{16}$ ft. 18. 4; 1 in.

Page 100. 1. \$51.06. 2. \$1.92. 3. \$8.80. 4. \$0.62. 5. \$0.07.

Page 102. 1. $\frac{1}{8}$. 2. \$10. 3. \$5; \$3.75; \$1.25.

Page 103. 1. $253\frac{1}{8}$ lb.; $1012\frac{1}{2}$ lb. 2. $42\frac{3}{16}$ lb.; $168\frac{3}{4}$ lb. 3. 1800 lb.; 300 lb.; yes. 4. Yes. 5. $126\frac{9}{16}$ lb.; $506\frac{1}{4}$ lb.

Page 104. 1. \$90. 2. \$73.40. 3. \$18.75. 4. \$16.95. 5. \$144.60. 6. \$378.20. 7. 2130; \$378.20.

Page 106. 1. 6; 18. 2. 14; 35. 3. $\frac{1}{4}$ mi. 4. $5\frac{1}{4}$. 5. $97\frac{1}{6}$; 176. 6. \$4.38. 7. $16\frac{15}{28}$ lb. 8. $63\frac{3}{4}$ lb. 9. 255 mi. 10. $396\frac{9}{16}$ mi. 11. $1\frac{35}{64}$ in. 12. $16\frac{5}{16}$ in. 13. \$1.26. 14. \$1.62. 15. \$0.84. 16. \$3.75. 17. \$37.38. 18. $\$1.12\frac{1}{2}$. 19. 4000. 20. 2700. 21. \$156.25; \$187.50. 22. 1760. 23. \$66; \$90; \$192; \$450. 24. $7\frac{1}{4}$ hr. 25. 496. 26. 640. 27. \$0.10.

Page 109. 1. 432. 2. 27 sq. ft. 3. $60\frac{1}{2}$ sq. yd.; $544\frac{1}{2}$ sq. ft. 4. 80 sq. rd.

Page 110. 1. 47,200; 295. 2. 100. 3. 1120. 4. 160 sq. ft. 5. 150 sq. ft. 6. 96 sq. ft.

Page 111. 9. $\frac{1}{4}$. 10. 1 in. to 4 ft. 11. 24 sq. in. 12. $\frac{1}{6}$.

Page 112. 1. 60 cu. in. 2. 90 cu. in. 3. 336 cu. in. 4. 1536 cu. in. 5. 266 cu. in. 6. 700 cu. ft. 7. 35 cu. ft. 8. 77 cu. ft. 9. 8000 cu. in. 10. 12,960 cu. in. 11. 198.

Page 113. 1. 5000; 3500; 7500. 2. 40; 44; 52. 3. 112; 116; 156. 4. $437\frac{1}{2}$. 5. 5500; 7000; 10,500. 6. $4\frac{3}{8}$; $5\frac{1}{4}$; 7. 7. $1\frac{1}{4}$; $1\frac{1}{2}$; $2\frac{1}{4}$. 8. 60; 68; 88.

Page 116. 1. 38. 2. $27\frac{1}{2}$. 3. 330. 4. 15.

Page 118. 1. 1008; 1392; 2304. 2. \$27. 3. \$0.60; \$1.50. 4. 3240. 5. The first; \$7. 6. \$20.83. 7. \$250.

Page 119. 1. 408; 8. 2. 240. 3. \$1.25. 4. \$1.19; \$0.81; \$1.50. 5. \$0.32.

Page 121. 1. 98 in. 2. 36 ft. 3. $91\frac{1}{2}$ ft. 4. 34 oz. 5. 11 qt. 6. 1280 cu. ft. 7. $54\frac{1}{2}$ ft. 8. 320 sq. rd. 9. 66″. 10. 16 ft. 11. 440 sq. rd. 12. 630 cu. ft. 13. 95. 14. 80. 15. 124. 16. 66. 17. 20. 18. 66; 528. 19. 1064. 20. 2176. 21. 200. 22. 126. 23. 59. 24. 16,200. 25. 84; 22,869. 26. 2750.

Page 123. 1. 8 ft. 2. 3 lb. 3. 11 lb. 4. 60 hr. 5. $6\frac{3}{4}$ bu. 6. $7\frac{2}{3}$ ft. 7. $2\frac{3}{4}$ yd. 8. 2 hr. 9. 16 rd. 11 ft. 10. 8 T. 11. 8 bu. 15 pt.; 263 qt. 1 pt. 12. 3 sq. ft. 68 sq. in. 13. 1425 ft.; $\frac{95}{352}$ mi. 14. $\frac{1}{16}$ ft.; $\frac{1}{48}$ yd. 15. $1466\frac{2}{3}$ yd.; $\frac{5}{6}$ mi. 16. $\frac{13}{24}$ ft.; $\frac{13}{72}$ yd. 17. 2 yd. 9 in.; $2\frac{1}{4}$ yd. 18. 3 ft. 6 in.; $3\frac{1}{2}$ ft.; 1 yd. 6 in. 19. 75 gal. 2 qt.; $75\frac{1}{2}$ gal. 20. 36 qt. 1 pt.; $36\frac{1}{2}$ qt.; 9 gal. 1 pt.; $9\frac{1}{8}$ gal. 21. 34 gal. 1 qt.; $34\frac{1}{4}$ gal. 22. $\frac{3}{4}$. 23. $64\frac{7}{12}$ ft.; $21\frac{19}{36}$ yd. 24. $2\frac{2}{3}$ ft.; 2 lb.; $10\frac{2}{3}$ yd.; 16 qt.; 8 gal. 25. 6 lb.; $2\frac{2}{3}$ yd.; 32 yd.; 1 hr. 36 min. 26. $7\frac{1}{2}$. 27. 1 sq. ft. 130 sq. in. 28. 270. 29. 4 sq. ft. 4 sq. in. 30. 18. 31. 1768.

Page 125. 1. 38 ft. 2. 248 sq. ft. 34 sq. in. 3. 63 yd. 20 in. 4. 1432 lb. 14 oz. 5. 82 ft. 1 in. 6. 188 sq. ft. 45 sq. in. 7. 131 rd. 8. 1443 lb. 2 oz.

Page 126. **1.** 46 ft. 10 in. **2.** 14 yd. 1 ft. **3.** 6 yd. 34 in. **4.** 13 gal. 2 qt. **5.** 52 ft. $8\frac{1}{4}$ in. **6.** 14 bu. 6 qt. **7.** 17 rd. $3\frac{1}{2}$ ft. **8.** 20 A. 116 sq. rd. **9.** 23 bu. 3 pk. **10.** 31 qt. 1 pt. **11.** 7 lb. 11 oz. **12.** 57 sq. ft. 109 sq. in.

Page 128. **1.** 22 ft. 6 in. **2.** 24 yd. 1 ft. **3.** 37 ft. **4.** 38 lb. 4 oz. **5.** 48 yd. 2 ft. **6.** 31 lb. 12 oz. **7.** 52 lb. 14 oz. **8.** 69 lb. 3 oz. **9.** 46 ft. 8 in. **10.** 31 ft. 8 in. **11.** 11 ft. 9 in. **12.** 14 yd. 9 in. **13.** 62 yd. 9 in. **14.** 108 lb. 12 oz. **15.** 32 mi. 24 ft. **16.** 365 ft. 10 in. **17.** 275 lb. 3 oz. **18.** 240 yd. 8 in. **19.** 7 gal. **20.** 76 gal. 2 qt. **21.** 47 gal. 1 qt. **22.** 13 mi. 19 rd. **23.** 44 mi. 123 rd. **24.** 15 mi. 3750 ft. **25.** 7 ft. $8\frac{1}{2}$ in. **26.** 9 ft. 10 in. **27.** 3000 lb. **28.** 109 ft. 8 in. **29.** 147 gal.

Page 129. **1.** 3 ft. $9\frac{1}{2}$ in. **2.** 1 ft. $8\frac{1}{2}$ in. **3.** 8° 31′. **4.** 2 hr. 45 min. **5.** 25 ft. 3 in. **6.** 18 ft. 4 in. **7.** 12 oz. **8.** 9 oz. **9.** 20 rd. **10.** 3 hr. 22 min. **11.** 1 wk. 3 da. **12.** 5 hr. 17 min. **13.** 17 ft. 11 in. **14.** 3 lb. 5 oz. **15.** 45 mi.; $52\frac{1}{2}$ mi. **16.** 20 mi. **17.** $12,313\frac{29}{67}$. **18.** 127 ft. 8 in. **19.** 154 cu. ft. 1152 cu. in. **20.** $22\frac{1}{2}$. **21.** \$0.32. **22.** 2. **23.** 62 lb. 8 oz. **24.** 43 min. **25.** 16 min. **26.** 35 min. **27.** 2 min. $15\frac{2}{3}$ sec.

Page 131. **1.** 660; 1320. **2.** 660; 1320. **3.** $27\frac{1}{2}$; $41\frac{1}{4}$. **4.** $26\frac{1}{3}$; $3\frac{5}{38}$. **5.** 45 ft. **6.** 184. **7.** 1080. **8.** 25. **9.** 1330. **10.** 60 in.; 5 ft. **11.** 4. **12.** $437\frac{1}{2}$. **13.** $11\frac{17}{77}$. **14.** $519\frac{3}{4}$. **15.** 34,408. **16.** $537\frac{3}{5}$; $33\frac{3}{5}$. **17.** 4301. **18.** 9. **19.** 720. **20.** 5. **21.** $42\frac{3}{5}$ mi. **22.** 5. **23.** \$0.48. **24.** 29,000 lb. **25.** $16,008\frac{1}{3}$. **26.** $15\frac{3}{11}$ min. **27.** 56 in.; 4 ft. 8 in.; $4\frac{2}{3}$ ft. **28.** 62 lb. 8 oz.; $62\frac{1}{2}$ lb. **29.** 2 min. $12\frac{4}{5}$ sec. **30.** 25,875 T. **31.** 1500; 90,000. **32.** 1207 ft. 7 in. **33.** 709 ft. 7 in. **34.** 799 ft. 10 in. **35.** 1089 ft. 1 in. **36.** 717 yd. 9 in. **37.** 330 gal. 3 qt. **38.** 548 bu. 3 pk. **39.** 285 bu. 3 pk. **40.** 8 lb. 15 oz. **41.** 3 ft. **42.** 7 rd. $4\frac{1}{6}$ yd. **43.** 6 bu. 3 pk. **44.** 282 ft. 4 in. **45.** 123 lb. 9 oz. **46.** 74 yd. 29 in. **47.** 275 bu. 2 pk. **48.** 4286 lb. 10 oz. **49.** 7972 lb. **50.** 18,237 ft. 6 in. **51.** 1704 lb. 6 oz. **52.** 1257 lb.; 2514 lb. **53.** 31,640 ft.; 15,820 ft. **54.** 21 lb. 14 oz. **55.** 162. **56.** 151 lb. 4 oz. **57.** 25 ft. 2 in. **58.** 31 lb. 11 oz. **59.** 109. **60.** 107 lb. 5 oz.; 53 lb. $10\frac{1}{2}$ oz. **61.** 17 ft. $\frac{1}{5}$ in.; 34 ft. $\frac{2}{5}$ in.

Page 136. **1.** \$2400.92. **2.** \$2600. **3.** \$1750.91. **4.** \$2700.54. **5.** 9,923,580; 19,501,992. **6.** 82,026,384; 24,169,992. **7.** 4,116,879; 6,831,162. **8.** 5,866,875; 8,846,875. **9.** 11,529,072; 5,764,536. **10.** $83\frac{43}{115}$; $42\frac{48}{113}$; $86\frac{226}{487}$; $293\frac{125}{287}$. **11.** $40\frac{307}{317}$; $24\frac{363}{526}$; $59\frac{8}{63}$; $39\frac{4}{19}$; 23; 46. **12.** \$1142.10. **13.** \$3128. **14.** \$308.75. **15.** \$126. **16.** 2842. **17.** \$156. **18.** \$220. **19.** \$80. **20.** \$77. **21.** 495. **22.** $8812\frac{1}{2}$. **23.** $68\frac{1}{12}$. **24.** $51\frac{7}{8}$. **25.** $\frac{6}{16}$; $\frac{12}{16}$; $\frac{14}{16}$; $\frac{3}{16}$; $\frac{12}{16}$; $\frac{8}{16}$; $\frac{5}{16}$; $\frac{8}{16}$.

Page 137. **1.** 1694. **2.** \$4.20. **4.** 400. **5.** \$1600. **6.** 14,501 ft.; $4833\frac{2}{3}$ yd. **7.** 24; 20. **8.** \$126. **9.** 5 mi.; 20 mi.; 50 mi. **11.** The first; \$14. **12.** 3514. **13.** $\frac{1}{16}$ mi.

Page 139. **1.** \$13.75. **2.** \$5.20. **3.** 44; \$1.10. **4.** 10,098. **5.** \$34.32; \$75.58.

Page 140. **1.** $4562\frac{26}{27}$; \$2281.48. **2.** \$12,320. **3.** \$8800. **4.** \$3369.60. **5.** \$289.34.

Page 145. 1. $\frac{4}{5}$. 2. $\frac{3}{5}$. 3. $\frac{1}{5}$. 4. $\frac{2}{5}$. 5. $\frac{7}{20}$. 6. $\frac{13}{20}$. 7. $\frac{9}{20}$. 8. $\frac{11}{20}$. 9. $\frac{1}{8}$. 10. $\frac{3}{8}$. 11. $\frac{5}{8}$. 12. $\frac{7}{8}$. 13. $1\frac{1}{4}$. 14. $2\frac{3}{4}$. 15. $5\frac{1}{8}$. 16. $10\frac{3}{8}$. 17. 0.30; 0.50; 0.10; 0.80; 4.00; 63.00. 18. 0.500; 0.250; 0.370; 0.090; 31.000. 19. 0.47; 0.68; 0.10; 0.04; 7.80. 20. $\frac{1}{8}$; $200. 21. $\frac{1}{3}$; 207. 22. $\frac{3}{8}$; 663.

Page 146. 1. 0.5. 2. 0.25. 3. 0.125. 4. 0.75. 5. 0.375. 6. 0.625. 7. 0.875. 8. 0.2. 9. 0.4. 10. 0.6. 11. 0.8. 12. 0.0625. 13. 0.8125. 14. 0.4375. 15. 0.5625. 16. 0.8125. 17. 0.03125. 18. 0.15625. 19. 0.333. 20. 0.666. 21. 0.166. 22. 0.833. 23. 0.28. 24. 0.36. 25. 2.44. 26. 4.12. 27. 0.1625. 28. 8.2625. 29. 4.36. 30. 8.175.

Page 147. 1. $10.90. 2. $41.67. 3. 44.88. 4. 75.72. 5. 81.201. 6. 120.32; 40.18. 7. 98.75; 35.75. 8. 72.41; 26.85. 9. 48.65; 22.75. 10. 76.82; 57.22. 11. 49.21; 35.93. 12. 55.02; 41.08. 13. 540.67; 539.33. 14. 165.48; 91.36. 15. 206.92; 77.08.

Page 148. 1. 9 in. 2. 91 A. 3. 27.75 A. 4. 386.06. 5. 400. 6. 43.759. 7. 4000. 8. 20.034. 9. 696.9; 277.7. 10. 89.36; 15.38. 11. 6.879; 1.127. 12. 2.999; 2.607. 13. 1.12; 1.066. 14. 35.8; 8.3. 15. 52.8; 25.72. 16. 77.18; 8.84. 17. $41.69\frac{3}{4}$; $8.85\frac{1}{4}$. 18. 56; $12.66\frac{2}{3}$. 19. 427.5; 69.9. 20. 50.45; 14.35. 21. 7.754; 2.388. 22. 5.627; 3.649. 23. 4.527; 0.889. 24. 1429.7; 172.9. 25. 79.8; 22.36. 26. 15.243; 2.779. 27. 7.425; 2.809. 28. 6.495; 4.745. 29. 1338.61; 485.65. 30. 42.17; 2.59. 31. 12.46; 3.764. 32. 6.7; 5.726. 33. 14.135; 2.089. 34. 11.84. 35. 10.75. 36. 22.82. 37. 5.15. 38. 6.658. 39. 85.6 mi. 40. 13.75 gal.

Page 150. 1. $5.55; $7.40. 2. $12.50; $7.50. 3. $2.70; $4.05. 4. $14.55. 5. $9.50. 6. $6.75. 7. $8.75. 8. $7.35. 9. $286.45. 10. $94.05. 11. $8.52. 12. $15.24. 13. $44.10. 14. $42.93. 15. $31.92. 16. $51.28. 17. $22.15. 18. $89.64. 19. $17. 20. $13.32. 21. $59.44. 22. $35.36. 23. $35.49. 24. $8.97. 25. $59.34. 26. $39.12. 27. $460.32. 28. $308.32. 29. $356.40. 30. $102.58. 31. $228.47. 32. $699.55. 33. $305.44. 34. $197.54. 35. $641.16. 36. $324.72. 37. $525.60. 38. $313.20. 39. $881.10. 40. $461.13. 41. $773.76. 42. $562.52. 43. $6281.08. 44. $5958.26. 45. $3850. 46. $1248.39. 47. $5087.40. 48. $778. 49. $3559.92. 50. $2402.70. 51. $164.79. 52. $102.85. 53. $332.15. 54. $199.35. 55. $332.88. 56. $4.05. 57. $27. 58. $14.58. 59. $131.85. 60. $3.10. 61. $856.50.

Page 152. 1. 681.25. 2. 2287.34. 3. 1895.4. 4. 2173.15. 5. 2087.424. 6. 2784.375. 7. 3416.25. 8. 2614.85. 9. 22,568.4. 10. 4364.025. 11. 1500.375. 12. 2558.409. 13. 127.44 ft. 14. 85.32 in. 15. 2440.94.

Page 153. 1. 3696 ft.; 1408 yd. 2. 3168; 4752. 3. 1232; 704; 880. 4. 3696.

Page 155. **1.** 1.58. **2.** 6.56. **3.** 5.25. **4.** 2.88. **5.** 6.03. **6.** 12.1. **7.** 3.12. **8.** 3.32. **9.** 43.38. **10.** 6.51. **11.** 1.89. **12.** 73.84. **13.** 75.6. **14.** 2.97. **15.** 164.5. **16.** 65.61. **17.** 38.4. **18.** 12.06. **19.** 67.2. **20.** 7.84. **21.** 100.5. **22.** 9.18. **23.** 59.2. **24.** 3.43. **25.** 105. **26.** 4.94. **27.** 28.8. **28.** 4.23. **29.** 174.8. **30.** 16.92. **31.** 35.04. **32.** 921.6. **33.** 18.9. **34.** 4259.52. **35.** 2424.558. **36.** 280.0027. **37.** 372.6437. **38.** 4392.63. **39.** 6348.861. **40.** 1815.268. **41.** 0.3. **42.** $184.795. **43.** $100. **44.** 1876.71456. **45.** $162.09. **46.** $495.25. **47.** 15.836 gal. **48.** 17.444 in. **49.** 42.432 ft. **50.** 3494.54 sq. ft. **51.** 132.795432 ft. **52.** 9120; 57; $4987.50. **53.** 27.232 mi.

Page 157. **1.** 269.2. **2.** 146.1. **3.** 588.6. **4.** 7776. **5.** 2920. **6.** 43,771. **7.** 5233.6. **8.** 33,372. **9.** 24,720. **10.** 1250. **11.** 13,450. **12.** 301,104. **13.** 622,062.5. **14.** 2,880,288. **15.** 3,319,180. **16.** 936,625. **17.** 1,428,280. **18.** 43,750. **19.** 936. **20.** 936. **21.** 936.

Page 158. **1.** $4.50. **2.** $7.58. **3.** $7.79. **4.** $6.85. **5.** $9.74. **6.** $10.54. **7.** $27.60. **8.** $11.54. **9.** $83.89. **10.** $1328.50. **11.** $378.45. **12.** $139.81. **13.** $45.84.

Page 160. **1.** $11.80. **2.** $24.50. **3.** $4.70. **4.** $8. **5.** $24.65. **6.** $14.50. **8.** $7.90. **9.** $11.30. **10.** $13.35. **11.** $31.25. **12.** $101.25. **13.** $56.60. **14.** $36. **15.** $101. **16.** $69.85. **17.** $106.35. **18.** $411.05.

Page 162. **1.** $2.33. **2.** $2.84. **3.** $2. **4.** $1.10. **5.** $1.55. **6.** $2.22. **7.** $1.70. **8.** $3.33. **9.** $3.77. **10.** $7.33. **11.** $4.99. **12.** $8.90. **13.** $3.06. **14.** $6.98. **15.** $6.84. **16.** $2.95. **17.** $5.17. **18.** $5.17. **19.** $4.18. **20.** $6.92. **21.** $5.47. **22.** $5.26.

Page 165. **1.** $1.05. **2.** $0.20. **3.** $1.65. **4.** $1.75. **5.** $3.10. **6.** $3.77. **7.** $3.75. **8.** $3.24. **9.** $6.32. **10.** $3.13.

Page 167. **1.** 412.6 in. **2.** 1.27 in. **3.** $4.41. **4.** 54.85. **5.** 417.4. **6.** 862.8. **7.** 712.8. **8.** 416.5. **9.** 658.0. **10.** 11.7. **11.** 5.2. **12.** 11.6. **13.** 5.8. **14.** 4.8. **15.** 1.1. **16.** 0.9. **17.** 0.3. **18.** 0.2. **19.** 0.2. **20.** 7.61. **21.** 10.45. **22.** 8.58. **23.** 4.02. **24.** 28.31. **25.** 23.61. **26.** 12.40. **27.** 12.08. **28.** 12.72. **29.** 8.37. **30.** 0.92. **31.** 1.08. **32.** 1.68. **33.** 1.71. **34.** 2.25. **35.** 1.85. **36.** 2.06. **37.** 1.68. **38.** 0.88. **39.** 2.18. **40.** 6.19. **41.** 6.83. **42.** 14.34. **43.** 6.88. **44.** 14.08. **45.** 6.57. **46.** 20.00. **47.** 18.61. **48.** 30.70. **49.** 10.47.

Page 169. **1.** 14 min. **2.** $\frac{1}{6}$. **3.** 5. **4.** 5. **5.** 5. **6.** 500. **7.** 500. **8.** 7.6. **9.** 21. **10.** 21. **11.** 2.1. **12.** 2.2. **13.** 3.4. **14.** 2.46. **15.** 2500. **16.** 10,000. **17.** 1700. **18.** 1.7. **19.** 50. **20.** 7.6.

Page 170. **1.** 1.9. **2.** 0.18. **3.** 0.47. **4.** 7.49. **5.** 7.9. **6.** 0.63. **7.** 2.28. **8.** 0.746. **9.** 0.816. **10.** 0.025. **11.** 0.031. **12.** 0.042. **13.** 1.44. **14.** 0.0024. **15.** $0.039\frac{7}{11}$. **16.** 2.904. **17.** $0.012\frac{5}{6}$. **18.** 0.031. **19.** 0.301. **20.** 0.301. **21.** 0.65.

Page 171. 1. 0.0345. 2. 0.01624. 3. 1.168. 4. 0.01031. 5. 0.0001052. 6. 0.00626. 7. 0.00131. 8. 0.0121. 9. 0.0311. 10. \$7.605. 11. \$14.6755. 12. \$4.1567. 13. 11.853. 14. 270.565. 15. 6.213. 16. 5.792. 17. 4.916. 18. 1568.889. 19. 45.351. 20. 3.115. 21. 19.142. 22. 1.893. 23. 781.072. 24. 11.326. 25. 66.104. 26. 13.529. 27. 26.865.

Page 172. 1. 80¢. 2. \$1.13. 3. \$1.79. 4. \$1.09. 5. \$5.60. 6. \$0.09$\frac{1}{2}$. 7. \$2.69. 8. \$2.40. 9. \$130. 10. 129$\frac{1}{4}$ mi. 11. \$0.23. 12. \$0.28.

Page 173. 1. 0.21. 2. 1$\frac{1}{3}$. 3. $\frac{6}{35}$. 4. 3. 5. 3.5. 6. 3$\frac{1}{3}$. 7. 56. 8. 1. 9. 0.1. 10. 0.3. 11. 0.1. 12. 1. 13. 2. 14. \$4. 15. \$1.95; \$2.60. 16. \$146.25. 17. \$19. 18. \$0.88; \$2.64. 19. \$2.70; \$8.10. 20. \$2.25; \$6.75. 21. \$5.70; \$11.40. 22. \$8.82; \$10.78. 23. \$10.95. 24. \$23.75. 25. \$15.75. 26. \$17.10; \$25.65. 27. \$2.70; \$3.60; \$4.50; \$7.20.

Page 175. 1. 339.2. 2. \$4680. 3. 51.5 mi. 4. 44.5 mi. 5. 19.64 mi. 6. 18.35 mi. 7. 36.44 mi.; 39.84 mi. 8. $\frac{1}{3}$. 9. Frank. 10. \$1743.75. 11. \$6370. 12. \$23,550. 13. \$20,685; \$23,167.20. 14. \$451; \$481.25. 15. \$716.85; \$741.15. 16. \$44,025; \$55,178. 17. \$3,234,436.92. 18. \$10,192,402.50. 19. 20,000. 20. 13.9. 21. The second; 5.08 mi. 22. 8.9 mi. 23. 213.5. 24. 46.25 ft. 25. 267$\frac{23}{30}$ A. 26. 9.16 ft. 27. 124.7 mi. 28. 510.8 mi. 29. 87.35 rd. 30. 186 lb. 31. 3000. 32. 943.9. 33. 944. 34. 212.25.

Page 179. 1. 1889. 2. 7,000,096.5. 3. \$78.01. 4. \$203.58. 5. \$140.10. 6. \$220.09. 7. \$158.97. 8. 69 ft.; 120 ft. 9 in.; 43 ft. 1$\frac{1}{2}$ in. 9. 2 ft. 11 in.; 2 ft. 4 in.; 4 ft. 8 in. 10. 44 rd. 1$\frac{1}{3}$ ft. 11. 120 sq. rd. 12. \$135.26. 13. $\frac{3}{4}$. 14. $\frac{1}{8}$. 15. 2. 16. $\frac{1}{4}$. 17. $\frac{3}{8}$. 18. 1$\frac{1}{2}$. 19. $\frac{5}{8}$. 20. $\frac{1}{16}$. 21. 4. 22. $\frac{3}{8}$. 23. $\frac{15}{32}$. 24. 1$\frac{1}{6}$. 25. 1511.1. 26. 264.11. 27. 237.51. 28. 221.25. 29. 329.93. 30. 19.958. 31. 0.8291. 32. 9.562. 33. 1.0192. 34. 2.4222. 35. 95.18; 85.46. 36. 259; 160.6. 37. 7.265; 5.281. 38. 88.02; 74.06. 39. 78.91; 22.95. 40. 1.911; 0.159. 41. 0.4398; 0.122. 42. 85.39; 79.572. 43. 101.064; 27.006. 44. 109.647; 30.489. 45. 9.296. 46. 259.488. 47. 317.2. 48. 185. 49. 830.025. 50. 2546.2116. 51. 196.54. 52. 235.68. 53. 225.024. 54. 329.4308. 55. 74. 56. 7.198. 57. 71.98. 58. 0.7198. 59. 7198.

Page 182. 1. 15 da.; \$60. 2. \$0.34$\frac{15}{23}$. 3. \$1.60; \$0.06$\frac{22}{23}$. 4. $\frac{4}{5}$. 5. \$240; \$48. 6. \$0.10. 7. \$0.05$\frac{1}{3}$. 8. \$0.04$\frac{2}{3}$. 9. \$0.15$\frac{5}{9}$. 10. \$13,000,000. 11. \$4,000,000; \$1,200,000. 12. \$61,776. 13. \$19,000,000. 14. \$0.04$\frac{1}{2}$; $\frac{9}{10}$ T. 15. \$0.22$\frac{1}{2}$. 16. \$144. 17. \$25,920.

Page 188. 7. 1200; 2400; 4800. 8. 2 yd.; 72 in. 9. 22$\frac{1}{2}$ ft.; 7$\frac{1}{2}$ yd. 10. 25 rd. 11. 280$\frac{1}{2}$ ft. 12. 62 rd. 13. 1012$\frac{1}{2}$.

Page 192. 1. 827 ft. 2. 117 ft. 8 in. 3. 119 ft. 4. 2. 5. 37 ft. 9 in. 6. 42 yd. 1 ft. 7. 29 ft. 5 in. 8. 27 ft. 7 in. 9. 10 ft. 6$\frac{7}{8}$ in. 10. 15 yd. 1$\frac{1}{4}$ ft. 11. 7 in. 12. 29 ft. 8 in. 13. 56 mi. 250 rd. 14. 33$\frac{1}{2}$ ft. 15. 17$\frac{3}{5}$ ft.; 35$\frac{1}{5}$ ft.; 35$\frac{1}{5}$ ft. 16. 8; 9; 7. 17. 30; 40; 50.

Page 193. 1. 1800 sq. ft. 2. 1400 sq. yd. 3. 7200 sq. ft. 4. 3200 sq. yd. 5. 20,000 sq. ft. 6. 81,000 sq. ft. 7. 450,000 sq. ft. 8. 8400 sq. in. 9. 684 sq. ft.

Page 195. 1. 688 sq. ft. 2. 34.365 sq. in. 3. 27.405 sq. in. 4. 71.44 sq. in. 5. $791\frac{9}{16}$ sq. ft. 6. 1404 sq. in. 7. $4\frac{4}{9}$ sq. ft. 8. 268.3125 sq. in. 9. 67.5 sq. ft. 10. 54.4 sq. rd.; 61.2 sq. in. 11. 600 sq. rd.; $3\frac{3}{4}$ A. 12. 3900.

Page 196. 1. 331.8 sq. in. 2. 4662.08 sq. ft. 3. $28\frac{1}{2}$ sq. ft. 4. 242.55 sq. ft. 5. 8.75 sq. in. 6. 136.125 sq. ft. 7. 425.88 sq. in. 8. 195 sq. ft. 9. 118.925 sq. in. 10. 142.725 sq. ft. 11. $5\frac{2}{5}$.

Page 197. 1. 26.13 sq. in. 2. 1181.5 sq. ft. 3. 121.68 sq. ft. 4. $7\frac{3}{4}$ sq. ft. 5. $13\frac{9}{22}$ sq. rd. 6. $39\frac{3}{4}$ sq. ft. 7. $64\frac{1}{8}$ sq. ft. 8. 43,815 sq. ft. 9. 16 sq. ft. 99 sq. in. 10. 89 sq. yd. 226 sq. in. 11. 7 in. 12. 11 ft. 13. 12 ft. 14. 24 ft. 15. 80 ft. 16. 6 ft. 17. 6 ft. 18. 6 ft. 19. 3 in. 20. 8 in. 21. 9 in. 22. 10 in. 23. 10 ft. 24. 12 yd.

Page 198. 1. $136\frac{1}{16}$ sq. in. 2. $233\frac{3}{4}$ sq. in. 3. 3.675 sq. in. 4. 7.875 sq. in. 5. $16\frac{5}{8}$ sq. ft. 6. 331.5 sq. rd. 7. 2 in. 8. 240 sq. rd.

Page 200. 1. 1292. 2. 1830. 3. $83.20. 4. $8\frac{3}{4}$; 24. 5. 112. 6. $33.33; $41.67. 7. $7486\frac{1}{9}$. 8. 129,600 cu. ft. 9. 168; 468; 168. 10. $111.11. 11. 1904; 65,280 lb.

Page 201. 1. $134\frac{2}{5}$. 2. 108. 3. $7\frac{1}{5}$. 4. 900. 5. 6. 6. $382\frac{13}{18}$. 7. $42\frac{6}{7}$. 8. $58\frac{17}{21}$. 9. $2\frac{1}{2}$. 10. $62\frac{2}{5}$. 11. $21\frac{1}{4}$. 12. 5.544. 13. $14\frac{22}{25}$. 14. $7\frac{7}{64}$. 15. $8\frac{15}{16}$. 16. 30. 17. 11,000. 18. $28\frac{1}{2}$. 19. $7\frac{367}{1232}$. 20. $1.80. 21. $85\frac{35}{99}$. 22. 8663. 23. $12.50. 24. 3834. 25. 49.58. 26. 159.58 cu. in. larger. 27. 489.58 cu. in. larger. 28. 9120. 29. The first; $370\frac{5}{32}$. 30. $1235\frac{1}{4}$. 31. The first; $50\frac{55}{64}$. 32. $869\frac{1}{2}$; $427\frac{31}{32}$; $1847\frac{11}{16}$; $1059\frac{45}{64}$.

Page 205. 1. 110,400. 2. 6 ft. 3. 15 in. 4. $112\frac{1}{2}$ sq. ft. 5. $43\frac{1}{3}$ ft. 6. $12\frac{1}{2}$ ft. 7. $27\frac{1}{2}$ ft. 8. 39 in. 9. 7.2 in. 10. 11 ft. 11. 12 ft. 12. 12 ft.; 12 ft.; $9\frac{1}{2}$ ft. 13. 16.4 ft.; 14.3 ft.; 9 ft. 14. $66\frac{1}{2}$. 15. $68\frac{68}{215}$. 16. 160; yes. 17. $3280.50. 18. 1620. 19. $26.58. 20. $7\frac{123}{275}$. 21. 60. 22. $691.20. 23. $291.55.

Page 207. 1. 30; 16. 2. $33\frac{1}{3}$. 3. $28\frac{4}{7}$; $35\frac{5}{7}$. 4. 35; 28. 5. $62\frac{1}{2}$. 6. $41\frac{2}{3}$. 7. $33\frac{1}{3}$. 8. $41\frac{2}{3}$. 9. $36\frac{4}{11}$. 10. $44\frac{4}{9}$. 11. $1\frac{3}{4}$. 12. $\frac{4}{7}$. 13. $\frac{15}{16}$; $\frac{3}{4}$.

Page 209. 1. 60,277,809. 2. 72.064. 3. $25.24. 4. $31.44. 5. $30.07. 6. $22.49. 7. $33.96. 8. $12.90. 9. $1\frac{1}{6}$. 10. $\frac{3}{8}$. 11. $1\frac{5}{8}$. 12. $\frac{5}{8}$. 13. $\frac{1}{6}$. 14. $\frac{11}{16}$. 15. $\frac{1}{16}$. 16. $\frac{9}{32}$. 17. $\frac{7}{8}$. 18. $\frac{3}{4}$. 19. $4\frac{3}{8}$. 20. $5\frac{5}{6}$. 21. $2\frac{3}{8}$. 22. $5\frac{1}{6}$. 23. $6\frac{1}{4}$. 24. 0.84. 25. 4.14. 26. 1.44. 27. 0.546. 28. 1.2.

Page 211. 1. $2\frac{1}{8}$ lb. 2. $2\frac{3}{8}$ lb. 3. $2\frac{7}{16}$ lb.

Page 212. 1. $1.49. 2. $4.08. 3. $2.90.

Page 214. 6. 48,320; 483,200; 24,160; 120,800; 604,000. 76,480; 764,800; 38,240; 191,200; 956,000. 91,760; 917,600; 45,880; 229,400; 1,147,000. 70,480; 704,800; 35,240; 176,200; 881,000. 61,120; 611,200; 30,560; 152,800; 764,000.

7. 525.6; 5256; 262.8; 1314; 6570. 71.76; 717.6; 35.88; 179.4; 897. 3488; 34,880; 1744; 8720; 43,600. 90.08; 900.8; 45.04; 225.2; 1126. 7.36; 73.6; 3.68; 18.4; 92. 8. 272.8; 2728; 136.4; 682; 3410. 84.16; 841.6; 42.08; 210.4; 1052. 2456; 24,560; 1228; 6140; 30,700. 70.24; 702.4; 35.12; 175.6; 878. 8.24; 82.4; 4.12; 20.6; 103. 9. 176.8; 1768; 88.4; 442; 2210. 68.08; 680.8; 34.04; 170.2; 851. 7656; 76,560; 3828; 19,140; 95,700. 20.08; 200.8; 10.04; 50.2; 251. 7.44; 74.4; 3.72; 18.6; 93. 10. 235,800; 63,666; 77,814. 180,500; 48,735; 59,565. 73,700; 19,899; 24,321. 156,700; 42,309; 51,711. 301,200; 81,324; 99,396. 11. 1417; 382.59; 467.61. 1257; 339.39; 414.81. 20,340; 5491.8; 6712.2. 183.5; 49.545; 60.555. 23.7; 6.399; 7.821. 12. 2712; 732.14; 894.96. 2042; 551.34; 673.86. 16,810; 4538.7; 5547.3. 242.1; 65.367; 79.893. 16.8; 4.536; 5.544. 13. 337; 90.99; 111.21. 2412; 651.24; 795.96. 9370; 2529.9; 3092.1. 186.8; 50.436; 61.644. 4.1; 1.107; 1.353.

Page 216. 6. 97.26. 7. 176.22. 8. 738.96. 9. 206.67. 10. 39.488. 11. 109.2. 12. 289.86. 13. 591.68. 14. 148.77. 15. 139.14. 16. 539.2. 17. 548.04. 18. 654.96. 19. 266.76. 20. 319.104. 21. 537.12; $1505.68; $5424.84; $17,107.72. 22. 137.6; $546.40; $2211.20; $2540. 23. 2091.6; $988.80; $1462.50; $8289.60. 24. 149; 169; 183. 25. 294; 198; 315. 26. 149,707.88.

Page 221. 1. $336. 2. $180. 3. $540. 4. $42.75. 5. $75. 6. $14.84. 7. $12.90. 8. $67.93. 9. $508.25. 10. $554.40. 11. $10.80. 12. 30. 13. $12\frac{1}{2}$.

Page 222. 1. $5.81. 2. 17. 3. 19. 4. $6250. 5. 354. 6. 625. 7. 125. 8. 8064. 9. $17\frac{73}{231}$ mi. 10. 54,912. 11. 6 min. 25 sec. 12. 7. 13. $337\frac{1}{2}$. 14. $15\frac{29}{45}$. 15. $3,000,000. 16. $12. 17. 14.67; 117.33. 10; 80.

Page 225. 1. $378.75. 2. $13.30. 3. $17.28. 4. \$0.08$\frac{5}{8}$; $23.72. 5. $24. 6. $27.05. 7. $1. 8. \$0.24$\frac{7}{8}$; $58. 9. $26.30.

Page 227. 1. 31. 3. 500.003. 6. 48 in. 7. 144 sq. in. 8. 1728. 9. \$14.42$\frac{1}{2}$. 10. 208. 11. $\frac{5}{16}$. 12. $\frac{11}{16}$. 13. $4\frac{3}{8}$. 14. $1\frac{3}{8}$. 15. $\frac{7}{16}$. 16. $\frac{1}{2}$. 17. $\frac{9}{16}$. 18. $1\frac{7}{9}$. 19. 77,664; 58,248; 659,716,848; 7. 20. 28,511; 20,365; 99,535,974; 6. 21. 50,000; 40,072; 225,810,729; 9. 22. 35.19; 6.21; 299.943; 1.4286. 23. 307.8; 239.4; 9357.12; 8. 24. 2.538; 1.692; 0.894645; 5. 25. $1\frac{3}{8}$; $\frac{3}{8}$; $\frac{7}{16}$; $1\frac{3}{4}$. 26. $\frac{7}{8}$; $\frac{3}{8}$; $\frac{5}{32}$; $2\frac{1}{2}$. 27. $1\frac{1}{8}$; $\frac{3}{8}$; $\frac{9}{32}$; 2. 28. 11,784.3. 29. 38 ft. 9 in.; 19 ft. 7 in. 30. 48 ft. 10 in.; 11 ft. 4 in. 31. 24 lb. 11 oz.; 7 lb. 13 oz. 32. 58 yd. 7 in.; 18 yd. 21 in. 33. 90 ft. 8 in.; 5 ft. 8 in. 34. 261 ft.; 7 ft. 3 in. 35. 180 yd. 20 in.; 7 yd. 8 in. 36. 148 rd. 6 ft.; 9 rd. $4\frac{1}{2}$ ft. 37. 62 lb. 8 oz.; 2 lb. 8 oz. 38. 215 yd. 2 in.; 4 yd. 14 in. 39. 440; 1100; 2200; 3300. 40. 560; 1400; 2800; 4200. 41. 720; 1800; 3600; 5400. 42. 960; 2400; 4800; 7200. 43. 1440; 3600; 7200; 10,800. 44. 2400; 6000; 12,000; 18,000. 45. 46 ft. 8 in.; 116 ft. 8 in.; 233 ft. 4 in.; 350 ft. 46. 166 yd. 24 in.; 416 yd. 24 in.; 833 yd. 12 in.; 1250 yd. 47. 15,552; 19,008. 48. 47,520; 58,080. 49. 83,520; 102,080. 50. 28,566; 34,914. 51. 111,285;

136,015. **52.** 133,263 ; 162,877. **53.** 237,078 ; 289,762. **54.** 281,583 ; 344,157. **55.** 1200 ; 12,000 ; 3000 ; 9000 ; 4000. **56.** 3600 ; 36,000 ; 9000 ; 27,000 ; 12,000. **57.** 2520 ; 25,200 ; 6300 ; 18,900 ; 8400. **58.** 4800 ; 48,000 ; 12,000 ; 36,000 ; 16,000. **59.** 12,000 ; 120,000 ; 30,000 ; 90,000 ; 40,000. **60.** 12,240 ; 122,400 ; 30,600 ; 91,800 ; 40,800. **61.** 48,360 ; 483,600 ; 120,900 ; 362,700 ; 161,200. **62.** 48,480 ; 484,800 ; 121,200 ; 363,600 ; 161,600.

Page 230. 1. $\frac{1}{4}$. 2. $\frac{1}{8}$. 3. $1\frac{1}{4}$ lb. 4. $2\frac{1}{2}$. 5. $\frac{1}{3}$. 6. $\frac{1}{2}$; $\frac{1}{3}$. 7. $7\frac{1}{2}$ lb.; $3\frac{3}{4}$ pt. 8. $5\frac{1}{3}$; $8\frac{1}{3}$.

Page 231. 1. $7.20 ; $367.20. 2. $3.75 ; $\frac{5}{12}$. 3. $5.55. 4. $53.25. 5. $18.40. 6. $5.50. 7. $0.30.

Page 232. 1. $18.20 ; $9.80. 2. $4.80. 3. $13.78. 4. $29.25 ; $2.73 less. 5. $12 ; $0.53. 6. $0.07. 7. $19\frac{23}{53}$.

Page 236. 1. 816. 2. 648. 3. 6185. 4. $854.20. 5. 489 bu. 6. 24 ft. 2 in. 7. $11.41. 8. $9.21. 9. 24 ; 24. 10. 16 ; 48. 11. 19 ; 133. 12. 176 ; 1232.

Page 237. 1. 297. 2. 474. 3. 84. 4. $6.85. 5. $26.62. 6. 498 rd. 7. 1629 bu. 8. 74 ft. 3 in. 9. 121 bu. 1 qt. 10. 94 lb. 10 oz.

Page 238. 1. $\frac{8}{25}$. 2. $\frac{17}{25}$. 3. $\frac{6}{25}$. 4. $\frac{1}{16}$. 5. $\frac{9}{25}$. 6. $\frac{5}{4}$. 7. $\frac{17}{100}$. 8. $\frac{15}{4}$. 9. $\frac{33}{40}$. 10. $\frac{1}{3}$. 11. $37\frac{1}{2}\%$. 12. $62\frac{1}{2}\%$. 13. $87\frac{1}{2}\%$. 14. 20%. 15. 40%. 16. 60%. 17. 80%. 18. $6\frac{1}{4}\%$. 19. $18\frac{3}{4}\%$. 20. $93\frac{3}{4}\%$.

Page 240. 1. $54,000. 2. $81,000. 3. 1200. 4. 1204.5. 5. $149.76. 6. $593.75. 7. $5980. 8. 4089 ft. 9. $2754. 10. 28. 11. 30; 35. 12. 304. 13. 341. 14. 41. 15. 182.4 lb. 16. 35,658. 17. 0.18 yd.; 35.82 yd. 18. $42\frac{1}{2}$¢. 19. 371.91 A. 20. 195. 21. $120. 22. 276. 23. 260. 24. 14. 25. 58. 26. 60. 27. 93 ; 279. 28. 154 ; 126. 29. 42.5 ft.; 62.5 ft.; 250 ft. 30. 37 lb.; 74 lb.; 111 lb. 31. 3.96 in. 32. $\frac{1}{10}$ in. 33. $5\frac{19}{20}$ in.; $4\frac{2}{3}$ in. 34. 3000. 35. $620,000. 36. 100. 37. $186,000,000. 38. 75. 39. 14,520. 40. 20. 41. 27. 42. $14. 43. $74.70. 44. 15 ft. $7\frac{1}{2}$ in.

Page 246. 1. $58.75. 2. $6.90. 3. $5.69. 4. $4.42. 5. $19.65. 6. $4.37. 7. $20.26. 8. $4.63. 9. $6.86.

Page 249. 1. $596.90. 2. $297.50. 3. $233.75. 4. $276.25. 5. $19.55. 6. $36.40. 7. $360.24. 8. $140.40. 9. $180.60. 10. $11.20. 11. $1.05. 12. $5.60. 13. $2.20. 14. $64.50. 15. $56.20. 16. $42.50. 17. $209.95. 18. $329. 19. $14.42. 20. $10.66. 21. $396. 22. $5.76. 23. $220. 24. $546.25. 25. $448.14. 26. $568.75. 27. $374. 28. $275.20. 29. $10.20. 30. $5333.72. 31. $40.03. 32. $13.16. 33. $3303.30. 34. $9.62. 35. $14.88. 36. $1.53. 37. $0.51. 38. $23.25. 39. $1.08 ; $0.96 ; $0.90. 40. $0.81 ; $0.72. 41. $125. 42. $650.25.

Page 252. 1. 20%. 2. $22\frac{22}{49}\%$. 3. $2\frac{2}{49}\%$. 4. $11\frac{1}{9}\%$ gain. 5. $15\frac{1}{5}\%$. 6. 10¢.

Page 253. 1. $21.25. 2. $66. 3. $90. 4. $1006.76. 5. $29.20 ; $262.80. 6. $196.

Page 254. 1. $473.60. 2. $21.48. 3. $168. 4. $4.88. 5. $66.70. 6. $787.50. 7. $25.53.

Page 255. 1. $22.50; $15. 2. $47.50; $28.50. 3. $9.40; $11.75. 4. $45; $37.50. 5. $0.81. 6. $503.50.

Page 256. 1. $26.08. 2. $18.75. 3. $0.95. 4. $9.75. 5. $13. 6. $1.89. 7. $11. 8. $18.01. 9. $1.88. 10. $43.50. 11. $13.76. 12. $3.78. 13. $26.75. 14. $31.26. 15. $2.40. 16. $2.05. 17. $3.50. 18. $4.38. 19. $2.25. 20. $8.13. 21. $10.13. 22. $0.90. 23. $13.50. 24. $10.13. 25. $0.35. 26. $5.93. 27. $12.96. 28. $14.25. 29. $18.75. 30. $26.70.

Page 257. 1. $23.46; $46.92; $70.38. 2. $9.12; $18.24; $27.36. 3. $26.16; $13.08; $8.72; $17.44; $21.80. 4. $18; $9; $15; $1.50; $13.50. 5. $21.60; $1.80; $0.90; $0.30; $0.06. 6. $7.02; $3.51; $2.34; $1.76; $0.59; $0.20. 7. $93; $46.50; $23.25 : $69.75; $11.63.

Page 258. 1. $7.50. 2. $11. 3. $1025. 4. $87.50. 5. $3.15. 6. $4.10. 7. $87.50. 8. $11.33. 9. $47.25. 10. $286.88. 11. $52.50. 12. $35. 13. $26.25. 14. $48.75. 15. $61.25. 16. $53.75. 17. $75. 18. $62.50. 19. $50. 20. $37.50. 21. $25. 22. $27.13. 23. $50.38. 24. $73.63. 25. $106.18. 26. $116.25. 27. $632.50. 28. $618.80. 29. $65.81. 30. $1265.63. 31. $743.13. 32. $9.50; $484.50.

Page 260. 1. $\frac{1}{10}$; 10%. 2. $\frac{3}{25}$; 12%. 3. $\frac{31}{50}$; 62%. 4. $\frac{5}{8}$; $62\frac{1}{2}$%. 5. $\frac{1}{32}$; $3\frac{1}{8}$%. 6. $\frac{33}{100}$; 33%. 7. $\frac{1}{5}$; 20%. 8. $\frac{1}{50}$; 2%. 9. $\frac{1}{6}$; $16\frac{2}{3}$%. 10. $\frac{3}{4}$; 75%. 11. $164; $1476. 12. 44. 13. 28. 14. $7.50. 15. $877.50. 16. $33.75. 17. $300. 18. 0.48; 168; 182. 19. 0.225; 5895. 20. $6720.

Page 262. 1. $\frac{1}{100}$; 1%. 2. $\frac{37}{100}$; 37%. 3. $\frac{3}{8}$; $37\frac{1}{2}$%. 4. $\frac{1}{16}$; $6\frac{1}{4}$%. 5. $\frac{31}{100}$; 31%. 6. $\frac{5}{16}$; $31\frac{1}{4}$%. 7. $\frac{1}{3}$; $33\frac{1}{3}$%; 8. $\frac{33}{50}$; 66%. 9. $\frac{2}{3}$; $66\frac{2}{3}$%. 10. $\frac{87}{100}$; 87%. 11. $\frac{7}{8}$; $87\frac{1}{2}$%. 12. $\frac{4}{25}$; 16%. 13. $\frac{3}{40}$; $7\frac{1}{2}$%. 14. $\frac{1}{12}$; $8\frac{1}{3}$%. 15. $\frac{3}{80}$; $3\frac{3}{4}$%. 16. 49. 17. 75; 150. 18. 16. 19. 42. 20. 138. 21. 27. 22. 42. 23. 1.7. 24. 2.4. 25. 15. 26. 19.2. 27. 121. 28. 134. 29. 276. 30. 345. 31. 306.25 lb. 32. 84,000 lb. 33. 304. 34. 26.55. 35. 18.

Page 264. 1. 9.54 ft. 2. 39.37. 3. 6082.56. 4. 64. 5. 108. 6. 105. 7. 96. 8. 4.8. 9. 9.2. 10. 3.75. 11. 7.05. 12. 255. 13. 19. 14. 543. 15. 543. 16. $200. 17. $174. 18. $116. 19. $62.25. 20. $211. 21. $18.66. 22. $57.45. 23. $37.17. 24. $23.10. 25. $248.40. 26. $35.30. 27. $185.29. 28. $292.90. 29. $493.032. 30. $22,687.50. 31. $6125. 32. $16,506. 33. 600; 600; 600. 34. 1320; 1320; 1320.

Page 265. 1. 83.5. 2. 48.2. 3. 306. 4. $8.52. 5. $23.87. 6. $37.44. 7. 177. 8. 177. 9. 177. 10. $360. 11. $318.75. 12. $360. 13. $621. 14. $735. 15. $666. 16. $800. 17. $1540. 18. $1050. 19. $65.60. 20. $836. 21. $1350. 22. The second; $11.60. 23. $21. 24. $108; $216; $27; $135; $243. 25 $165.

Page 267. 1. MDCCCCXXI or MCMXXI; MDCCCCXXVII or MCMXXVII; 69; 88. **2.** 8708; 4354; 14,217,987; 3. **3.** 21,987; 15,705; 59,195,286; 6. **4.** 33,768; 25,326; 124,717,887; 7. **5.** 36,234; 24,156; 182,347,605; 5. **6.** 63,090; 49,070; 393,120,800; 8. **7.** 35,505; 21,303; 201,696,804; 4. **8.** 34,560; 27,648; 107,495,424; 9. **9.** 28,301; 20,215; 98,075,094; 6. **10.** 77,770; 62,216; 544,335,561; 9. **11.** $1\frac{5}{12}$; $\frac{1}{12}$; $\frac{1}{2}$; $\frac{8}{9}$; $1\frac{1}{8}$. **12.** $\frac{17}{24}$; $\frac{1}{24}$; $\frac{1}{8}$; $\frac{8}{9}$; $1\frac{1}{8}$. **13.** $1\frac{7}{24}$; $\frac{1}{24}$; $\frac{5}{12}$; $1\frac{1}{15}$; $\frac{15}{16}$. **14.** $\frac{19}{24}$; $\frac{13}{24}$; $\frac{1}{12}$; $5\frac{1}{3}$; $\frac{3}{16}$. **15.** $\frac{7}{8}$; $\frac{5}{8}$; $\frac{3}{32}$; 6; $\frac{1}{6}$. **16.** $1\frac{3}{8}$; $\frac{1}{8}$; $\frac{15}{32}$; $1\frac{1}{5}$; $\frac{5}{6}$. **17.** $\frac{11}{16}$; $\frac{5}{16}$; $\frac{3}{32}$; $2\frac{2}{3}$; $\frac{3}{8}$. **18.** $\frac{31}{48}$; $\frac{1}{48}$; $\frac{5}{48}$; $1\frac{1}{15}$; $\frac{15}{16}$. **19.** 3.36. **20.** 6.75. **21.** 5.82. **22.** 0.666. **23.** 0.544. **24.** 0.95. **25.** 2.3. **26.** 0.8. **27.** 0.45. **28.** 0.24. **29.** 4 ft. 10 in. **30.** 3 ft. 7 in. **31.** 2 ft. $1\frac{1}{2}$ in. **32.** 2 lb. 7 oz. **33.** 3 lb. 9 oz.

Page 269. 1. $3000. **2.** $1649.13. **3.** $3000. **4.** $1823.31. **5.** $775.39. **6.** $1628.38. **7.** $1888.89. **8.** $2873.18. **9.** 312. **10.** 423. **11.** 294. **12.** 13.2. **13.** 19.05. **14.** 34.04. **15.** 95,844. **16.** 10,028.3. **17.** 2471.04. **18.** 150.102. **19.** 60.2154. **20.** 5.4273. **21.** 11.963. **22.** 0.0432. **23.** 0.0252. **24.** 43.8511. **25.** 417.658. **26.** 56.4. **27.** 0.63. **28.** 108.89. **29.** 0.21. **30.** 1.25. **31.** 0.38. **32.** 2.23. **33.** 468.33. **34.** 1865.63. **35.** 0.11. **36.** $13\frac{11}{16}$. **37.** $1\frac{43}{93}$. **38.** 5.565.

Page 271. 1. $258. **2.** $16.50. **3.** $15. **4.** $45.90. **5.** $$68\frac{1}{3}$ greater. **6.** $150. **7.** $353.60. **8.** $812.25. **9.** $564.50; $$385.58\frac{1}{3}$; the automobile.

Page 272. 1. $26. **2.** $22.30. **3.** $31.70. **4.** $27. **5.** $5.91. **6.** $90.61. **7.** $135.92. **8.** $1155.28. **9.** No.

Page 274. 1. $6\frac{13}{24}$; $14\frac{3}{8}$; $15\frac{5}{12}$; $32\frac{1}{3}$. **2.** $2\frac{23}{48}$; $5\frac{7}{48}$; $\frac{23}{24}$; $6\frac{1}{24}$. **3.** $15\frac{1}{3}$; $66\frac{23}{32}$; $50\frac{5}{8}$; 2121. 2; 5; $1\frac{1}{2}$; $\frac{122}{147}$. **4.** 800 rd.; 5 rd.; $2\frac{2}{5}$ T.; 588 in.; $2\frac{1}{2}$ sq. yd.; 3 cd. **5.** 211; 276; 432. 543; 3780; 371.91. **6.** $765; $210; $150. $675; $300; $180. **7.** $168; $880; $1620. $812; $1375; $25. **8.** 100; 200; 100. **9.** $125; $125; $125. **10.** 100.

Page 275. 1. MDCCCCXXI; nineteen hundred twenty-one; one thousand nine hundred twenty-one. **2.** $115.41. **3.** $4938. **4.** $27,983. **5.** $216. **6.** $180. **9.** 410. **10.** $226.10.

Page 276. 1. $3. **2.** 27.84. **3.** 4320 bu. **4.** $\frac{3}{8}$. **5.** 57. **6.** $675. **7.** The first. **8.** $175. **9.** $171.45. **10.** 2 min.

Page 277. 1. 0.1; 0.4; 0.232; 0.723; 0.3. **2.** 1.3; 1.5; 2.25; 3.007; 2.8. **3.** 237.06; 17.78. 766.44; 40.78. 842.57; 182.85. 1059.06; 187.78. 1441.08; 165.72. **4.** 104; 0.5; 111; 111; 11; 1.85. **5.** 20. **6.** 0.03125. **7.** 146.61 ft. **8.** 24,500. **9.** 122,044 lb. **10.** $772.80; the first is $373.20 greater.

Page 278. 1. 62.4; 179.8; 272; 567; 978; 282.2. **2.** 55.76; 67.76; 40.128; 65.794; 15.7768; 37.4376. **3.** 0.25; 0.09; 0.13; 1.39; 0.32; 2143.13. **4.** 4.988; 14.071; 1377.097. **5.** $464,000. **6.** 130,004,632 lb. **7.** 174. **8.** 25¢. **9.** $189,383,000. **10.** 32.

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